CLOUD PHYSICS - tutorial 3 Activation

Köhler and κ-Köhler curves

1. **Kelvin effect**

   The equilibrium saturation over a pure water droplet is given by:

   $$S_{eq} = exp\left(\frac{A(T)}{r}\right) \qquad \text{where} \quad A(T) = \frac{2\sigma}{\rho_l R_v T}$$

   For $A(T)/r << 1$, the equilibrium saturation takes the approximate form:

   $$S_{approx} = 1 + \frac{A(T)}{r}$$

   Plot $S_{eq}(r)$ and $S_{approx}(r)$ for a given temperature (e.g. $T = 0°C$). Calculate the relative error $\left(S_{eq} - S_{approx}\right)/S_{eq}$. For which values of r is the relative error smaller than 1% and 0.1%?
   Assume that the surface tension of water is independent of temperature.

2. **Raoult/solute effect**

   The equilibrium saturation ratio over an aqueous solution droplet is given by:

   $$S = a_w exp\left(\frac{A(T)}{r}\right),$$

   where a_w is the activity of water in the solution (the Raoult effect) and is given by:

   $$a_w = \frac{r^3 - r_d^3}{r^3 - r_d^3(1-\kappa)},$$

   or, in its approximate form, as:

   $$a_{w,appr} = 1 - \kappa\frac{r_d^3}{r^3}.$$

   r is the droplet radius, r_d is the dry radius describing the amount of solute (CCN), and κ is the hygroscopicity parameter.

   Plot $a_w(r)$ and $a_{w,appr}$ to illustrate the Raoult (solute) effect for NaCl ($\kappa = 1.28$) and NH_4NO_3 (ammonium nitrate, *azotan amonu*, $\kappa = 0.67$) for different dry radii ($r_d = 0.02,\ 0.05,\ 0.1\ \mu m$).
   Plot the relative error $(a_w - a_{w,appr})/a_w$ for different dry radii and different κ values.

3. **Köhler curve**

   The κ-Köhler equation:

$$S(r, \kappa, r_d, T) = \frac{r^3 - r_d^3}{r^3 - r_d^3(1-\kappa)} exp\left(\frac{A(T)}{r}\right)$$

   describes the equilibrium saturation ratio over a droplet containing a soluble nucleus.

   Plot the Köhler curves for NaCl and NH_4NO_3i for:

   - $r_d = 0.01,\ 0.03,\ 0.1\ \mu m$
   - $T = 0,\ 20^o\ C$

   For sufficiently large droplets (with radius R), the equilibrium saturation converges to 100%. This means that the curvature and solute terms in the Köhler equation become negligible. Assume two scenarios in which these effects are negligible: when the equilibrium supersaturation $(S-1)$ is less then 0.1% or 0.01%) For $\kappa = 1.28$ and $\kappa = 0.2$ plot the relationship between the dry radius r_d and R.

4. **Critical radius and critical saturation**

   Show how the critical radius and critical supersaturation depend on the dry radius, r_d, and the hygroscopicity parameter, κ. Assume the temperature remains constant.