STABILITY IN THE ATMOSPHERE

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LECTURE OUTLINE

1. Stratifiction
2. Hydrostatic stability; parcel method
3. Conditional stability

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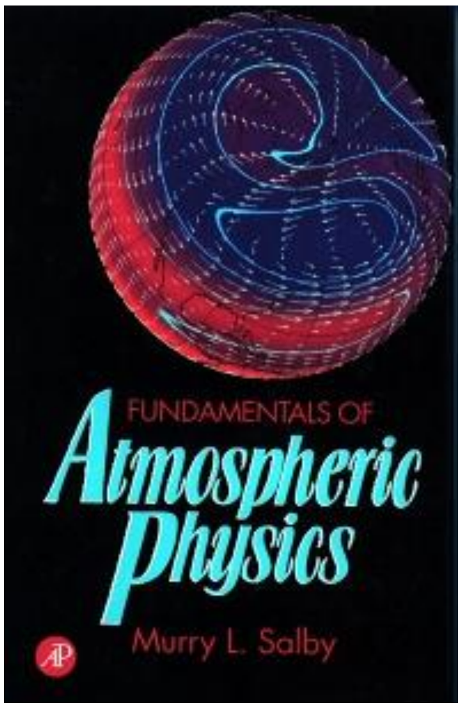
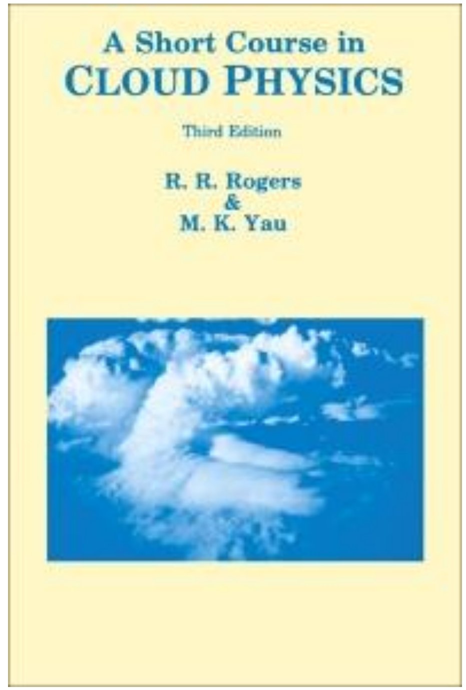
Salby, Chapter 6



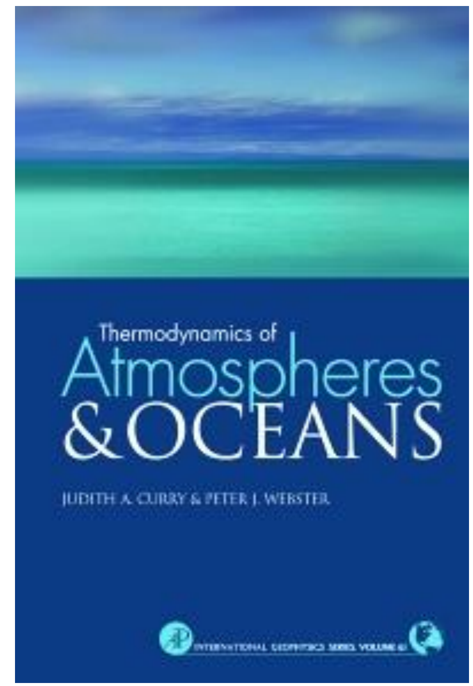
Fundamentals of Atmospheric Physics,
M.L. Salby; Salby

R&Y, Chapter 4



A Short Course in Cloud Physics,
R.R. Rogers and M.K. Yau; R&Y

C&W, Chapter 7



Thermodynamics of Atmospheres
and Oceanes,
J.A. Curry and P.J. Webster; C&W

Stratification

The hydrostatic balance is given by: $$vdp = -gdz$$

The ideal gas law transforms this into: $$dz = -\frac{R_d T}{g} d\ln p$$

Neither Γ_d nor Γ_s has a direct relationship to the temperature of the surroundings because a displaced parcel is thermally isolated under adiabatic conditions.

Thermal properties of the environment are dictated by the history of the air residing at a given location, for example, by its trajectory and the thermodynamic influences it has experienced.

The environmental lapse rate is defined as: $\Gamma = -\frac{dT}{dz}$, where T refers to the ambient temperature

The hydrostatic balance equation can be expressed as:

$$\frac{d\ln T}{d\ln p} = \frac{R_d}{g}\Gamma$$

$$\frac{d\ln T}{d\ln p} = \frac{\Gamma}{\Gamma_d}\kappa$$

$$\Gamma_d = \frac{g}{c_{pd}}$$

$$\kappa = \frac{R_d}{c_{pd}}$$

Lagrangian interpretation of stratification - 1

- Hydrostatic equilibrium applies in the presence of motion as well as under static conditions.
- Interpreting thermal structure in terms of the behavior of individual air parcels provides some insight into the mechanisms controlling the mean stratification.
- For a layer of constant lapse rate, the relationship between temperature and pressure

$$\frac{d\ln T}{d\ln p} = \frac{\Gamma}{\Gamma_d}\kappa \longrightarrow \frac{T}{T_s} = \left(\frac{p}{p_s}\right)^{\kappa\left(\Gamma/\Gamma_d\right)}$$

resembles that implied by Poisson's relation $Tp^{-\kappa} = const$

Lagrangian interpretation of stratification -2

For polytropic process

$$\delta q = c dT$$

$$(c_v - c)dT + pdv = 0 \qquad c_v \longrightarrow (c_v - c)$$

$$(c_p - c)dT - vdp = 0 \qquad c_p \longrightarrow (c_p - c)$$

Air parcels moving vertically exchange heat with their surroundings at such a rate that their temperature varies linearly with height.

$$\kappa \longrightarrow \frac{R}{c_p - c} = \frac{g}{c_p}\frac{c_p}{g}\frac{R}{c_p - c} = \kappa\frac{\Gamma}{\Gamma_d} \qquad \longrightarrow \qquad c = c_p\left(1 - \frac{\Gamma_d}{\Gamma}\right)$$

$$\Gamma = const \neq 0$$

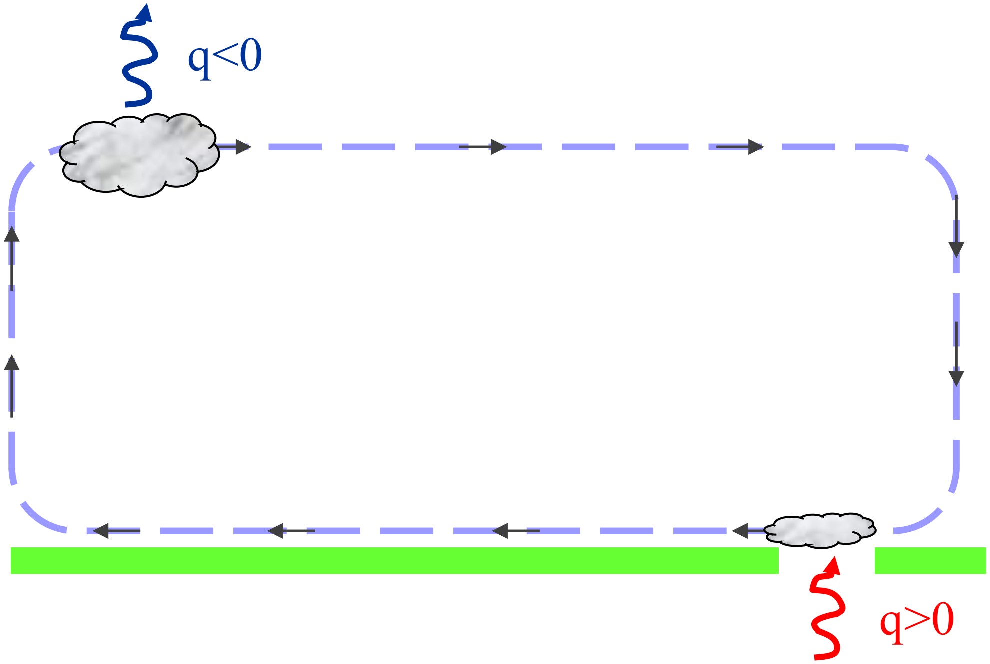
$$\frac{T}{T_s} = \left(\frac{p}{p_s}\right)^{\kappa\left(\Gamma/\Gamma_d\right)}$$

Adiabatic stratification (1)

$$\Gamma = \Gamma_d$$

$$c = c_p\left(1 - \frac{\Gamma_d}{\Gamma}\right) \longrightarrow c = 0$$

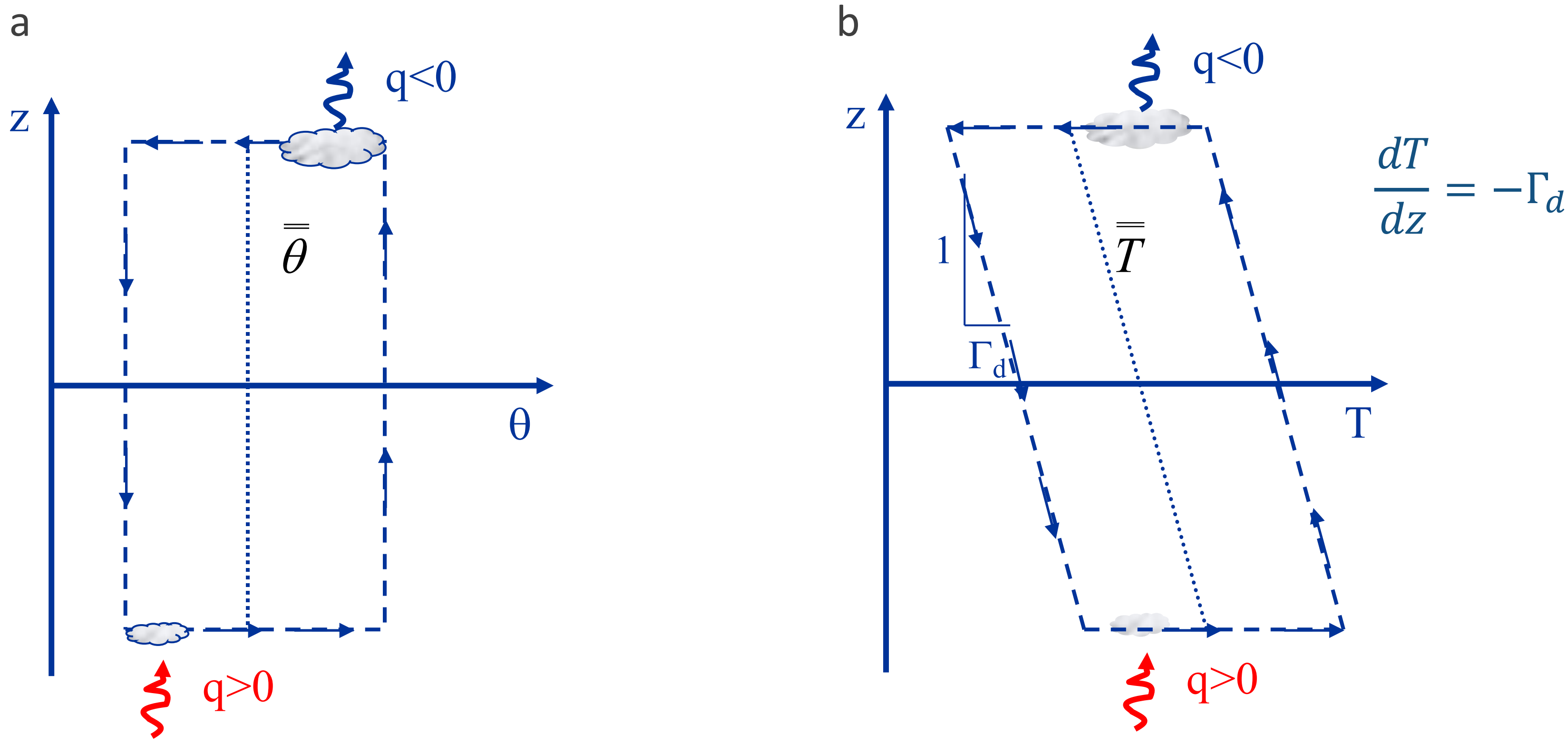
$$\delta q = \delta\theta = 0$$



$$\overline{\theta(z)} = const$$

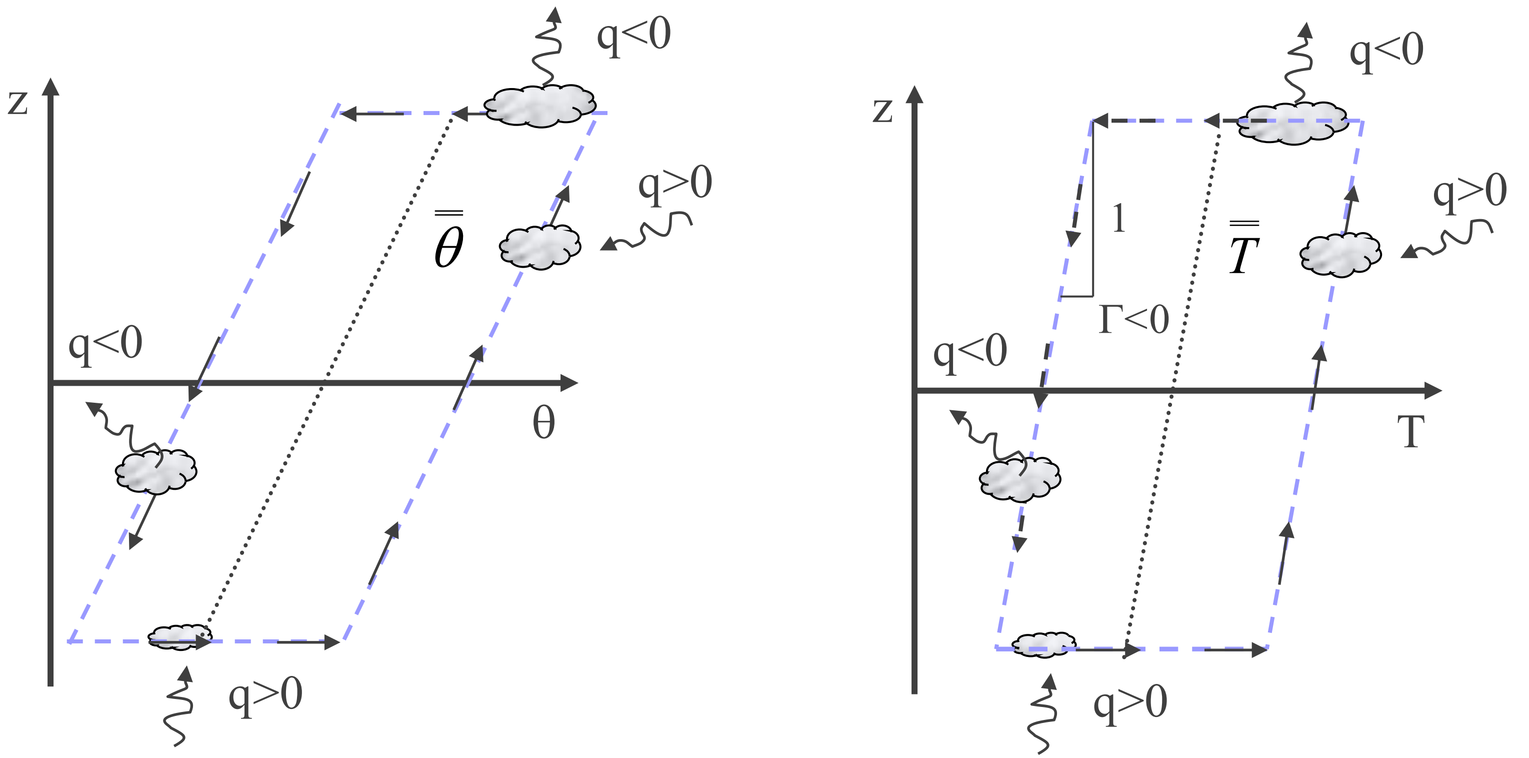
An idealized circuit followed by an air parcel during which it absorbs heat at the base of a layer and rejects heat at its top, with adiabatic vertical motion between. Fig. 6.6 Salby.

Adiabatic stratification (2)



The thermodynamic cycle followed by an air parcel in terms of (a) potential temperature and (b) temperature. The horizontally averaged behavior for a layer composed of many such parcels is indicated by dotted lines. Fig. 6.7 Salby

Diabatic stratification



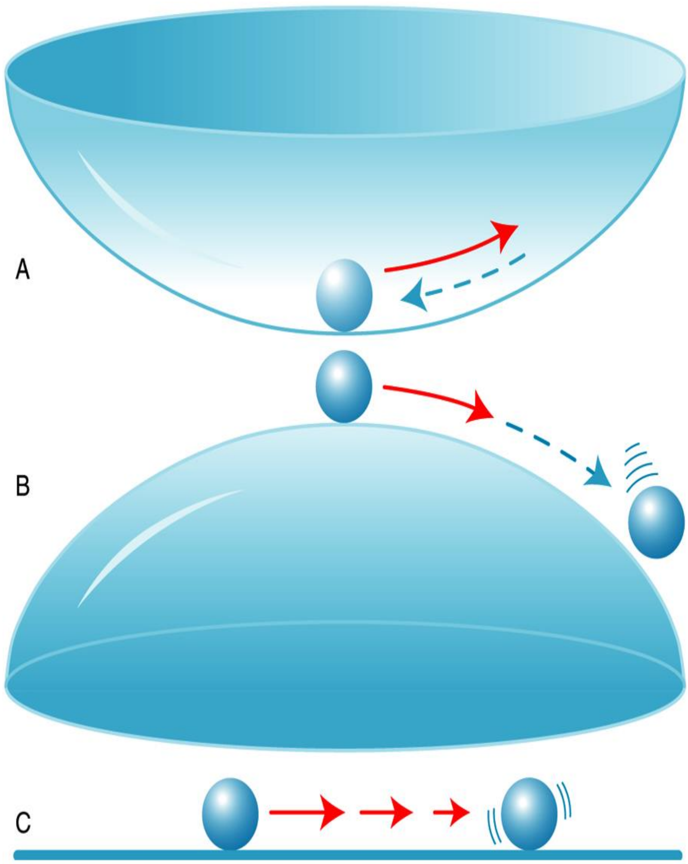
The thermodynamic cycle followed by an air parcel whose vertical motion is diabatic and whose temperature increases with height. (a) potential temperature and (b) temperature. The horizontally averaged behavior for a layer composed of many such parcels is indicated by dotted lines. Fig. 6.8 Salby

Stability of vertical motion

- We will examine vertical displacements in a fluid that is in hydrostatic balance.
- A parcel moving vertically within the fluid is subject to adiabatic expansion or compression, and hence the temperature will change.
- As the parcel moves vertically, it may become warmer or cooler than the surrounding fluid at a particular level.
- The parcel is subject to the Archimedes' force (buoyancy).

Stability of vertical motion

- Depending on the effect of the buoyancy force acting on the displaced mass:
  - If it returns the mass to its initial position, the fluid is statically stable (A).
  - If it accelerates the mass away from its initial position, the fluid is statically unstable (B).
  - If the mass remains in balance with its surroundings, the fluid is in a state of neutral equilibrium (C).



Parcel method

- We consider a small mass, or parcel, that is displaced vertically in a fluid at rest and in hydrostatic equilibrium.
- Simplifying assumptions adopted in the parcel method are:
  - The parcel retains its identity and does not mix with its environment.
  - The parcel's motion does not disturb its environment.
  - The pressure of the parcel adjusts instantaneously to the ambient pressure of the surrounding fluid.
  - The parcel moves isentropically, so that its potential temperature remains constant.

Why do we study stability in the atmosphere?

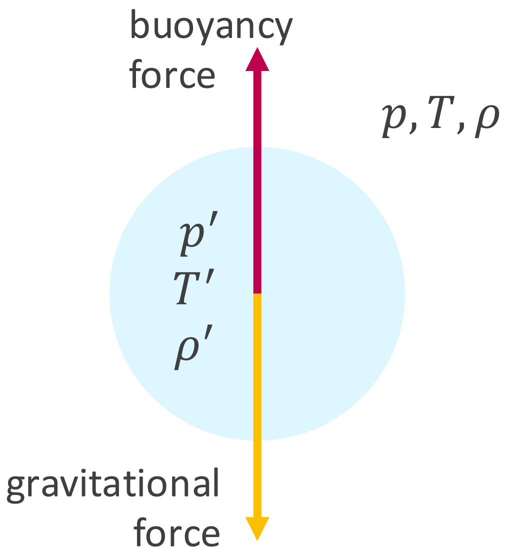
- The static stability of the atmosphere is important in the explanation and prediction of:
  - Cumulus convection and severe storms.
  - Precipitating systems.
  - Boundary-layer turbulence.
  - Large-scale atmospheric dynamics.

Stability criteria

Primed parameters will describe the properties of a parcel; unprimed parameters describe the properties of the parcel's environment.

The fluid environment is assumed to be in hydrostatic equilibrium; the pressure gradient force is balanced by the gravitational force.

$$0 = -g - \frac{1}{\rho}\frac{\partial p}{\partial z}$$

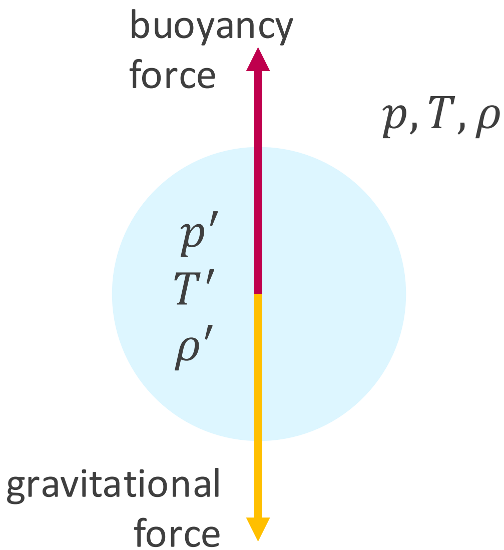


We will consider a small displacement of the parcel in the vertical direction.

From Newton's second law of motion, the acceleration of the parcel must be equal to the sum of the gravitational and pressure gradient forces.

$$\frac{d^2 z}{dt^2} = -g - \frac{1}{\rho'}\frac{\partial p'}{\partial z}$$

Hydrostatic balance - 1



$$f_b = \left(\frac{\rho - \rho'}{\rho'}\right) g$$

the net buoyancy force per unit mass

$$\frac{d^2 z}{dt^2} = -g - \frac{1}{\rho'}\frac{\partial p'}{\partial z}$$

the environment is in hydrostatic balance

$$0 = -g - \frac{1}{\rho}\frac{\partial p}{\partial z}$$

the pressure p' adjusts instantaneously to the ambient pressure p

$$p = p' \quad \longrightarrow \quad \frac{\partial p}{\partial z} = \frac{\partial p'}{\partial z}$$

$$\frac{d^2 z}{dt^2} = \frac{\rho - \rho'}{\rho'} g$$

If the parcel is less dense than its surrounding fluid, then it will accelerate upwards.

Hydrostatic balance - 2

$$\frac{d^2z}{dt^2} = \frac{\rho - \rho'}{\rho'} g$$

We will write this equation in terms of vertical density gradients by considering a small vertical displacement of the parcel from its initial location.

Let $z = 0$ be the initial location, where the parcel density is equal to the density in the surroundings.

$$\rho_0' = \rho_0$$

We expand the density of the parcel and the density of the environment about the initial location using a Taylor's series expansion. We will ignore higher-order terms involving powers of z, assuming the vertical displacement is small.

$$\rho' = \rho_0' + \left(\frac{d\rho'}{dz}\right) z + \cdots \qquad \rho = \rho_0 + \left(\frac{d\rho}{dz}\right) z + \cdots$$

$$\frac{d^2z}{dt^2} = \frac{g}{\rho'}(\rho - \rho') = \frac{g}{\rho'}\left[\left(\frac{d\rho}{dz}\right) - \left(\frac{d\rho'}{dz}\right)\right] z \approx \frac{g}{\rho_0}\left[\left(\frac{d\rho}{dz}\right) - \left(\frac{d\rho'}{dz}\right)\right] z$$

Hydrostatic balance - 3

The Brunt-Väisälä frequency, N, is defined as:

$$N^2 = \frac{g}{\rho_0}\left[\left(\frac{d\rho'}{dz}\right) - \left(\frac{d\rho}{dz}\right)\right]$$

It is also referred to as the buoyancy frequency.

The equation of parcel's motion becomes:

$$\frac{d^2 z}{dt^2} + N^2 z = 0$$

$$z = A_1 \exp(iNt) + B_2 \exp(-iNt)$$

$$z = A_1 \cos(Nt) + B_1 \sin(Nt)\ , \qquad N^2 \quad > 0$$

$$z = A_1 exp(|N|t) + B_1 exp(-|N|t)\ , \quad N^2 \quad < 0$$

Criteria of static stability - 1

$$\frac{d^2z}{dt^2} + N^2 z = 0$$

$N^2 > 0$: stable; period of oscilation: $\tau_g = \frac{2\pi}{N}$

$N^2 = 0$: neutral

$N^2 < 0$: unstable

Criteria of static stability - 2

$$N^2 = \frac{g}{\rho_0}\left[\left(\frac{d\rho'}{dz}\right) - \left(\frac{d\rho}{dz}\right)\right]$$

For a moist (but unsaturated) atmosphere, using the ideal gas law and neglecting pressure fluctuations, we can express the density changes as:

in the parcel

$$p' = R_d T_v' \rho'$$

$$\frac{d\rho'}{dz} = -\frac{p'}{R_d T_v'^2}\frac{dT_v'}{dz} = \frac{\rho'}{T_v'}\Gamma_d$$

in the surroundings

$$p = R_d T_v \rho$$

$$\frac{d\rho}{dz} = -\frac{p}{R_d T_v^2}\frac{dT_v}{dz} = -\frac{\rho}{T_v}\frac{dT_v}{dz}$$

$$N^2 = \frac{g}{\rho_0}\left[\frac{\rho'}{T_v'}\Gamma_d + \frac{\rho}{T_v}\frac{dT_v}{dz}\right]$$

$$T_v' = T_v = T_0$$

$$\rho' = \rho = \rho_0$$

$$N^2 = \frac{g}{T_0}\left[\Gamma_d + \frac{dT_v}{dz}\right]$$

Criteria of static stability - 3

From the definition of virtual potential temperature:

$$\theta_v = T_v \left(\frac{p_0}{p}\right)^{\kappa}$$

$$\frac{1}{\theta_v}\frac{d\theta_v}{dz} = \frac{1}{T_v}\frac{dT_v}{dz} - \frac{R}{c_{pd}}\frac{1}{p}\frac{dp}{dz}$$

$$\frac{1}{\theta_v}\frac{d\theta_v}{dz} = \frac{1}{T_v}\left(\frac{dT_v}{dz} + \frac{g}{c_{pd}}\right)$$

$$N^2 = \frac{g}{\theta_o}\frac{d\theta_v}{dz}$$

stable	$N^2 > 0$	$\frac{d\theta_v}{dz} > 0$	or $-\frac{dT_v}{dz} < \Gamma_d$
neutral	$N^2 = 0$	$\frac{d\theta_v}{dz} = 0$	or $-\frac{dT_v}{dz} = \Gamma_d$
unstable	$N^2 < 0$	$\frac{d\theta_v}{dz} < 0$	or $-\frac{dT_v}{dz} > \Gamma_d$

$$\frac{d^2z}{dt^2} = \frac{\rho - \rho'}{\rho'} g$$

$$p = RT\rho\,, \qquad p = RT'\rho'$$

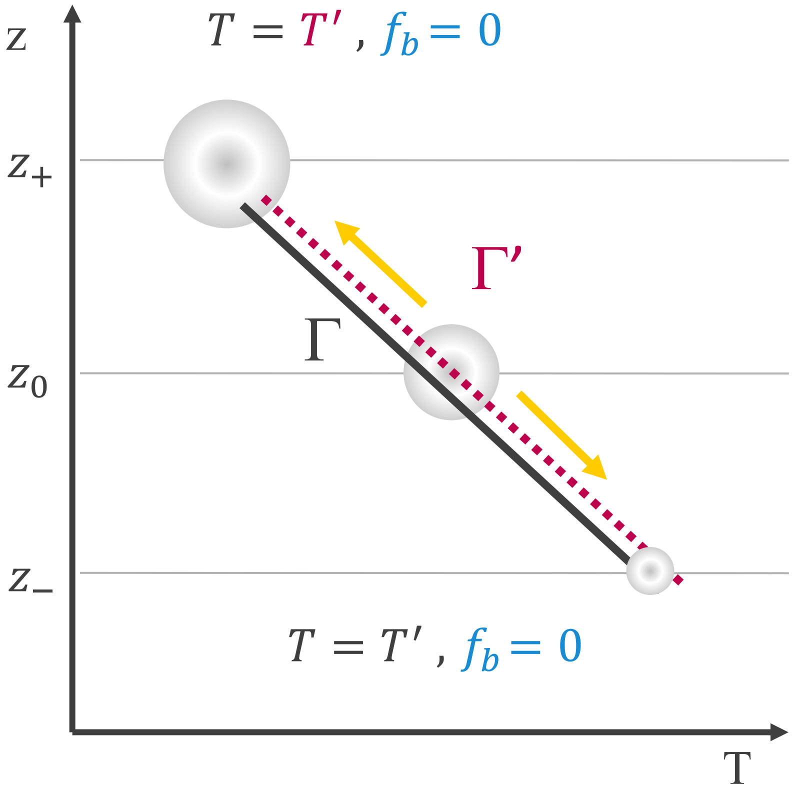
$$\frac{d^2z}{dt^2} = \frac{T' - T}{T} g$$

$$T' = T_0 - \Gamma' z \qquad \Gamma' = \Gamma_d \quad \text{or} \quad \Gamma' = \Gamma_s$$

$$T = T_0 - \Gamma z$$

$$\frac{d^2z}{dt^2} = \frac{g}{T}(\Gamma - \Gamma')z$$

Neutral stability, Γ=Γ’



$$\frac{d^2z}{dt^2} = \frac{g}{T}(\Gamma - \Gamma')z$$

f_b – buoyancy force

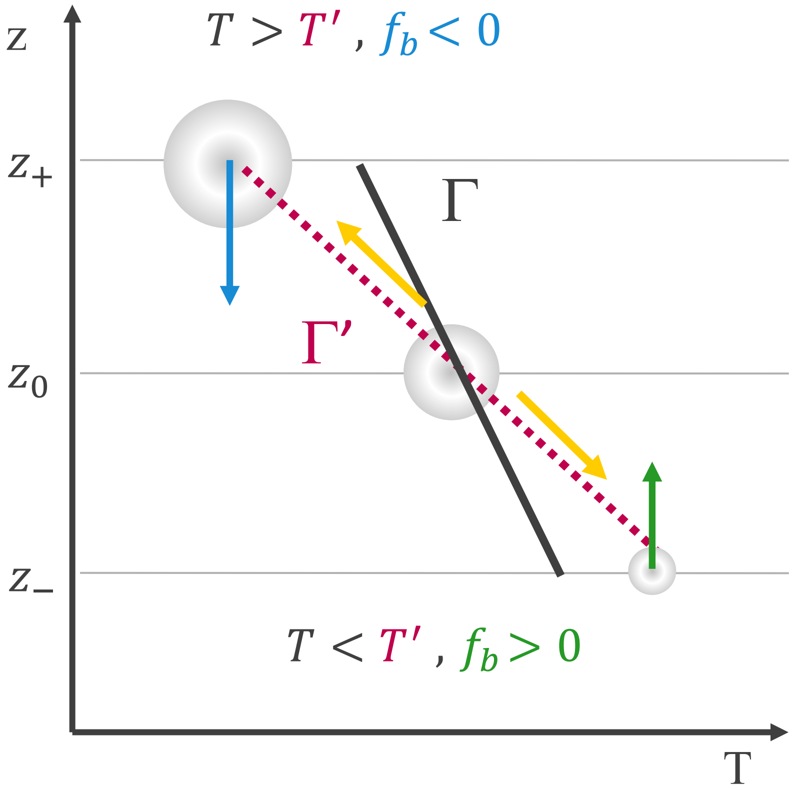
The parcel’s temperature changes:

- at the dry adiabatic lapse rate (~1°C/100m) if the parcel is unsaturated; $\Gamma' = \Gamma_d$
- at the saturated adiabatic lapse rate if the parcel is saturated; $\Gamma' = \Gamma_s$

Γ - the lapse rate in the environment (Γ=Γ’)

If the temperature lapse rate in the environment is equal to the parcel’s temperature lapse rate (either dry or moist adiabatic), then that parcel (dry or moist) does not experience a buoyancy force.

Stable/positive stability, Γ<Γ’



$$\frac{d^2z}{dt^2} = \frac{g}{T}(\Gamma - \Gamma')z$$

f_b – buoyancy force

The parcel’s temperature changes:

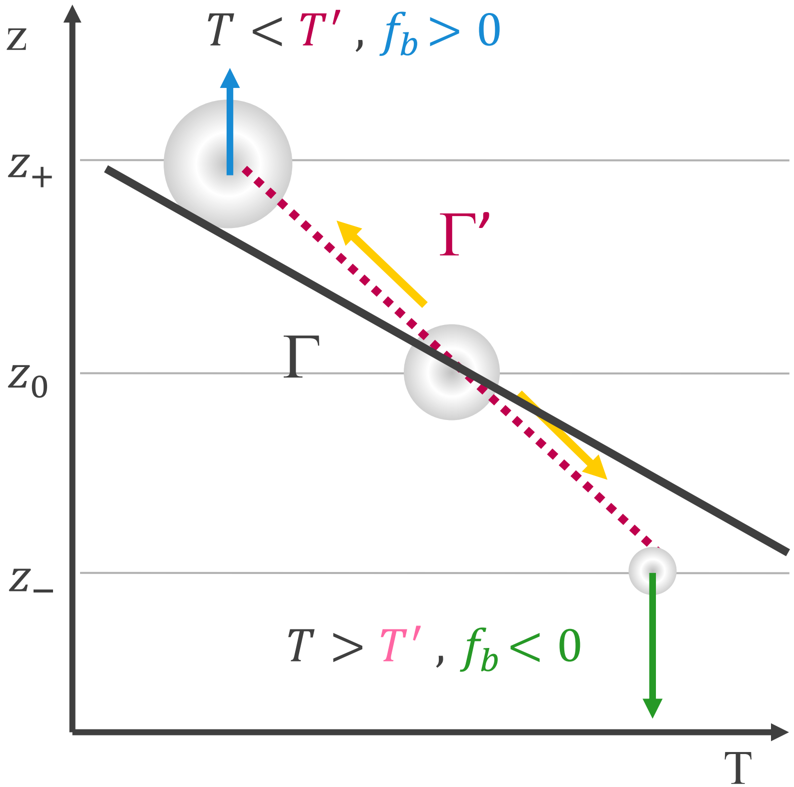
- at the dry adiabatic lapse rate (~1°C/100m) if the parcel is unsaturated; $\Gamma'=\Gamma_d$
- at the saturated adiabatic lapse rate if the parcel is saturated $\Gamma'=\Gamma_s$

Γ - the lapse rate in the environment (Γ=Γ’)

If the temperature lapse rate in the environment is smaller than the parcel’s temperature lapse rate (either dry or moist adiabatic), then that parcel (dry or moist) experiences a buoyancy force that opposes the displacement.

If the parcel is displaced upward (downward), it becomes heavier (lighter) than its surroundings and thus negatively (positively) buoyant.

Unstable/negative stability, Γ>Γ’



$$\frac{d^2z}{dt^2} = \frac{g}{T}(\Gamma - \Gamma')z$$

f_b – buoyancy force

The parcel’s temperature changes:

- at the dry adiabatic lapse rate (~1°C/100m) if the parcel is unsaturated; $\Gamma' = \Gamma_d$
- at the saturated adiabatic lapse rate if the parcel is saturated $\Gamma' = \Gamma_s$

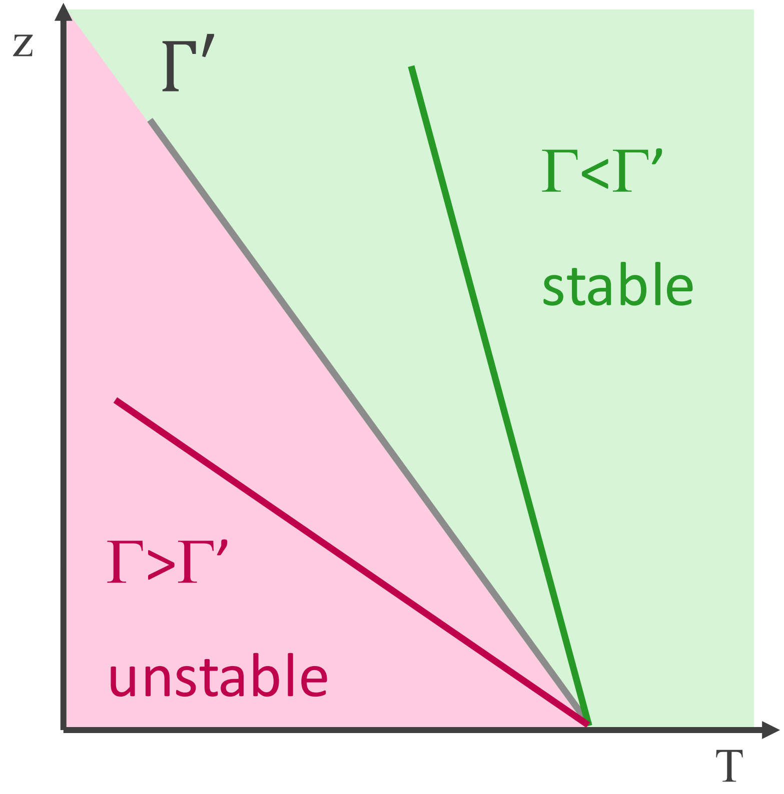
Γ - the lapse rate in the environment (Γ=Γ’)

If the temperature lapse rate in the environment is greater than the parcel’s temperature lapse rate (either dry or moist adiabatic), then that parcel (dry or moist) experiences a buoyancy force that reinforces the displacement.

If the parcel is displaced upward (downward), it becomes lighter (heavier) than its surroundings and thus positively (negatively) buoyant.

Stability of a dry parcel

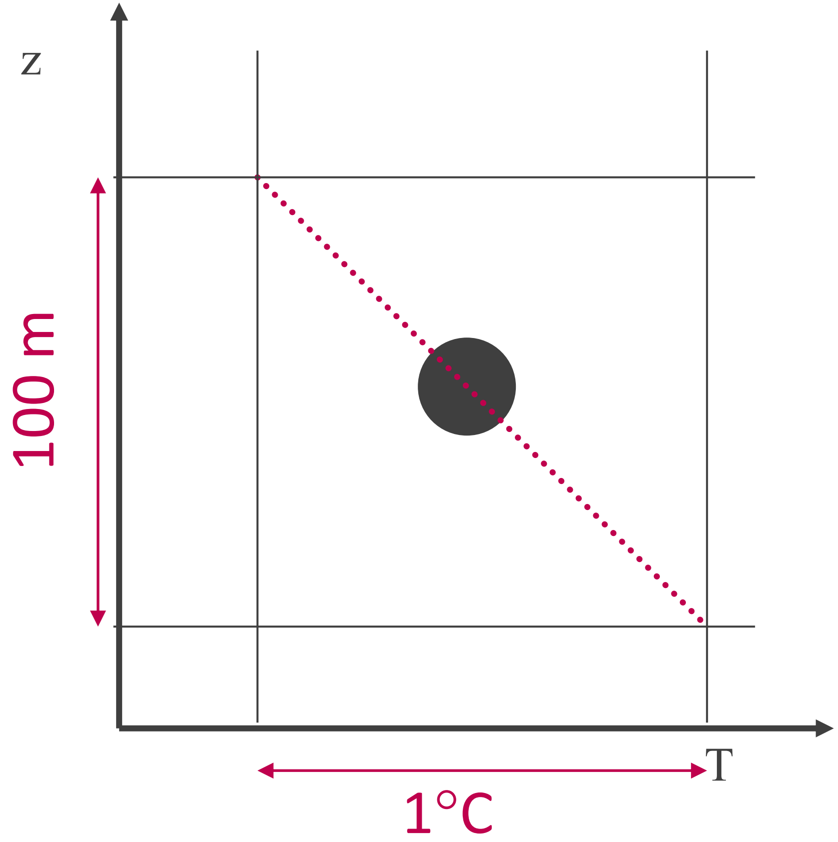
Γ' – the temperature lapse rate for a dry parcel (Γ_d)
Γ - the environmental lapse rate



Dry and wet (saturated) parcels

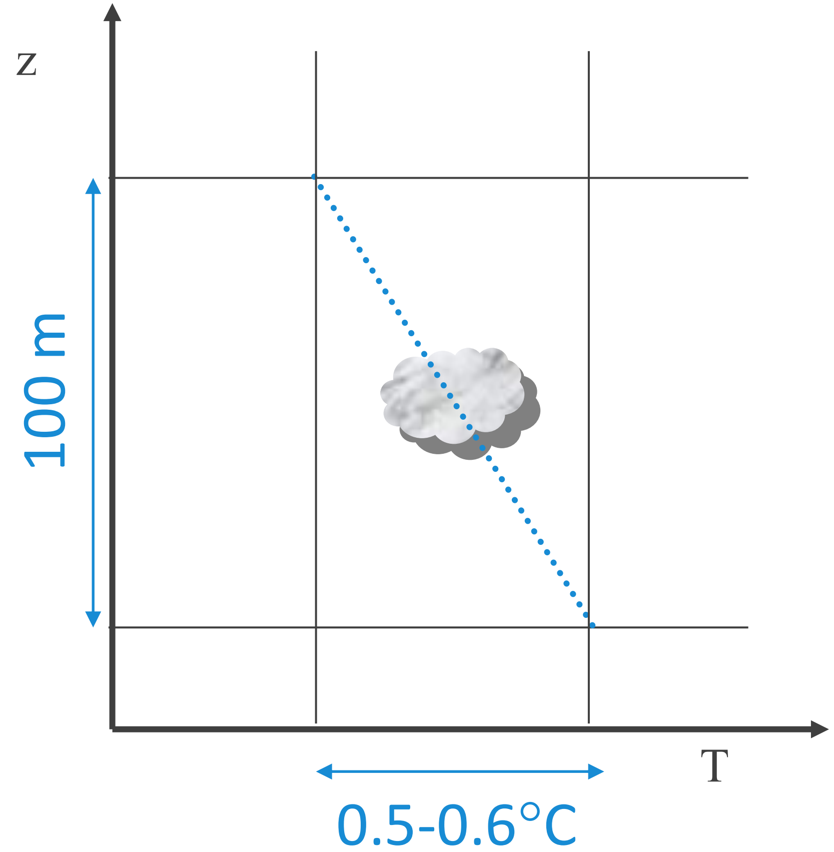
$$\Gamma' = \Gamma_d$$

dry adiabatic lapse rate



$$\Gamma' = \Gamma_s$$

wet adiabatic lapse rate



Stability of a moist (saturated) parcel

Vertical displacements of air parcels frequently result in phase changes (releasing latent heat), which affect the buoyancy of the air and thus the static stability criteria.

When a saturated parcel of air is displaced vertically, its temperature changes according to the saturated adiabatic lapse rate (Γ'_s).

The Brunt-Väisälä frequency is defined similarly to the case of a dry parcel.

$$N^2 = \frac{g}{T_0}\left(\frac{dT_\rho}{dz} + \Gamma_s'\right)$$

To account for the condensed water, density temperature is used rather than virtual temperature T_v.

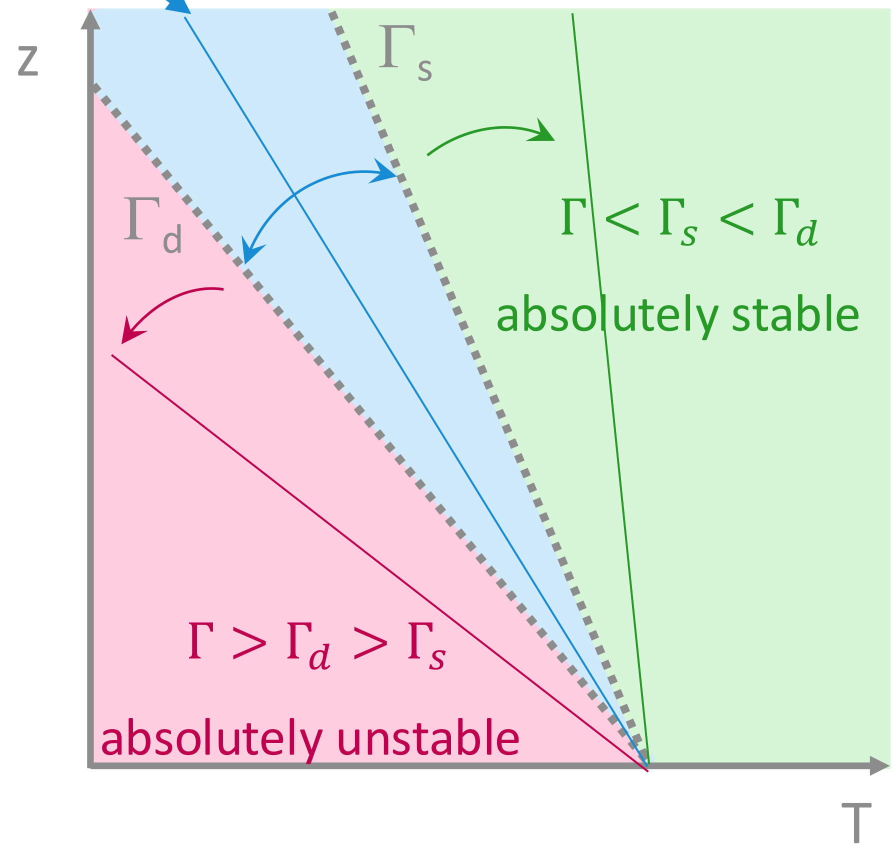
$$T_\rho = T(1 + 0.608q_v - q_l)$$

Note that $T_\rho = T_v$ when $q_l = 0$.

Stability criteria for dry and saturated parcels

$\Gamma_s < \Gamma < \Gamma_d$

Conditionally stable
- Stable for dry particles
- Unstable for moist/saturated particles



Stability criteria

absolutely stable $$-\frac{dT_\rho}{dz} < \Gamma_s$$

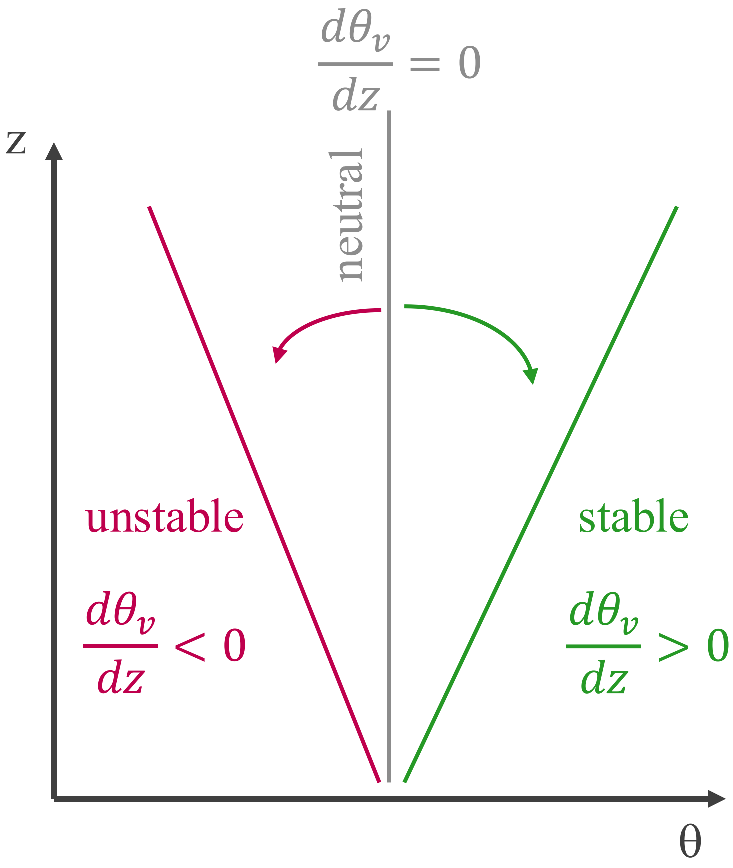
neutral for saturated particles $$-\frac{dT_\rho}{dz} = \Gamma_s$$

conditionally stable/unstable $$\Gamma_s < -\frac{dT_\rho}{dz} < \Gamma_d$$

neutral for unsaturated particles $$-\frac{dT_\rho}{dz} = \Gamma_d$$

absolutely unstable $$-\frac{dT_\rho}{dz} > \Gamma_d$$

Stability criteria for dry particles in terms of potential temperature



$$\frac{d\theta_v}{dz} = \frac{\theta_v}{T}(\Gamma_d - \Gamma)$$

stable	$\frac{d\theta_v}{dz} > 0$
neutral	$\frac{d\theta_v}{dz} = 0$
unstable	$\frac{d\theta_v}{dz} < 0$

Stability criteria for saturated particles in terms of equivalent potential temperature

If an air parcel is saturated, the stability criteria are similar to those for a dry parcel, but the virtual potential temperature has to be replaced by the equivalent potential temperature.

stable $\frac{d\theta_e}{dz} > 0$

neutral $\frac{d\theta_e}{dz} = 0$

unstable $\frac{d\theta_e}{dz} < 0$

The equivalent potential temperature can be written in an approximate form:

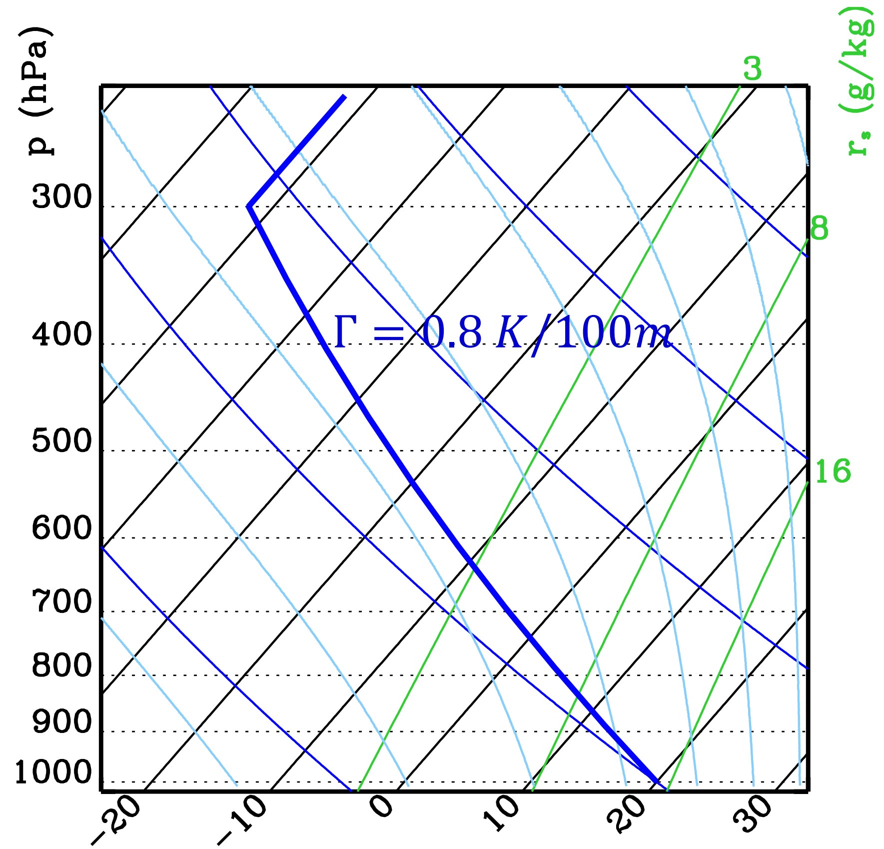
$$\theta_e = \theta \exp\left(\frac{L_{lv} q_s}{c_{pd} T}\right) \approx \theta\left(1 + \frac{L_{lv} q_s}{c_{pd} T}\right) = \theta + \frac{L_{lv} q_s}{c_{pd}}\left(\frac{p_0}{p}\right)^{\kappa}$$

For a layer to be unstable to wet-adiabatic displacements, the potential temperature must decrease with altitude and/or the specific humidity must decrease with altitude.

Conditional instability

Consider an unsaturated air parcel displaced upward within a conditionally unstable layer.

This profile represents a typical stratification of the tropical atmosphere: a constant, conditionally unstable lapse rate ($\Gamma_s<\Gamma<\Gamma_d$) in the troposphere, and a zero lapse rate in the lower stratosphere.



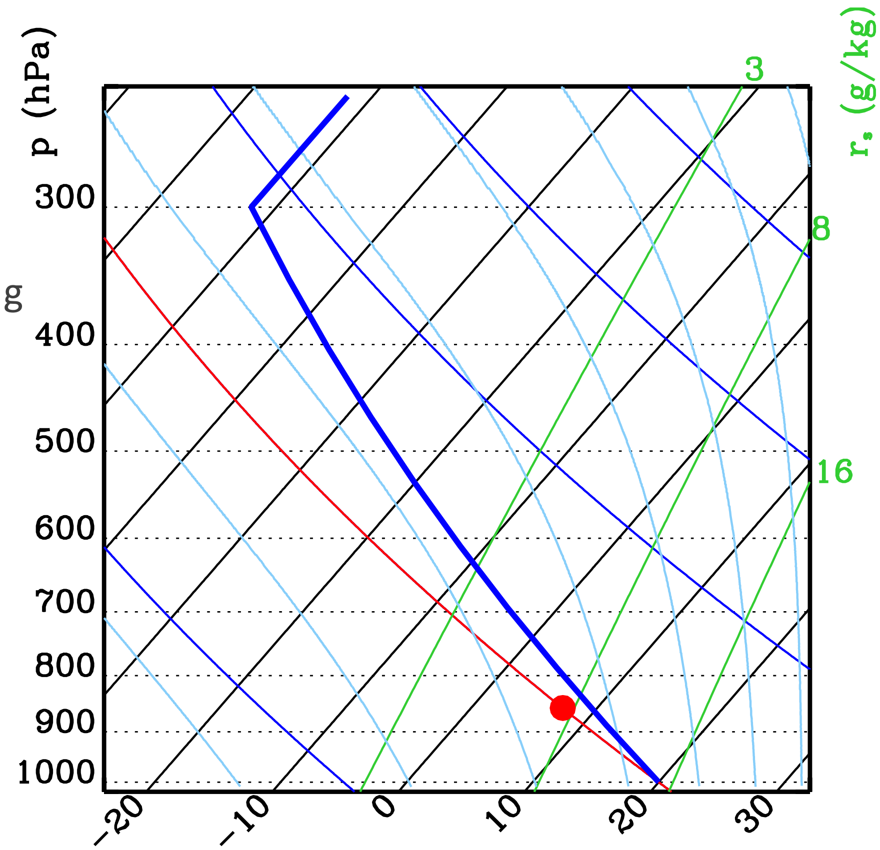
$$p_0 = 1000\,hPa$$

$$T_0 = 20°\mathrm{C}$$

$$q_s = 7.48\,g/kg$$

Below the LCL, the displaced air parcel cools at the dry adiabatic lapse rate (Γ_d), and, therefore cools more rapidly than its surroundings ($\Gamma<\Gamma_d$).

Consequently, the parcel experiences a positive restoring force that increases with upward displacement.



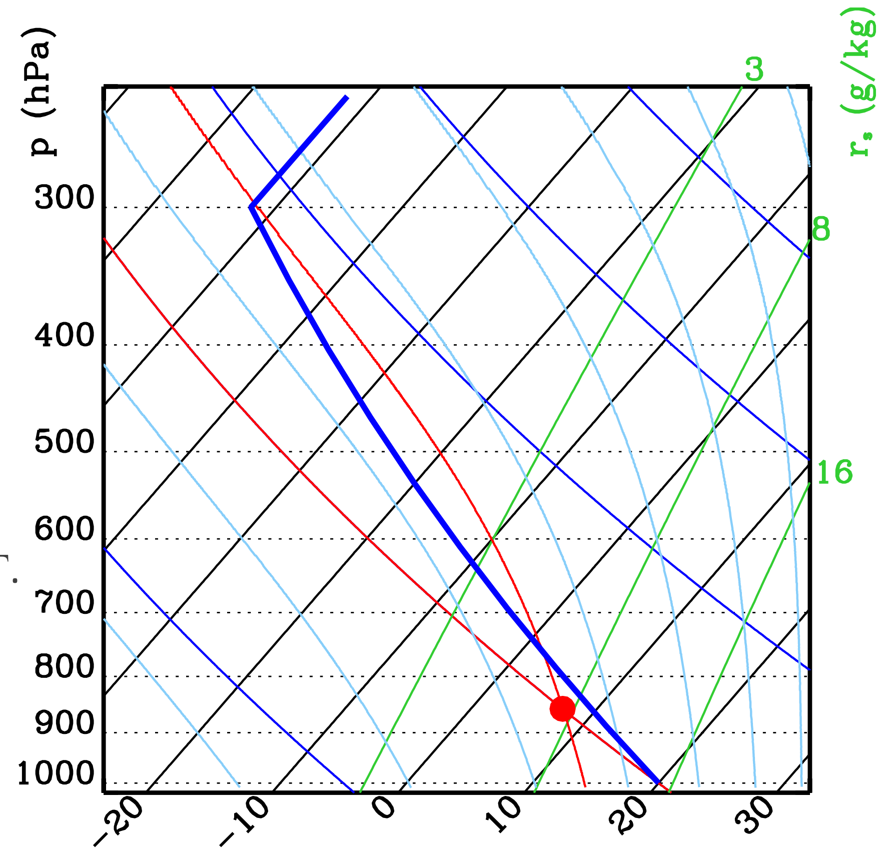
$$T_{LCL} = 7.27°C$$
$$p_{LCL} = 856\ hPa$$

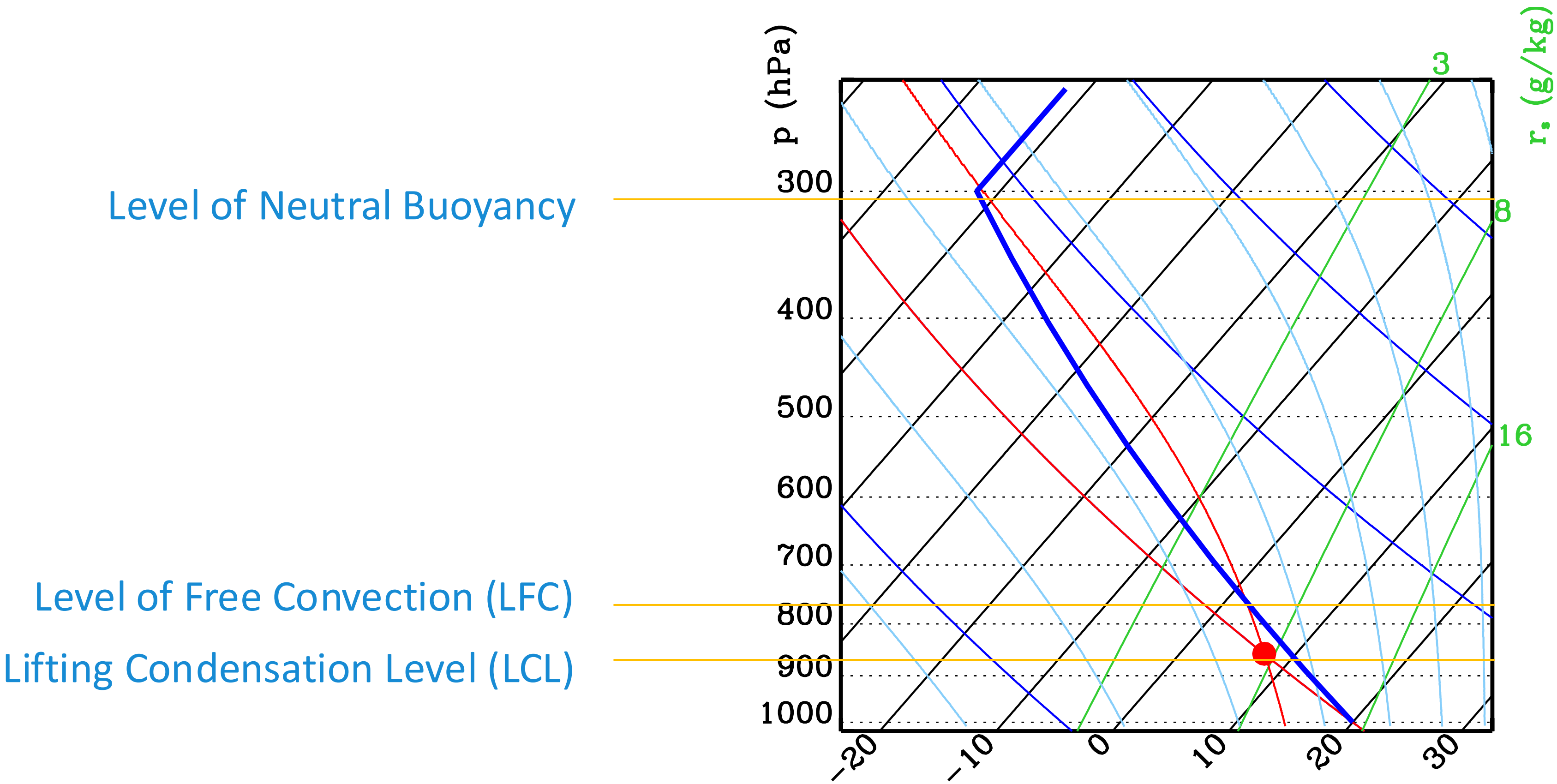
Below the LCL the displaced parcel cools at the dry adiabatic lapse rate (Γ_d) and therefore cools more rapidly than its surroundings ($\Gamma<\Gamma_d$).

The parcel experiences a positive restoring force, one which increases with upward displacement.

Above the LCL, the parcel cools more slowly at the saturated adiabatic lapse rate (Γ_s) due to the release of latent heat. Because the layer is conditionally unstable, the parcel cools more slowly than its surroundings, $\Gamma_s<\Gamma$.

Consequently, the temperature difference between the parcel and its environment and - hence the positive restoring force of buoyancy - diminishes with height.





A level where the parcel's temperature profile crosses the environmental temperature profile is the Level of Free Convection (LFC). Above this point, the parcel becomes warmer than its surroundings, experiencing a negative restoring force that allows it to ascend on its own accord.

In the lower stratosphere, the temperature does not vary with altitude. Here, the temperature of the parcel crosses the environmental profile a second time at a crossing level, p_c,called the Level of Neutral Buoyancy. Above this level, the parcel is once again cooler than its environment, meanng buoyancy opposes further ascent.

Under adiabatic conditions, the buoyancy force is conservative (the work performed along a cyclic path vanishes).

The potential energy P of the displaced parcel is: $dP = \delta w_b = -f_b dz$

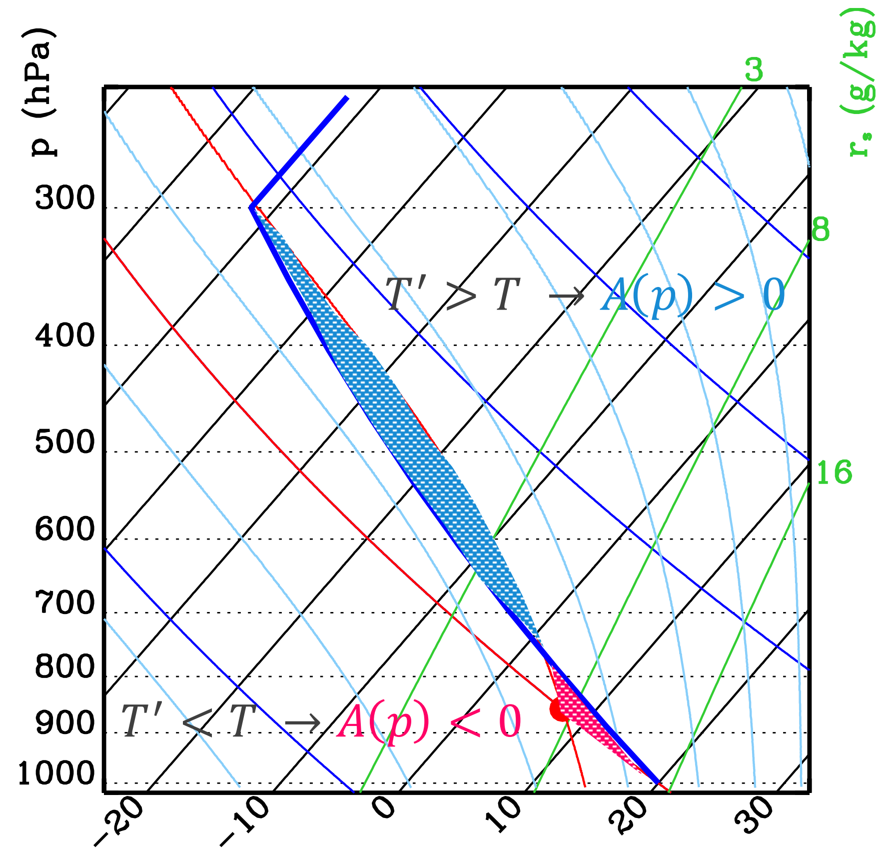
δw_b is the incremental work performed against buoyancy.

Let's define a reference value of zero potential energy at the undisturbed height z_0.

$$P = \int_{z_0}^{z} \left(\frac{\rho - \rho'}{\rho'}\right) g dz = \int_{p_0}^{p} (v' - v) dp$$

$$dp = -\rho g dz \qquad pv = RT \qquad pv' = RT'$$

$$P(p) = -R \int_{p_0}^{p} (T' - T)(-d \ln p) = -R \cdot A(p)$$



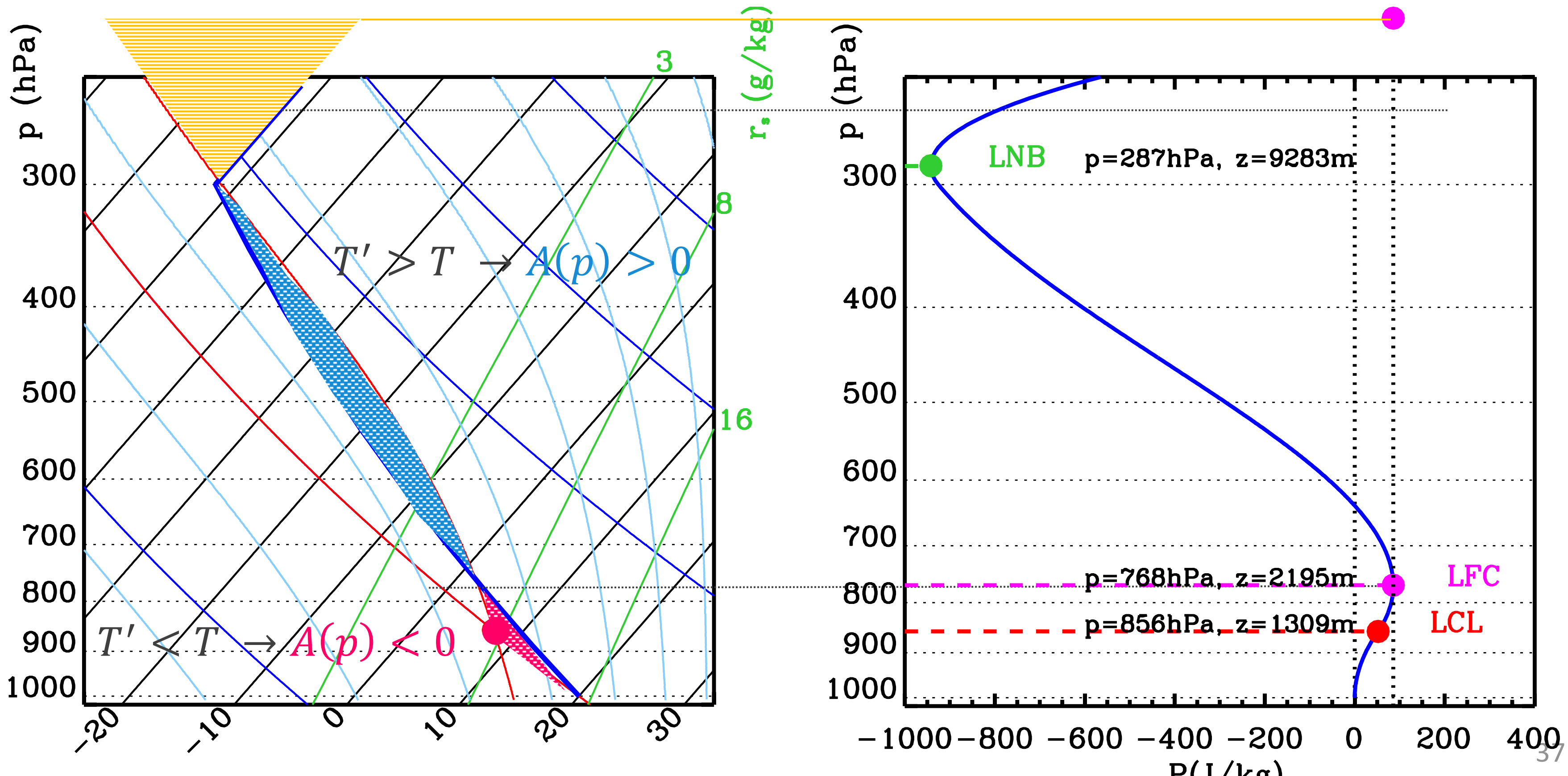
A(p) is the cumulative area between the temperature profile of the parcel T' and that of the environment T to the level p. A(p) can be positive or negative.

Below the LCL, T‘ <T, so potential energy increases upward.

Work must be performed against the positive restoring force of buoyancy to liberate the parcel from this potential well.

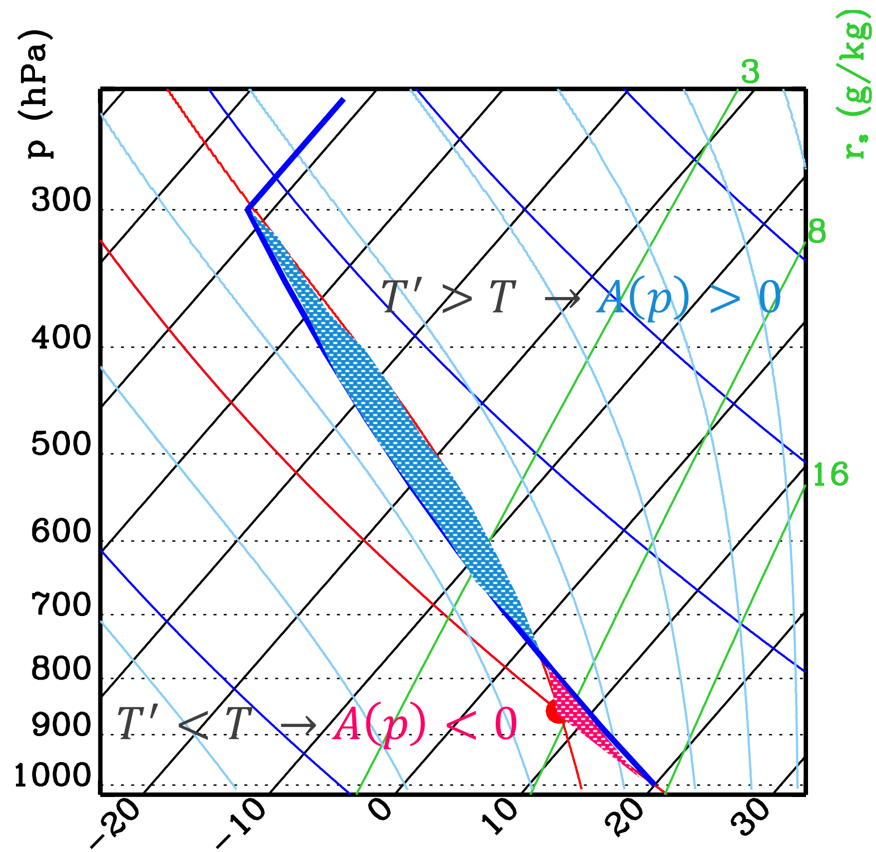
Above the LCL (where P is a maximum) the work that has been performed below is available for conversion to kinetic energy K. Above the LCL, T’>T, so P decreases upward. Under conservative conditions ΔK=-ΔP.

Decreasing P above the LCL represents a conversion of potential energy to kinetic energy, which drives deep convection through buoyancy work.



Convective Available Potential Energy (CAPE)

The total potential energy available for conversion to kinetic energy is termed the convective available potential energy (CAPE). It is represented by the blue area in the figure.



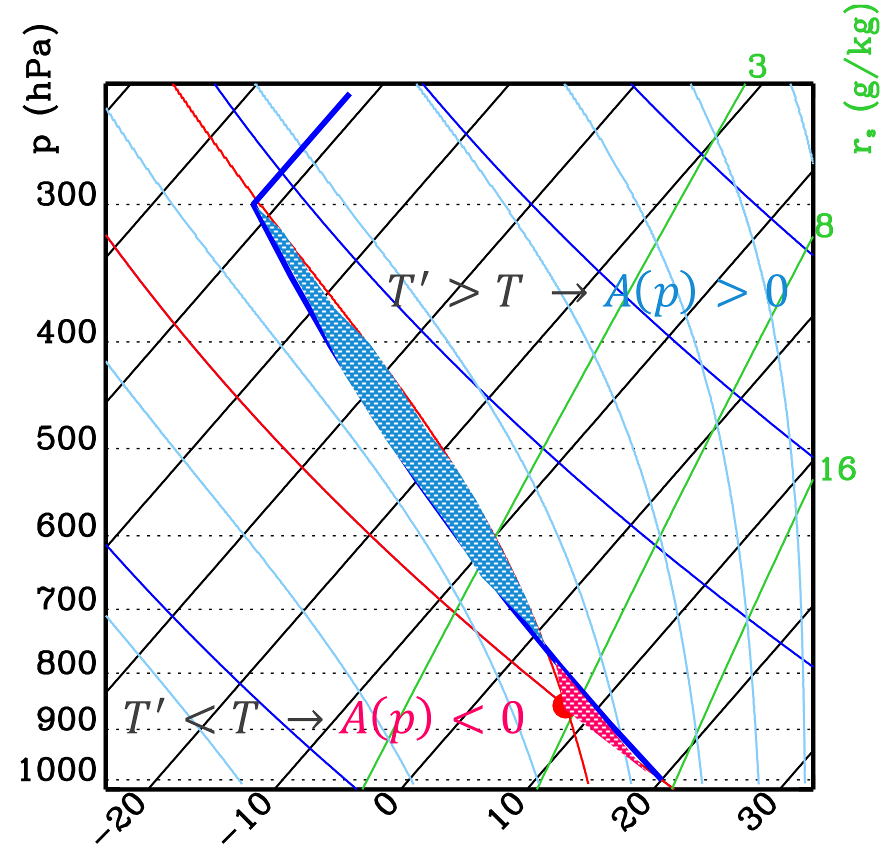
CAPE=1027 J/kg

Since the parcel's temperature crosses the environmental temperature profile a second time CAPE is necessarily finite, as is the kinetic energy that can be acquired by the parcel.

An upper bound on the parcel's kinetic energy is:

$$\frac{w'^2}{2} = P(p_{LFC}) - P(p_c)$$

$$= R \int_{p_{LFC}}^{p_c} (T' - T)(-d\ln p) = \text{CAPE}$$



In practice mixing with the surroundings makes the behavior inside convection inherently nonconservative, so the upward velocity presented above is seldom observed.

Above the crossing level (LNB – level of neutral buoyancy) T' and T are again reversed, so P increases upward above p_c. The parcel becomes negatively buoyant and is bound in another potential well.

Despite opposition by buoyancy, the parcel overshoots its new equilibrium level p_c due to the kinetic energy it acquired above the LCL. Because T diverges rapidly from T' the penetration into the stable layer aloft is shallow compared to the depth traversed through the conditionally unstable layer below. Like the maximum updraft the estimate of the penetration level is only an upper bound.

Convective domes overshooting: a photo and simulation results

Figure 3. (Left) Jumping cirrus photographed by Martin Setvák on 24 May 1996 late afternoon from an airplane above Alabama and Georgia (Courtesy of Martin Setvák). (Right) RHi 30% contour surface of the simulated storm at t = 1440 s. The vertical dimension is enhanced to match the perspective view of the photograph.

Wang, P.K., 2004: A cloud model interpretation of jumping cirrus above storm top. Geophys. Res. Let. Vol. 31, L18106, doi: 10.1029/2004GL020787.

Overshooting tops

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