STABILITY IN THE ATMOSPHERE

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LECTURE OUTLINE

1. Conditional stability
2. Entrainment
3. Modification of stability

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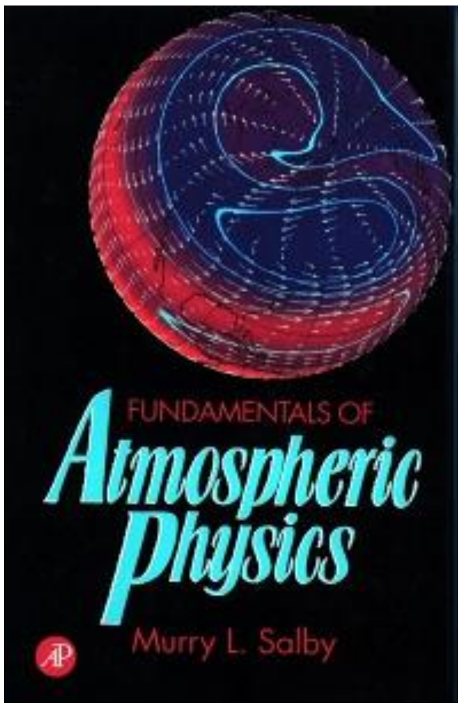
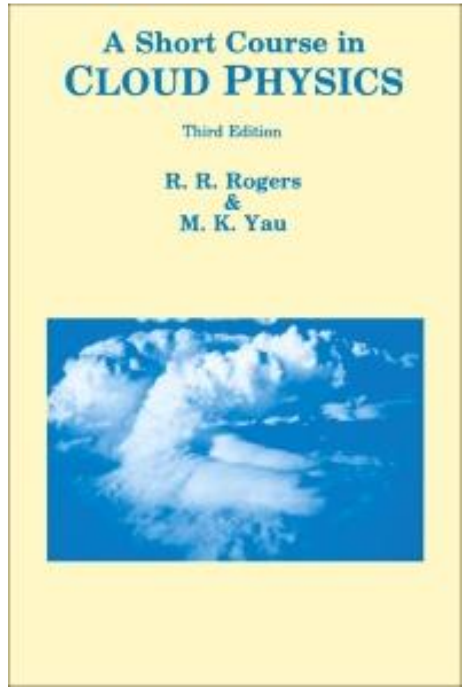
Salby, Chapter 6



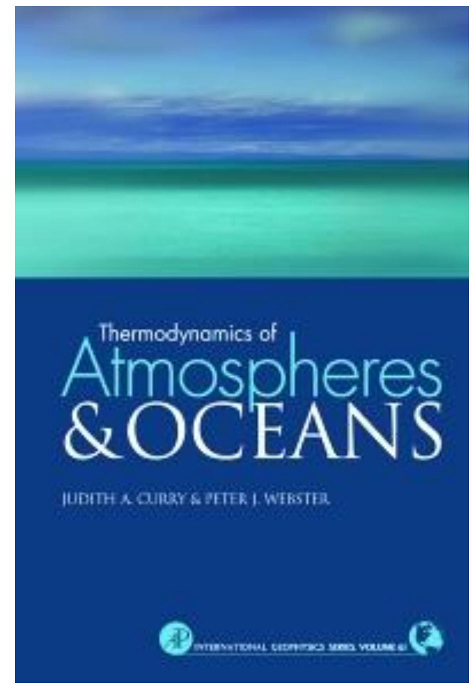
Fundamentals of Atmospheric Physics,
M.L. Salby; Salby

R&Y, Chapter 4



A Short Course in Cloud Physics,
R.R. Rogers and M.K. Yau; R&Y

C&W, Chapter 7



Thermodynamics of Atmospheres
and Oceanes,
J.A. Curry and P.J. Webster; C&W

Implication of stability for vertical motion - 1

- The stability of a layer determines its ability to support vertical motion, and thus the transfer of heat, momentum, and atmospheric constituents.
- Since vertical motion must be compensated by horizontal motion to conserve mass, hydrostatic stability also influences horizontal transport.
- Three-dimensional (3-D) turbulence that disperses atmospheric constituents involves both vertical and horizontal motion. Suppressing vertical motion also suppresses the horizontal component of 3-D eddy motion, thereby inhibiting turbulent dispersion.

Implication of stability for vertical motion - 2

- A stably stratified layer inhibits vertical motion. Small vertical displacements introduced mechanically (by flow over elevated terrain) or thermally (through isolated heating) are then opposed by the positive restoring force of buoyancy.
- Conversley, an unstable stratified layer promotes vertical motion through a negative restoring force.
- Work performed by or against buoyancy reflects a conversion between potential and kinetic energy.

Implication of stability for vertical motion - 3

The vertical momentum balance for an unsaturated air parcel:

$$\frac{d^2 z}{dt^2} + N^2 z = 0, \qquad N^2 = g\,\frac{d\ln\theta}{dz}$$

Equation was derived for small vertical displacements. Under positive or neutral stability, air displacements remain small enough for the stratification of the layer to be preserved.

An unstably stratified layer (negative stability) evolves differently. The solution to the equation takes the form:

$$z(t) = Ae^{\hat{N}t} + Be^{-\hat{N}t}, \qquad N^2 = -\hat{N}^2 < 0$$

Implication of stability for vertical motion - 4

$$z(t) = Ae^{\hat{N}t} + Be^{-\hat{N}t}\,, \qquad N^2 = -\hat{N}^2 < 0$$

The parcel's displacement grows exponentially with time.

The first term, which dominates the long-term behavior, violates the linear analysis applied in the derivation of the equation.

Except for very small values of N, displacements amplify exponentially – even in the presence of friction.

Small initial disturbances then evolve into fully developed convection, in which nonlinear effects limit further amplification by modifying the stratificationof of the layer.

By rearranging mass, convective cells alter N^2 and, hence, the buoyancy force experienced by individual air parcels. At this point, the simple linear description breaks down.

Implication of stability for vertical motion - 5

The amplifying motion is fueled by the conversion of potential energy (associated with the vertical distribution of mass) into kinetic energy (associated with convective motions).

These air motions, in turn, modify the stratification of the layer.

Fully developed convection, which results in efficient vertical mixing, rearranges the conserved property θ (θ_e) into a distribution that is statistically homogeneous. This limiting distribution corresponds to a state of neutral stability.

Thus, small disturbances to an unstable layer amplify and eventually evolve into fully developed convection, which neutralizes the instability by mixing θ (θ_e) into a uniform distribution.

In that limiting state, no more potential energy is available for conversion to kinetic energy, causing convective motions to decay through frictional dissipation.

Entrainment -1

Cooler and drier environmental air that is entrained into and mixed with a moist thermal depletes ascending parcels of their positive buoyancy and kinetic energy.

Only in the cores of broad convective towers are adiabatic values ever approached.

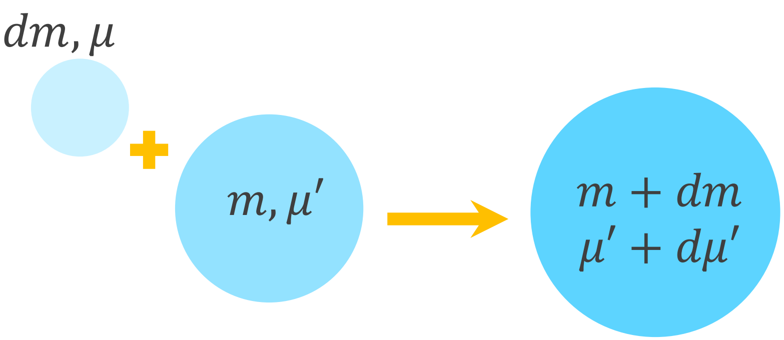
Mixing also modifies the surroundings of a cumulus tower, thereby altering the temperature and moisture gradients that control buoyancy.

Let us consider a process where entrained environmental air is uniformly mixed with the ascending air of a convective cell. Through mixing, additional mass is incorporated into the parcel.

Entrainment -2

If μ is a conserved property (e.g. $\mu = q_v$ beneath the LCL), then $m\mu$ represents the total amount of property μ associated with the parcel under consideration.

After a mass dm of environmental air has been mixed into it, the parcel will have mass $m + dm$ and a new property $\mu' + d\mu'$.



Primes distinguish the properties of the environment from those of the parcel.

Conservation of μ requires:

$$(m + dm)(\mu' + d\mu') = m\mu' + \mu dm$$

Higher-order terms are neglected:

$$\cancel{m}\mu' + m d\mu' + \mu' dm + dm d\mu' = \cancel{m}\mu' + \mu dm$$

$$d\mu' = \frac{dm}{m}(\mu - \mu')$$

If the process occurs over a time interval dt:

$$\frac{d\mu'}{dt} = \frac{d\ln m}{dt}(\mu - \mu')$$

Entrainment -3

If μ is not conserved but is instead produced per unit mass at a rate of S_μ, its evolution within the parcel is governed by the equation:

$$\frac{d\mu'}{dt} = \frac{d\ln m}{dt}(\mu - \mu') + S_\mu$$

The time rate of change is directly related to the parcel's vertical velocity: $\frac{d}{dt} = w\frac{d}{dz}$

$$w\frac{d\mu'}{dz} = w\frac{d\ln m}{dz}(\mu - \mu') + S_\mu$$

$$\frac{1}{H_e} = \frac{d\ln m}{dz} \quad \longrightarrow \quad w\frac{d\mu'}{dz} = \frac{w}{H_e}(\mu - \mu') + S_\mu$$

H_e denotes the mass entrainment scale height.
H_e reflects the rate at which the entraining thermal expands due to the incorporation of environmental air.
Physically, it represents the vertical distance required for the upwelling mass to increase by a factor of e.

Exemple: $\mu = \ln\theta_e$

The source term $S_{\theta e} = 0$ because θ_e is conserved.

$$\frac{d\ln\theta_e'}{dz} = \frac{1}{H_e}(\ln\theta_e - \ln\theta_e')$$

$$\frac{d\ln\theta_e'}{dz} = \frac{1}{H_e}\left[\ln\frac{\theta}{\theta'} + \frac{L_{lv}}{c_{pd}}\left(\frac{q_s}{T} - \frac{q_s'}{T'}\right)\right]$$

$$w\frac{d\mu'}{dz} = \frac{w}{H_e}(\mu - \mu') + S_\mu$$

$$\theta_e = \theta\exp\left(\frac{L_{lv}q_s}{c_{pd}T}\right)$$

$$\ln\theta_e = \ln\theta + \frac{L_{lv}q_s}{c_{pd}T}$$

Assuming that the air within both the parcel and the environment is unsaturated, q_s must be replaced by the actual mixing ratios, and the temperatures should correspond to those at the LCL.

Note that temperaturę differences are typically small compared to moisture differences.

Exemple: $\mu = \ln \theta_e$

$$\frac{d \ln \theta_e'}{dz} \cong \frac{1}{H_e}\left[\ln\frac{\theta}{\theta'} + \frac{L_{lv}}{c_{pd}T}(q_v - q_v')\right] = \frac{1}{H_e}\left[\ln\frac{T}{T'} + \frac{L_{lv}}{c_{pd}T}(q_v - q_v')\right]$$

The air in the ascending thermal is usually :

- **warmer than in the environment**: temperatures at the LCL fulfil also: $T' > T$. Therefore, the first term on the right-hand side is negative: $\ln(T/T') < 0$
- **more humid than in the environment**: $q_v' > q_v$. Therefore, the second term on the right-hand side i negative: $(q_v - q_v') < 0$.

Therefore, in the ascending thermal that mixes with the environment: $\frac{d \ln \theta_e'}{dz} < 0$

Entrainment acts as a brake on convection by reducing the parcel's temperature and mixing ratio toward environmental values.

The equivalent potential temperature of the parcel, θ_e', decreases with height as it ascends. This contrasts with the parcel's behavior under conservative conditions (e.g. in the absence of entrainment), where its mass is fixed and $\theta_e' = const$.

The two sinks of θ_e' on the right-hand side of the equation reflect transfers of sensible and latent heat to the environment.

Example: $\mu = w$ (specific momentum)

Suppose μ represents the vertical velocity w (which corresponds to the specific vertical momentum): $\mu' = w$

In the environment $w = 0$: $\mu = 0$

$$w\frac{d\mu'}{dz} = \frac{w}{H_e}(\mu - \mu') + S_\mu$$

$$w\frac{dw}{dz} = -\frac{w^2}{H_e} + S_w$$

$$\frac{1}{2}\frac{dw^2}{dz} = -\frac{w^2}{H_e} + S_w \qquad K' = \frac{w^2}{2}$$

$$\frac{dK'}{dz} = -\frac{2}{H_e}K' + \frac{S_K}{w}$$

K' represents the specific kinetic energy of the parcel (energy per unit mass).

The source term for kinetic energy (S_K) and vertical momentum (S_w) are related by: $S_k = wS_w$.

Example: $\mu = w$ (specific momentum) -2

Even in the absence of entrainment ($H_e \to \infty$), Kinetic energy (K) is not conserved because the parcel's kinetic energy changes due to the work (δw_b) performed on it by the buoyant force.

$$dP = \delta w_b = -f_b dz \qquad \Delta K = -\Delta P \qquad \frac{dK'}{dz} = f_b$$

The production term can be evaluated for the case where $H_e \to \infty$. This term does not change even if entrainment is present.

$$S_K = w f_b = wg\left(\frac{T' - T}{T}\right)$$

$$\frac{dK'}{dz} = -\frac{2}{H_e}K' + \frac{S_K}{w} \quad \longrightarrow \quad \frac{dK'}{dz} = g\left(\frac{T' - T}{T}\right) - \frac{2}{H_e}K'$$

The sink of K' on the right-hand side of the equation represents turbulent drag exerted on the ascending parcel due to the incorporation of momentum from the environment.

Entrainment reduces the parcel's acceleration from what it would be under the action of buoyancy alone, thereby limiting its penetration into a stable layer to approximately one entrainment height, H_e.

lapse rate modification due to entrainment -1

Consider a mass m of saturated cloudy air which rises from a level z, entraining a mass dm of the environmental air over a distance dz.

The cloudy air has a temperature T', and the environmental air has a temperature T.

We will apply the First Law of Thermodynamics to the mixture $m + dm$ assuming that no heat transfer mechanisms occur other than condensation, evaporation, and mixing.

$$dh = \delta q + vdp$$

$$c_p dT' - RT' \frac{dp}{p} = \delta q$$

The latent heat consumed to evaporate just enough liquid water from the cloudy air to fully saturate the mixed parcel.

$$m\left(c_{pd} dT' - R_d T' \frac{dp}{p}\right) = -mL_{lv} dq_s - c_{pd}(T' - T)dm - L_{lv}(q_s' - q_v)dm$$

The latent heat released by the cloudy parcel during ascent.

Sensible heat required to warm the entrained environmental air.

lapse rate modification due to entrainment - 2

By following a procedure similar to that used in the derivation of the saturated adiabatic lapse rate, we obtain an expression for the lapse rate of a saturated parcel subject to entrainment.

$$\Gamma_m = \Gamma_s + \frac{\frac{1}{m}\frac{dm}{dt}\left[(T' - T) + \frac{L_{lv}}{c_{pd}}(q_s' - q_v)\right]}{1 + \frac{\varepsilon L_{lv}^2 q_s}{c_{pd} R_d T^2}}$$

Γ_m reduces to Γ_s if $dm/dz = 0$, i.e., if no entrainment takes place.

When $dm/dz > 0$ and the parcel is warmer than the environment ($T' > T$), then $\Gamma_m > \Gamma_s$.

In effect, the mixing of cloudy air with cooler, dry environmental air reduces the density difference between the parcel and its environment, thereby diminishing the buoyancy force. Since the lapse rate in an entrained cloud is greater than the saturated adiabatic lapse rate, an entraining cloud achieves a smaller vertical velocity compared to that predicted by simple parcel theory.

The process of environmental air entrainment into vertically developing clouds is not adequately understood at present, making difficult to determine the entrainment rate, dm/dz.

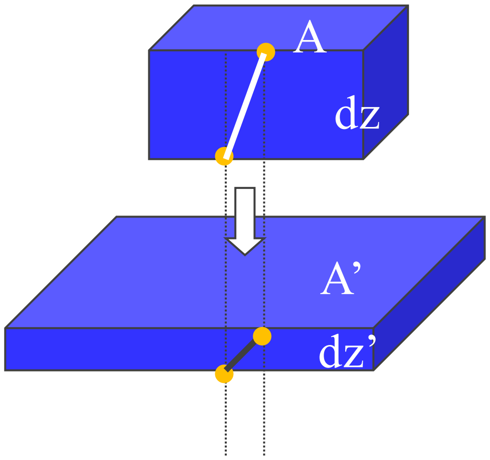
Processes producing changes in stability

The static stability of an atmospheric layer is modified by primary two factors:

- vertical motion within the layer
- differential heating or cooling of the layer.

Modification of stability due to vertical motions in a layer -1

Consider the large-scale ascent of a dry atmospheric layer, during which the mass of the layer remains constant (i.e., there is no horizontal or vertical mass convergence).



From the definition of potential temperature: $\theta = T\left(\frac{p_0}{p}\right)^{\kappa}$

$$\frac{d\ln\theta}{dz} = \frac{1}{T}\frac{dT}{dz} - \frac{R_d}{c_{pd}}\frac{1}{p}\frac{dp}{dz}$$

$$\frac{dp}{dz} = -\rho g$$

$$p = R_d T \rho$$

$$\frac{1}{\theta}\frac{d\theta}{dz} = \frac{1}{T}\frac{dT}{dz} + \frac{1}{T}\frac{g}{c_{pd}}$$

$$\frac{1}{\theta}\frac{d\theta}{dz} = -\frac{\Gamma_d - \Gamma_{env}}{T}$$

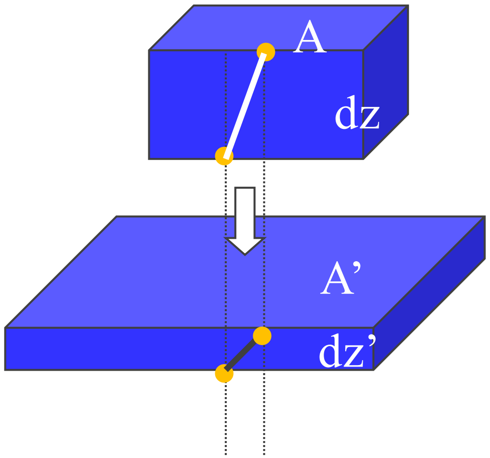
Γ_{env} denotes the environmental lapse rate.

For simplicity, virtual temperature corrections are neglected by assuming dry air.

Modification of stability due to vertical motions in a layer -2

In the preceding equation, we express dz in terms of dp using the hydrostatic equation.

$$\frac{1}{\theta}\frac{d\theta}{dp} = -\frac{R_d}{g}\frac{\Gamma_d - \Gamma_{env}}{p}$$



During ascent or descent of the layer, the derivative $d\theta/dp$ remains constant following the motion, since θ is conserved in dry adiabatic process and the mass of the layer is invariant.

Hence, we can write:

$$\Gamma_d - \Gamma_{env} = C_1 p$$

C_1 is constant

As a layer ascents, its lapse rate approaches the dry adiabatic lapse rate (Γ_d).

An initially stable layer becomes less stable.

An initially unstable layer becomes more stable

The reverse occurs during the descent of a layer as pressure increases, causing the layer's lapse rate to deviate further from Γ_d.

Potential instability -1

Changes in the layer's thermodynamic properties alter its stability.

The vertical stratification of moisture plays a key role in this proces, as different levels do not necessarily achieve saturation simultaneously.

Consider an unsaturated layer in which :

- potential temperature increases with height (a stably stratified layer)
- equivalent potential temperature decreases with height.

$$\frac{d\theta}{dz} > 0\,, \qquad \frac{d\theta_e}{dz} < 0 \quad \longrightarrow \quad d(\theta_e - \theta) < 0$$

The difference $\theta_e - \theta$ for an individual air parcel represents the total latent heat energy available for release through condensation.

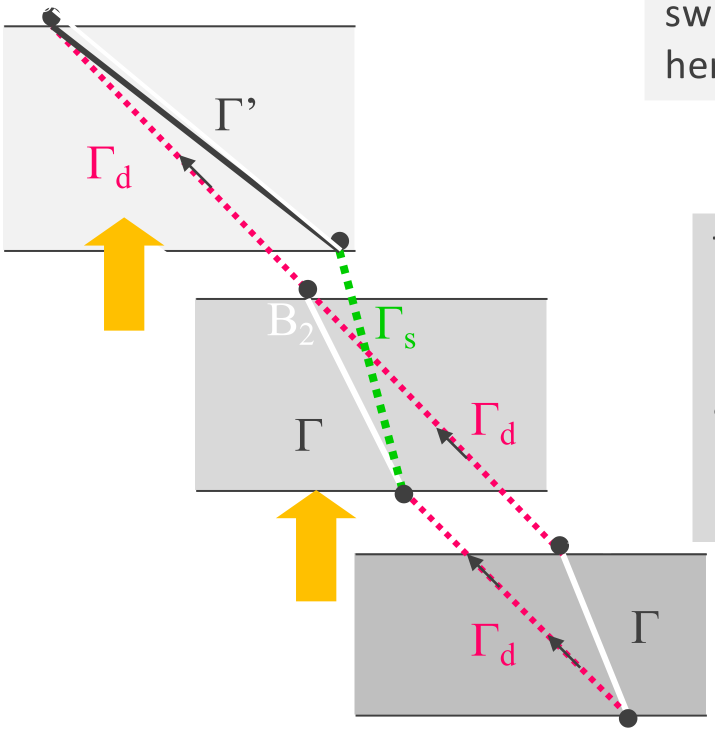
$$\theta_e - \theta = \theta \frac{L_{lv} q_v}{c_{pd} T}, \qquad \theta_e - \theta = \frac{L_{lv} q_v}{c_{pd}} \left(\frac{p_0}{p}\right)^{\kappa}$$

$$\theta_e = \theta \exp\left(\frac{L_{lv} q_s}{c_{pd} T}\right) \cong \theta \left(1 + \frac{L_{lv} q_s}{c_{pd} T}\right)$$

The specific humidity decreases with height.

$$\frac{d}{dz}(\theta_e - \theta) < 0 \;\longrightarrow\; \frac{dq_v}{dz} < 0$$

Potential instability -2



Differential cooling between lower and upper levels swings the temperature profile counterclockwise and hence destabilizes the layer.

The lower levels achieve saturation sooner than upper levels because they possess greater water vapor mixing ratios. Consequently, the lower levels cool more slowely at the saturated adiabatic lapse rate (Γ_s), whereas the upper levels continue to cool at the dry adiabatic lapse rate (Γ_d).

The layer is initially unsaturated. Air parcels along a vertical section all cool at the dry adiabatic lapse rate, Γ_d. The layer’s temperature profile shape is preserved through the displacement, as are the layer’s lapse rate (Γ) and stability.

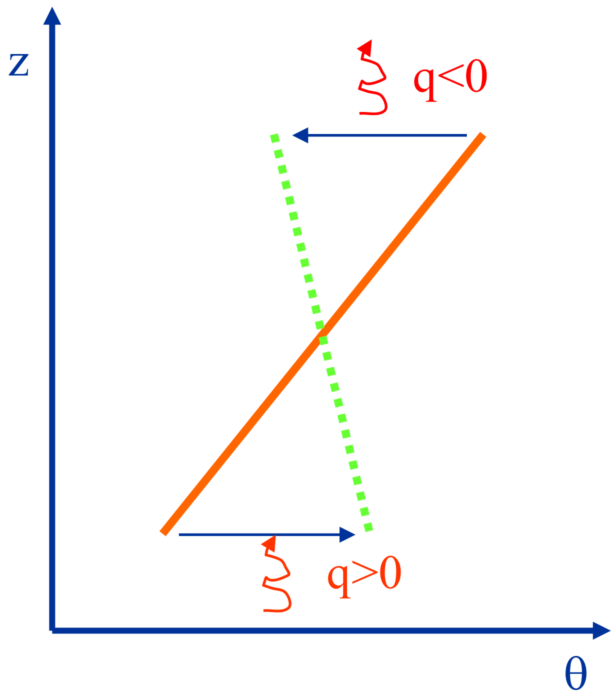
Potential instability -3

A layer characterized by $\frac{d\theta}{dz} < 0$, $\frac{d\theta_e}{dz} > 0$ is potentially stable.

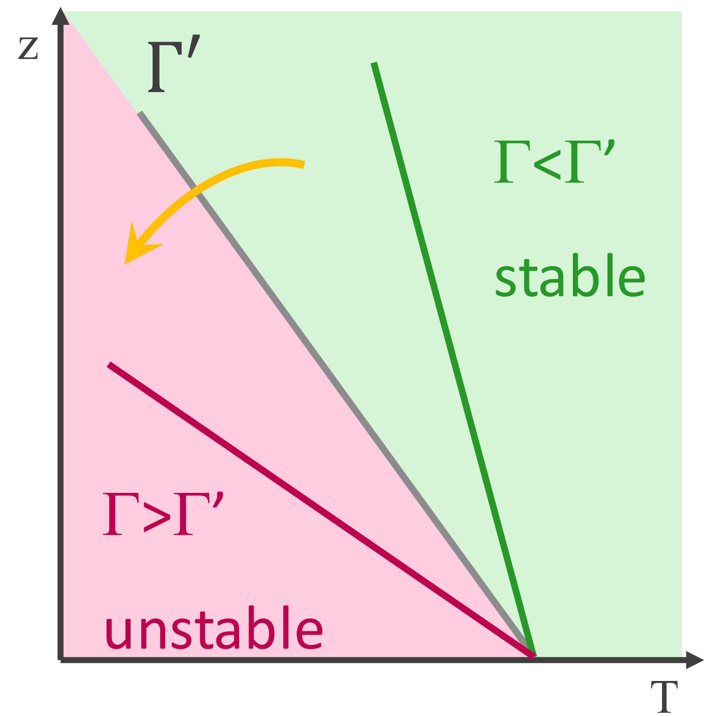
A layer characterized by $\frac{d\theta}{dz} > 0$, $\frac{d\theta_e}{dz} < 0$ is potentially unstable.

Destabilizing influences

The emission of longwave radiation at upper levels cools the troposphere from above.

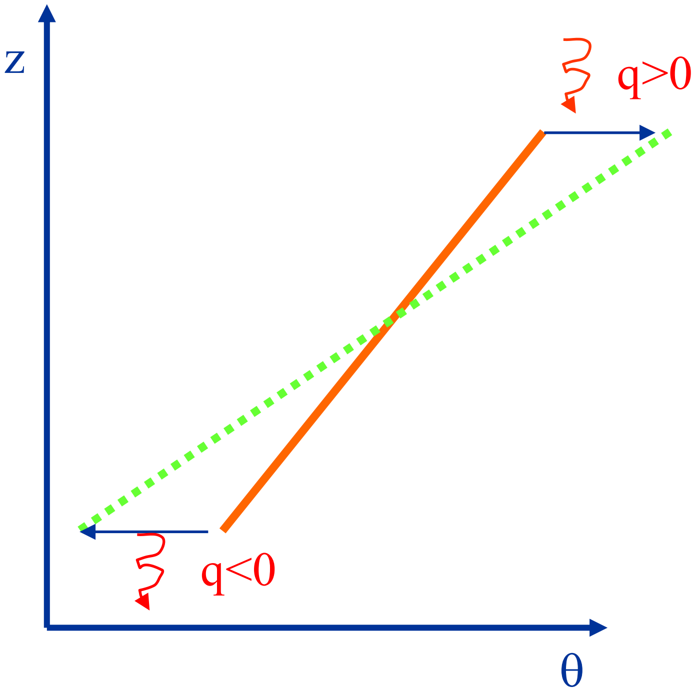


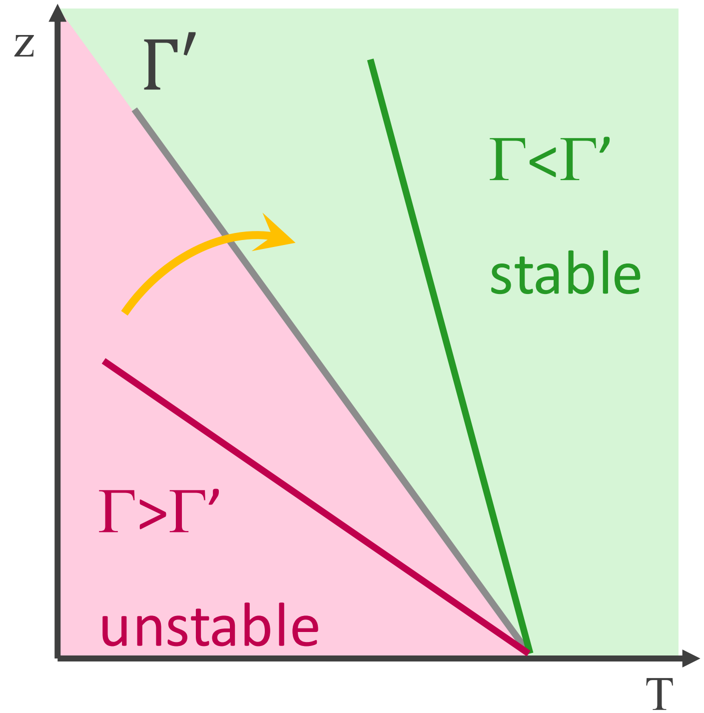
The absorption of longwave radiation and transfer of latent and sensible heat from the Earth's surface warm the troposphere from below.



Heating from below and cooling from above act to rotate the potential temperaturę profile counterclockwise, thereby destabilizing the layer.

Stabilizing influences





Cooling from below and heating from above act to rotate the potential temperature profile clockwise, thereby stabilizing the layer.

Factors modyfying the stability