**Photonics – series 3** (Jones calculus, light polarization)

**Total: 36 points (27 points + 9 „beauty" points) + BONUS**

**Before solving the problems prepare functions returning:**
- Jones vector for arbitrary polarization vector (specifically right-handed and left-handed polarizations and arbitrary linear polarization).
- Jones matrix for linear polarizer rotated under arbitrary angle
- Jones matrix for phase retarder with arbitrary phase retardation and element rotation

**Problem 1 (3 points):**
*(Relation between polarization and electromagnetic wave propagation)*
Write a function that plots temporary electric field amplitude $\boldsymbol{E}(x,t)=\Re(\boldsymbol{E_0}\exp(i(kx-\omega t)))$ and magnetic field amplitude $\boldsymbol{B}(x,t)=\Re(\boldsymbol{B_0}\exp(i(kx-\omega t)))$ with $\boldsymbol{B_0}=\omega^{-1}\boldsymbol{k}\times\boldsymbol{E_0}$ for a given polarization. Plot the trajectories of electric and magnetic field vectors for given point in space.

**Problem 2 (3 points):**
*(Representation of polarization in circle-polarization basis)*
Unpolarized light was sent through a linear polarizer rotated at 45 degrees angle with respect to x axis, and then through a quarter-wave plate rotated at 30 degrees angle with respect to x axis. Represent this polarization as a linear combination of right- and left-circular polarization.

**Problem 3 (3 points):**
*(Polarization change during perpendicular reflection)*
Electric field amplitude of linearly polarized wave reflected from perpendicular surface changes by a factor $r=\frac{1-n}{1+n}$, which is reflection coefficient calculated via Fresnel equations. Prove that even though Jones vector is not conserved, linear polarization does not change. How is it possible? How does circular or elliptic polarization change?

**Problem 4 (3 points):**
*(Nematic liquid crystal model)*
Consider two linear polarizers with relative rotation angle θ. Calculate intensity transmission coefficient of this system illuminated with linearly polarized light with polarization axis matched to the rotation angle of the first polarizer. How this coefficient changes if, in between these polarizers, we locate N linear polarizers rotated by an angle θ/(N+1) to previous (and next) polarizer? Plot transmission coefficient as function of number of polarizers N. Find number of polarizers for which transmission reaches 0.95.

**Problem 5 (3 points):**
*(Liquid crystal light modulator)*
Prove that the system consisting of two perpendicular linear polarizers and a phase retarder between the polarizers allows to regulate light intensity by changing phase retardation. How does the modulation range changes with phase retarder rotation angle?

**Problem 6 (3 points):**
*(Circular dichroism)*
Circular dichroism appears when material absorption depends on circular polarization direction. Find a Jones matrix that turns arbitrary polarization into right-handed polarization.

**Problem 7 (3 points):**
*(„Antiglare”)*
Antiglare surface on a computer screen consists of linear polarizer and quarter-wave plate rotated by 45 degrees angle. How does this system work?

**Problem 8 (3 points):**
*(Arbitrary polarization transformation)*
Show that arbitrary polarization can be transformed into any other linear polarization using a system consisting of a quarter-wave plate and half-wave plate.

**Problem 9 (3 points):**
*(Faraday insulator)*
Magnetooptic Faraday effect in some materials leads to linear polarization rotation under influence of strong magnetic field parallel to propagation direction. Furthermore, polarization rotation direction doesn’t depend on propagation direction. Consider a system consisting of two linear polarizers and material between them, which rotates polarization under angle $\beta$ due to magnetooptic Faraday effect. Show how to construct system which transmits light in one direction and blocks light in opposite direction

**Problem 10 (BONUS - 10 points to ANY series):**
Show that arbitrary polarization can be transformed into any other polarization using only quarter-wave plates (QWP) and half-wave plates (HWP).
*Hint: 1) use QWP to tranform polarization into linear*
*2) use HWP to rotate linear polarization*
*3) use QWP to finish transformation*
Is it possible to solve the problem by using only two elements (QWP and HWP)?