LECTURE OUTLINE

1. Ways of reaching saturation
   - Isobaric processes:
     - Isobaric cooling (dew-point temperature)
     - Isobaric and adiabatic cooling by evaporation of water (wet-bulb temperature)
     - Adiabatic and isobaric mixing

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Ways of reaching saturation
formation and dissipation of clouds

For simplicity, we assume that clouds form in the atmosphere when water vapor reaches saturation, and that f=100% (in reality, this value should be greater than 100%).

$$f = \frac{e}{e_s(T)} \cong \frac{q_v}{q_s(p,T)}$$

An increase in relative humidity can be accomplished by:

- increasing the amount of water vapor in the air (i.e., increasing q_v); evaporation of water from a surface or from rain falling through unsaturated air,
- cooling of the air (i.e., decreasing of $q_s(p,T)$); isobaric cooling (e.g. radiative), adiabatic cooling of rising air,
- mixing of two unsaturated parcels of air.

THE COMBINED FIRST AND SECOND LAWS WILL BE APPLIED TO IDEALIZED THERMODYNAMIC REFERENCE PROCESSES ASSOCIATED WITH THE PHASE CHANGE OF WATER:

1. isobaric cooling
2. adiabatic isobaric processes
3. adiabatic and isobaric mixing

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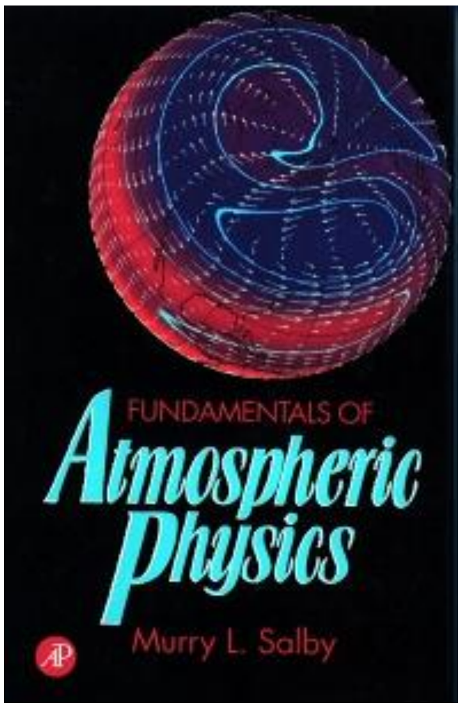
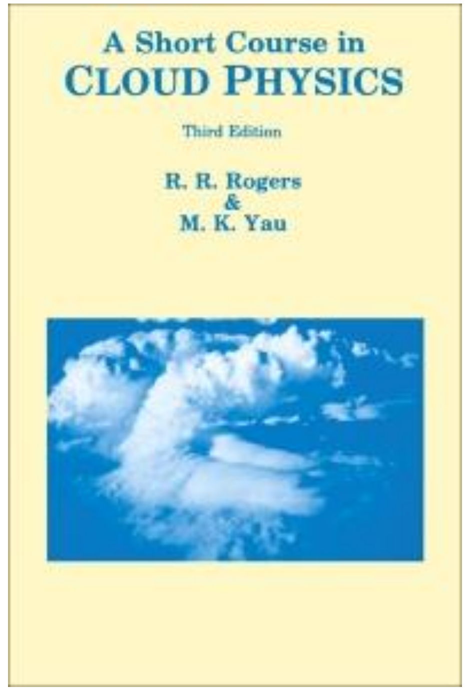
Salby, Chapter 5



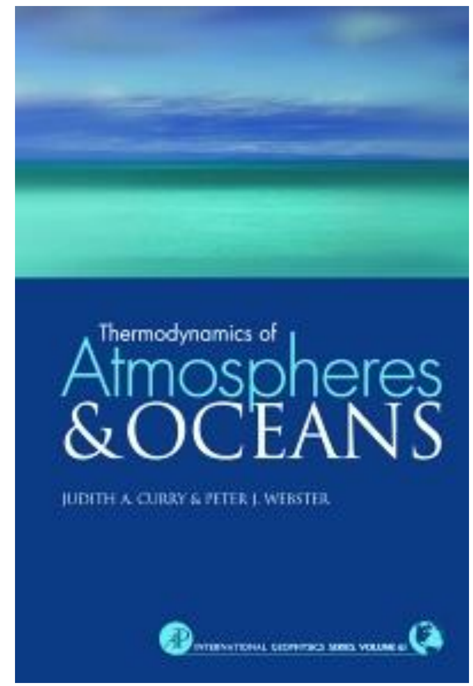
Fundamentals of Atmospheric Physics,
M.L. Salby; Salby

R&Y, Chapter 2



A Short Course in Cloud Physics,
R.R. Rogers and M.K. Yau; R&Y

C&W, Chapter 6



Thermodynamics of Atmospheres
and Oceanes,
J.A. Curry and P.J. Webster; C&W

ISOBARIC PROCESSES LEADING TO SATURATION OF THE AIR WITH WATER VAPOR

1. Isobaric cooling ($p=const$, $q_v=const$)
   - Dew point temperature
   - Isobaric cooling with condensation ($p=const$, $q_t=const$)
2. Isobaric and adiabatic cooling/heating by evaporation/condensation of water (*p=const, a source of water vapor / water)*
   - Wet bulb temperature
   - Equivalent temperature
3. Isobaric and adiabatic mixing ($p=const$, $q_t=const$)

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ISOBARIC COOLING

If vertical movement in the atmosphere is insignificant and the departure from the initial state is small, the processes can be concidered isobaric.

If there is no condensation, the First Law of Thermodynamics is:

$$\delta q = dh = c_p dT$$

The value of c_p for moist air is given by $c_p = c_{pd}(1 + 0.87 q_v)$, although it can be approximated by its dry air equivalent c_{pd}.

If the air is cooled, the relative humidity (f) will increase:

$$f = \frac{e}{e_s(T)} \cong \frac{q_v}{q_{vs}(p, T)}$$

- q_v does not change;
- the temperature decreases, therefore the saturation water vapor pressure (e_s) decreases
- because the process is isobaric ($p = const$), the same is true for q_s, i.e., q_s decreases.

Cooling leads to saturation (f=1) and then condensation starts.

Dew point temperature - T_d

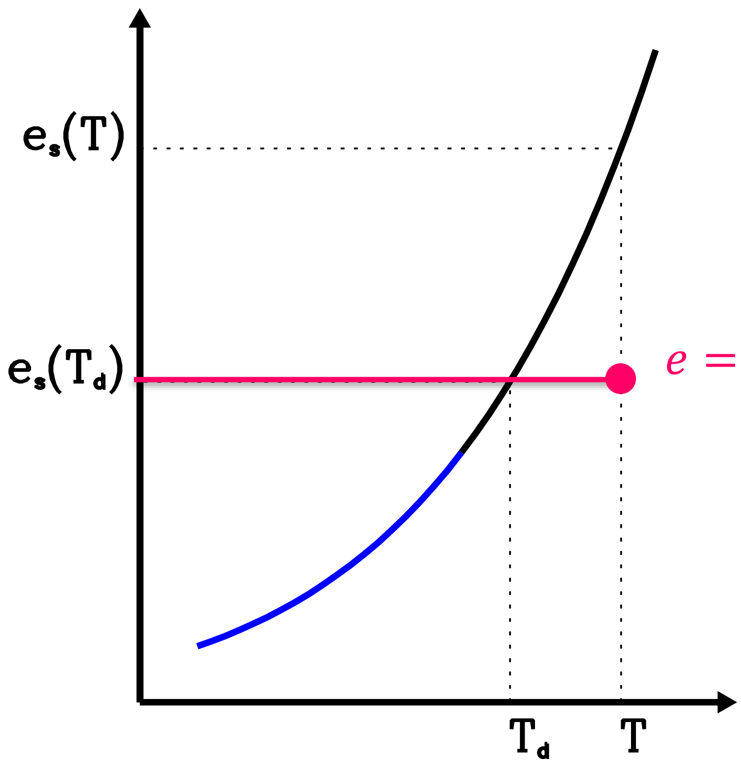
The dew point T_d is defined as the temperature to which the system must be cooled isobarically to achieve saturation

The dew point temperature remains unchanged during an isobaric process without condensation.

If saturation occurs below 0°C, this temperature is called the frost point T_f .

The dew point temperature is defined as: $e = e_s(T_d)$ or $q_v = q_s(T_d)$, where e is the actual pressure of water vapor in the air at temperature T, $q_v = const$ is the actual specific humidity.

T_d can be calculated from the equation: $$e = e_s(T)\exp\left[\frac{L_{lv}}{R_v}\left(\frac{1}{T} - \frac{1}{T_d}\right)\right]$$



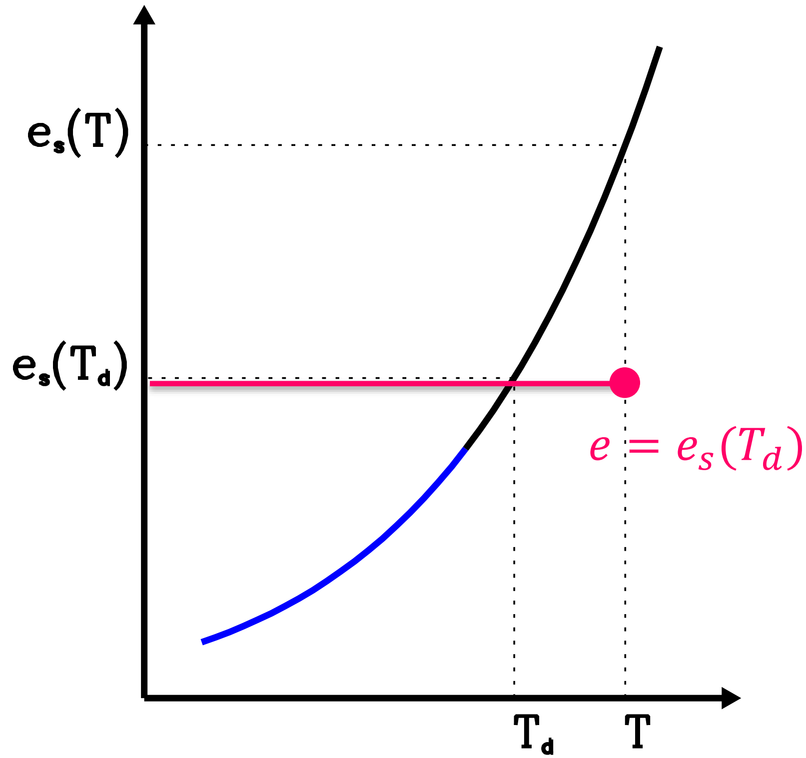
$e = e_s(T_d)$

e is the actual water vapor pressure in the air at temperature T

The unit of dew point temperature unit is the kelvin.

Dew point temperature is not a measure of temperature, but rather a measure of humidity in the air (relative humidity, f) .

We will integrate the Clausius-Clapeyron equation between temperatures T and T_d .



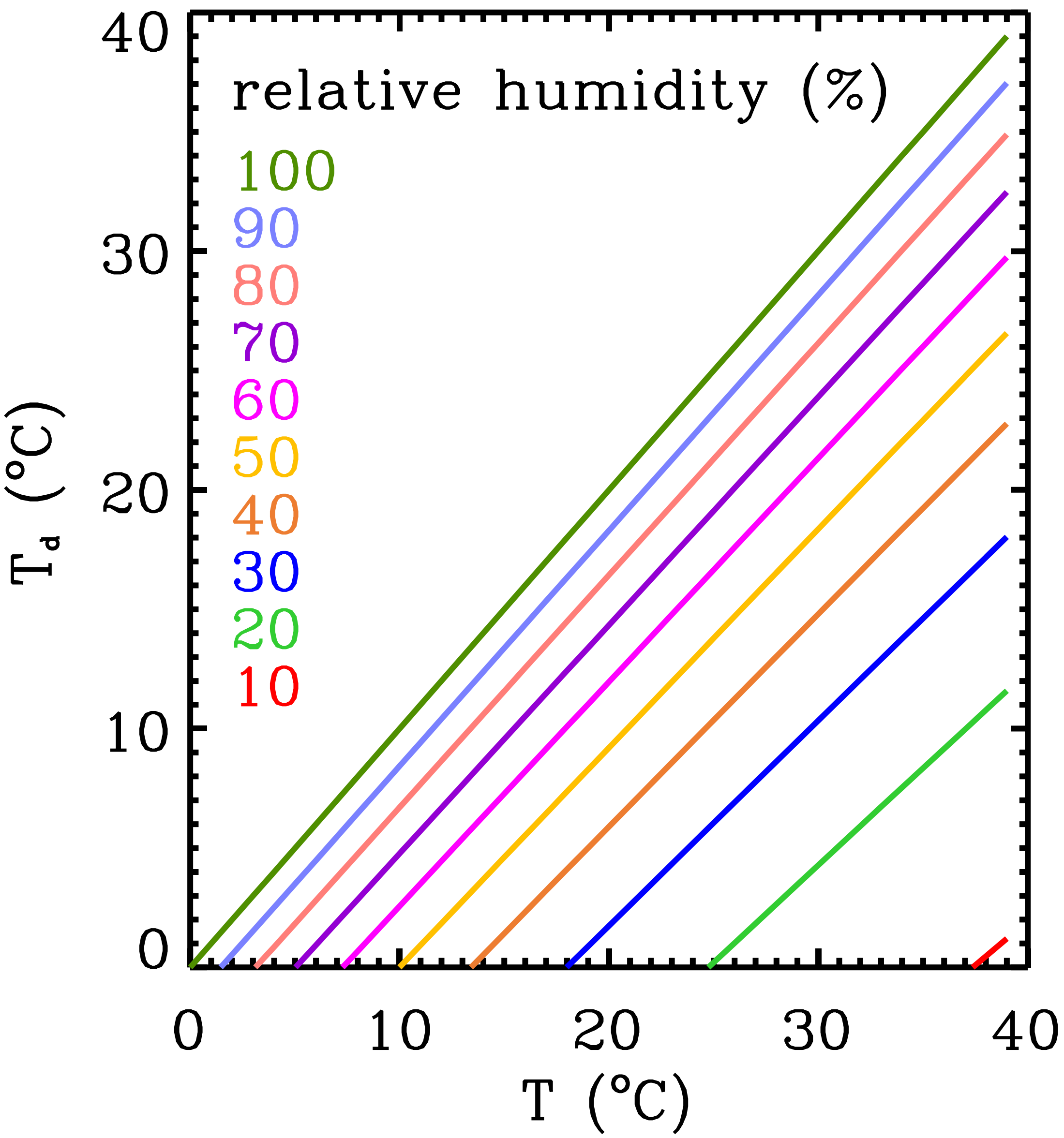
$$\frac{d\ln p}{dT} = \frac{L_{lv}}{R_v T^2}$$

$$\int_{e_{s(T)}}^{e} d\ln p = \int_{T}^{T_d} \frac{L_{lv}}{R_v T^2} dT$$

$$\ln\left(\frac{e}{e_s(T)}\right) = \ln f = \frac{L_{lv}}{R_v}\left(\frac{1}{T} - \frac{1}{T_d}\right)$$ We assume $L_{lv} = const.$

$$f = \exp\left[-\frac{L_{lv}}{R_v}\left(\frac{T - T_d}{T \cdot T_d}\right)\right]$$

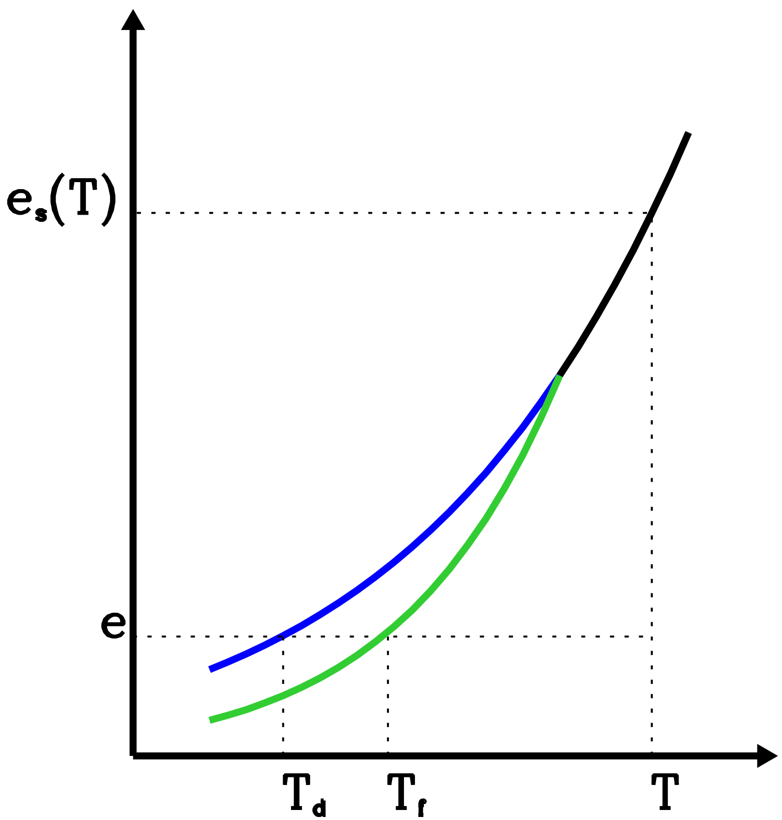
$T - T_d$ is known as the dew point depression or the dew point spread.


relative humidity (%)
100
90
80
70
60
50
40
30
20
10
0
10
20
30
40
0
10
20
30
40
T_d (°C)
T (°C)

Dew point temperature: $e = e_{s0} \exp\left[\frac{L_{lv}}{R_v}\left(\frac{1}{T_0} - \frac{1}{T_d}\right)\right]$

Frost point temperature: $e = e_{s0} \exp\left[\frac{L_{iv}}{R_v}\left(\frac{1}{T_0} - \frac{1}{T_f}\right)\right]$

T_f: $e = e_{si}(T_f)$ or $q_v = q_{si}(T_f)$



Isobaric cooling with condensation

If the air is cooled isobarically below the dew point temperature (T_d), condensation begins.

First Law of Thermodynamics : $dh = \delta q + v dp$

Enthalpy for two-component system: $dh = c_p dT + L_{lv} dq_v$

For isobabaric process dp=0: $dh = \delta q$

$$\delta q = c_p dT + L_{lv} dq_v$$

Let's assume that condensation always occurs at saturation (f=1):

- the specific humidity is equal to its value at saturation ($q_v = q_s$),
- $q_t = q_s + q_l$

In a closed system q_t does not change (i.e., $dq_t = 0$) , and $dq_l = -dq_s$.

The amount of water, dq_l, that condenses during isobaric cooling:

$$dq_l = -dq_s \cong -\frac{\varepsilon}{p} de_s = -\frac{\varepsilon}{p} \frac{L_{lv} e_s}{R_v T^2} dT = -\frac{\varepsilon^2 L_{lv} e_s}{p R T^2} dT$$

$$q \cong \varepsilon \frac{e_s}{p}$$

$$\frac{de_s}{dT} = \frac{L_{lv} e_s}{R_v T^2}$$

$$\varepsilon = \frac{R}{R_v}$$

In the First Law equation for an isobaric process $\delta q = c_p dT + L_{lv} dq_v$

- we express dq_v as a function of dT (see previous slide): $dq_v = \frac{\varepsilon^2 L_{lv} e_s}{pRT^2} dT$

to find how the temperature changes if heat δq is added:

$$\delta q = \left(c_p + \frac{\varepsilon^2 L_{lv}^2 e_s}{pRT^2} \right) dT$$

Before condensation occurs $\delta q = c_p dT$.

Once condensation begins, the temperature decreases more slowly during isobaric cooling due to the release of latent heat of condensation.

$$dT = \frac{\delta q}{c_p + \frac{\varepsilon^2 L_{lv}^2 e_s}{pRT^2}}$$

In the First Law equation for an isobaric process $\delta q = c_p dT + L_{lv} dq_v$

- we express dT as a function of dq_l (see previous slide): $dT = -\frac{pRT^2}{\varepsilon^2 L_{lv} e_s} dq_l$

to determine how much heat must be removed from the system to condense a given amount of water, dq_l.

$$\delta q = -c_p \frac{pRT^2}{\varepsilon^2 L_{lv} e_s} dq_l - L_{lv} dq_l$$

$$dq_l = -\left(\frac{\varepsilon^2 L_{lv} e_s}{c_p pRT^2 + \varepsilon^2 L_{lv}^2 e_s}\right) \delta q$$

This relation must be integrated numerically because e_s is temperature-dependent.

Once condensation begins, the dew-point temperature decreases, since the water vapor mixing ratio decreases as water condenses.

Relative humidity remains constant, at $f = 1 \longrightarrow T_d = T$

$$\left(f = \exp\left[-\frac{L_{lv}}{R_v}\left(\frac{T - T_d}{T \cdot T_d}\right)\right]\right)$$

Isobaric cooling is a primary formation mechanism for certain types of fog and stratus clouds.

The derived equations are equally applicable to isobaric heating. In this instance, an existing cloud or fog can be dissipated by evaporation that ensues from isobaric heating (e.g., solar radiation).

ADIABATIC AND ISOBARIC COOLING AND MOISTENING BY WATER EVAPORATION

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wet-bulb temperature, T_w

Consider a system composed of unsaturated moist air and rain falling through it. Because the air is undersaturated, the rain will evaporate.

Assuming there are no external heat sources (Δq=0) and the evaporation occurs isobarically (dp=0), the enthalpy of the system is conserved (dh=0):

$$dh = c_p dT + L_{lv} dq_v$$

$$0 = c_{pd} dT - L_{lv} dq_l$$

$$dh = 0$$
$$dq_v = -dq_l$$

$c_p = c_{pd}(1 + 0.87 q_v)$ or can be approximated as the dry-air value c_{pd}

If we allow just enough liquid water from the rain to evaporate so that the air becomes saturated ($q_v = q_s$), we can integrate the above equation:

- from the state where there is q_l of liquid water at temperature T
- to the state where all liquid evaporates ($q_l = 0$) and the temperature decreases to T_w.

$$c_p \int_T^{T_w} dT = \int_{q_l}^{0} L_{lv} dq_l$$

q_l is the amount of water that must be evaporated to bring the air to saturation at temperature T_w.

During the evaporation process, latent heat is drawn from the atmosphere, and the final temperature, referred to as the wet-bulb temperature, T_w , is cooler than the original temperature.

The wet-bulb temperature – the temperature to which air may be cooled by evaporating water into it at constant pressure, until saturation is reached.

$$c_p \int_{T}^{T_w} dT = \int_{q_l}^{0} L_{lv} dq_l$$

$$c_p(T_w - T) = -L_{lv} q_l, \qquad q_l = q_s(T_w, p) - q_v$$

$$T_w = T - \frac{L_{lv}}{c_p} [q_s(T_w, p) - q_v]$$

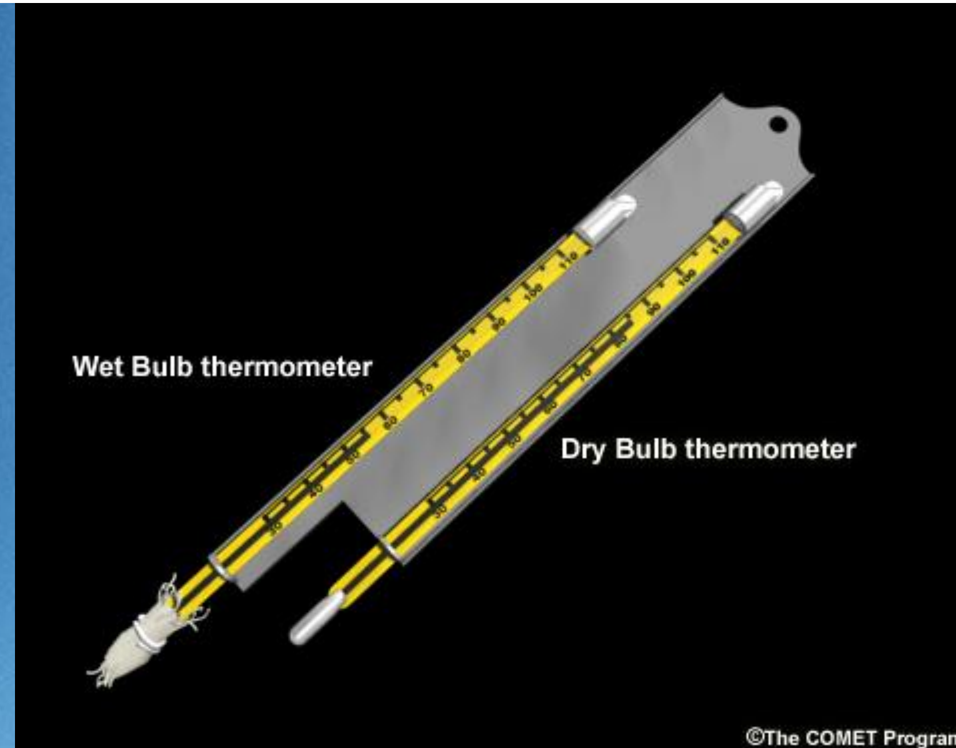
The temperature dependence of $L_{lv}(T)$ has been neglected. Given q_v, T, and p, this expression is implicit for T_w and must be solved numerically.

However, if T and T_w are given, then q_v is easily determined.

The wet-bulb temperature T_w is measured using a psychrometer.

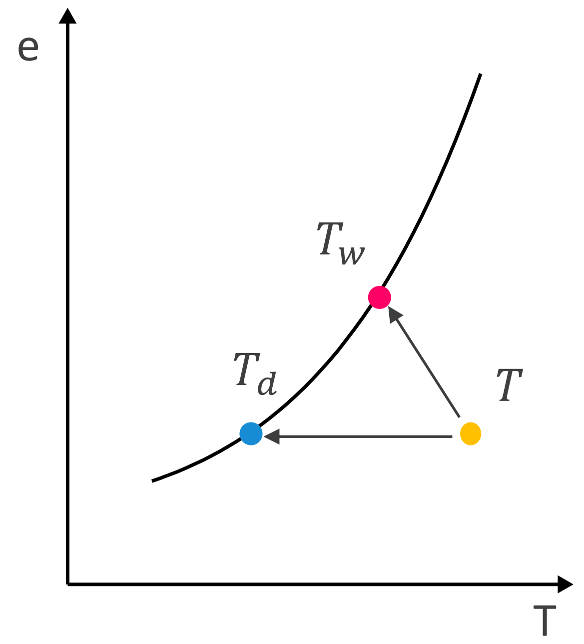
It consists of two thermometers, one of which is covered by a wet cloth and ventilated.

Assman’s psychrometer



The wet-bulb temperature in the atmosphere is conservative with respect to the evaporation of falling rain.

Calculations for given values of T and q_v show that $T_d < T_w < T$.



This can be illustrated graphically. Since e increases while T decreases as T approaches to T_w, the Clausius-Clapeyron diagram is as follows:

If ice is the evaporating phase, the ice-bulb temperature T_i can be determined analogously:

$$T_i = T - \frac{L_{iv}}{c_p}[q_s(T_i) - q_v]$$

It can be easily shown that $T_i > T_w$

equivalent temperature, T_e

Equivalent temperature (T_e) is defined as the temperature a sample of moist air would attain if all the moisture were condensed out at constant pressure without any heat transfer from/to the environment.

We use the same equation as before:

$$dh = c_p dT + L_{lv} dq_v$$

$$0 = c_{pd} dT + L_{lv} dq_s$$

$$dh = 0$$

$$dq_v = dq_s$$

The equation is integrated

- from a saturated state with water vapor q_s at temperature T
- to a state in which all water vapor condenses ($q_s = 0$) and the temperature increases to T_e:

$$c_p \int_T^{T_e} dT = -\int_{q_s}^{0} L_{lv} dq_s$$

$$c_p(T_e - T) = L_{lv} q_s$$

$$T_e = T + \frac{L_{vl} q_s}{c_p}$$

Equivalent potential temperature

We defined the equivalent potential temperature that is conserved in moist adiabatic/pseudo-adiabatic processes.

$$\theta_e = T\left(\frac{p_0}{p}\right)^{\frac{R_e}{c_{pe}}} \Omega_e \exp\left(\frac{q_v L_{vl}}{c_{pe} T}\right)$$

$$\Omega_e = \left(\frac{R}{R_e}\right)^{\frac{R_e}{c_{pe}}} \left(\frac{e}{e_s}\right)^{-\frac{q_v R_v}{c_{pe}}}$$

$$c_{pe} = c_{pd} + q_t(c_l - c_{pd})$$

$$R_e = (1 - q_t)R_d$$

e/e_s defines the relative humidity. The term Ω_e is close to 1 and depends only very weakly on the thermodynamic state.
The expression for θ_e in the full form is complicated, but it is rarely used in this form for practical applications.
For many purposes far simpler expressions capture much of the essential physics.

$$\theta_e = \theta \exp\left(\frac{q_s L_{vl}}{c_p T}\right)$$

Equivalent temperature: $T_e = T + \frac{L_{vl} q_s}{c_p}$

Equivalent potential temperature $\theta_e \cong \theta \exp\left(\frac{q_s L_{vl}}{c_p T}\right)$

For small $L_{lv} q_s / c_p T$:

$$\exp\left(\frac{L_{vl} q_s}{c_p T}\right) \cong 1 + \frac{L_{vl} q_s}{c_p T} = \frac{1}{T}\left(T + \frac{L_{vl} q_s}{c_p}\right)$$

$$\theta_e = \frac{\theta}{T}\left(T + \frac{L_{lv} q_s}{c_p}\right) = \left(T + \frac{L_{lv} q_s}{c_p}\right) \cdot \left(\frac{p_0}{p}\right)^{R_d/c_{pd}} = T_e \left(\frac{p_0}{p}\right)^{R_d/c_{pd}}$$

$$\theta_e \cong T_e \left(\frac{p_o}{p}\right)^{R_d/c_{pd}}$$ It is a potential equivalent temperature.

ADIABATIC AND ISOBARIC MIXING

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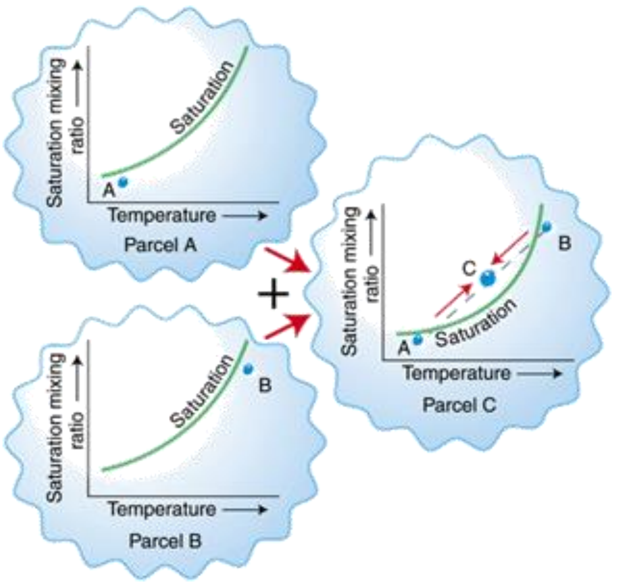
Uner certain conditions, the isobaric mixing of different unsaturated air masses can lead to fog formation (saturation).

Consider the isobaric mixing of two unsaturated air masses (Y_1 and Y_2) with different temperatures and water vapor contents.

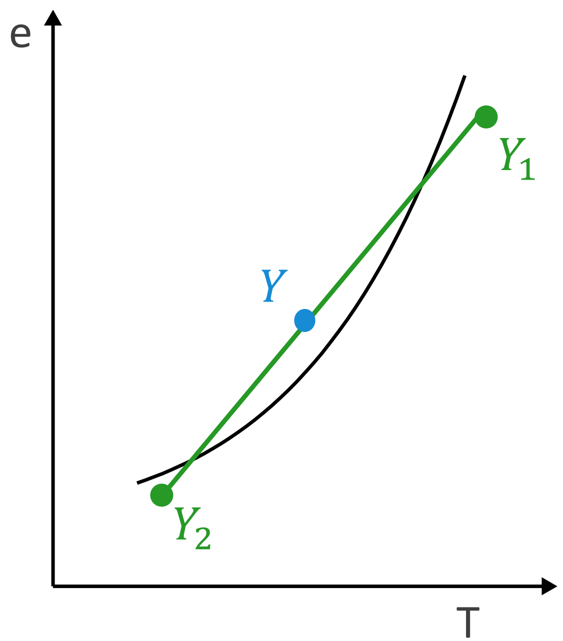
Assume no condensation occurs.



To describe the adiabatic and isobaric mixing, we use the First Law of Thermodynamics (in enthalpy form).

$$dH = \delta q + V dp$$

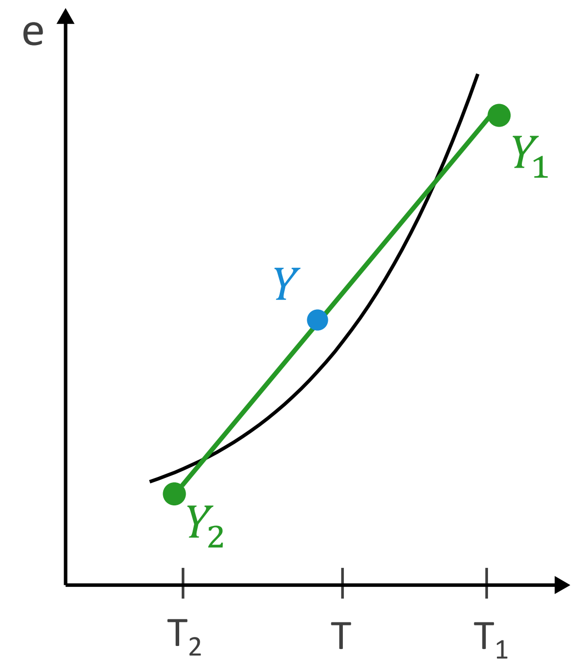
$$0 = dH \approx m_1 c_{pd} dT_1 + m_2 c_{pd} dT_2$$



dT_1 and dT_2 correspond to the temperature changes that occurs in response to the mixing process.

We neglect the contribution of water vapor to the heat capacity.

We will integrate the equation from the initial state defined by temperatures T_1 and T_2 to the final at temperature T.



$$m_1 c_{pd}(T - T_1) + m_2 c_{pd}(T - T_2) \approx 0$$

$$T \approx \frac{m_1}{m_1 + m_2} T_1 + \frac{m_2}{m_1 + m_2} T_2$$

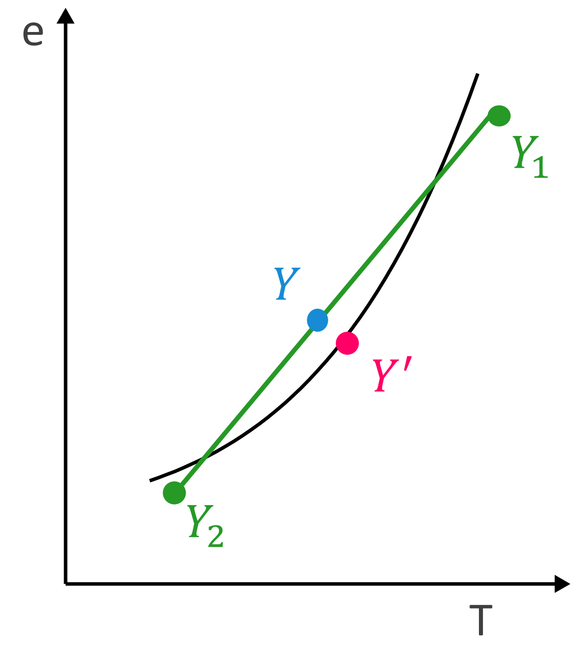
The total mass $m = m_1 + m_2$ of the system is conserved during mixing. The specific humidity of the mixture, q_v , is the weighted average of initial specific humidities q_{v1} and q_{v2}.

$$q_v = \frac{m_1}{m_1 + m_2} q_{v1} + \frac{m_2}{m_1 + m_2} q_{v2}$$

The partial pressures of water vapor also mix linearly because $q_v \approx \varepsilon\, e/p$, and the process is isobaric.

The values (e, T) at point Y' can be found by solving the former equation and the Clausius- Clapeyron equation simultaneously.

$Y \rightarrow Y'$ proces is adiabatic and isobaric.



The amount of water vapor condensed in this process is:

$$\Delta q_l = \frac{\varepsilon}{p}[e(Y) - e(Y')]$$

An example of cloud formation resulting from adiabatic and isobaric mixing:

- contrails
- mist/haze formed in exhaled air

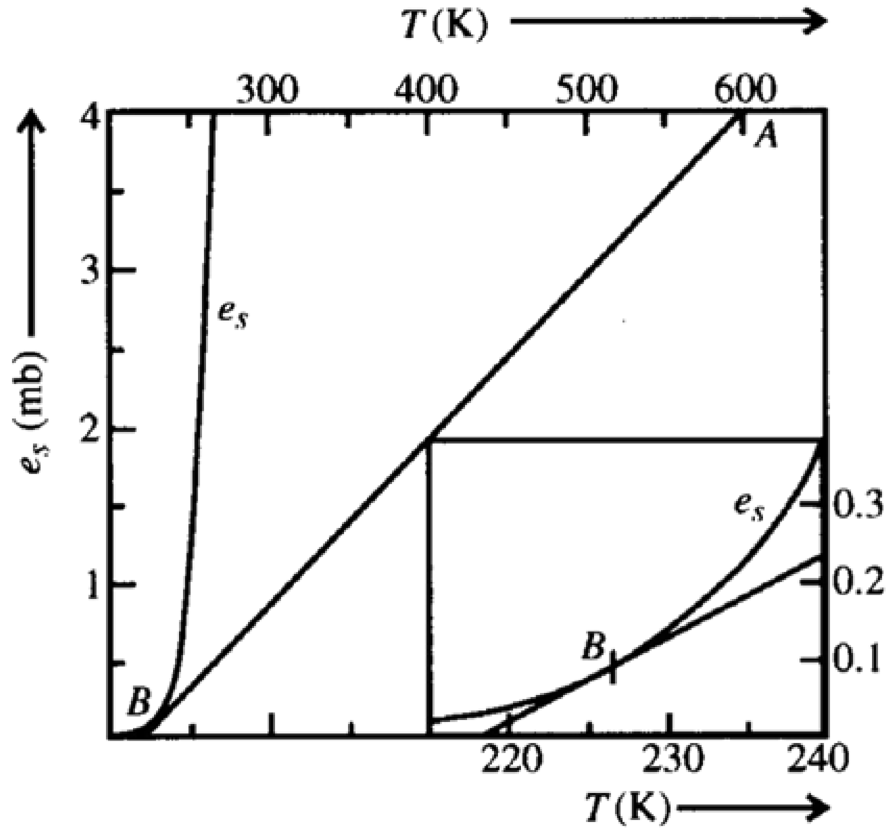


Fig. 6.4 C&W

A plane flying at 200 mb ejects water vapor into the atmosphere at the temperature and water vapor pressure represented by point A (600K, 4mb). For atmospheric temperatures less than -47^{o}C (226 K), the water vapor will condense forming condensation trails.