

Energy cascades in the convective atmospheric boundary layer: analysis of structure functions

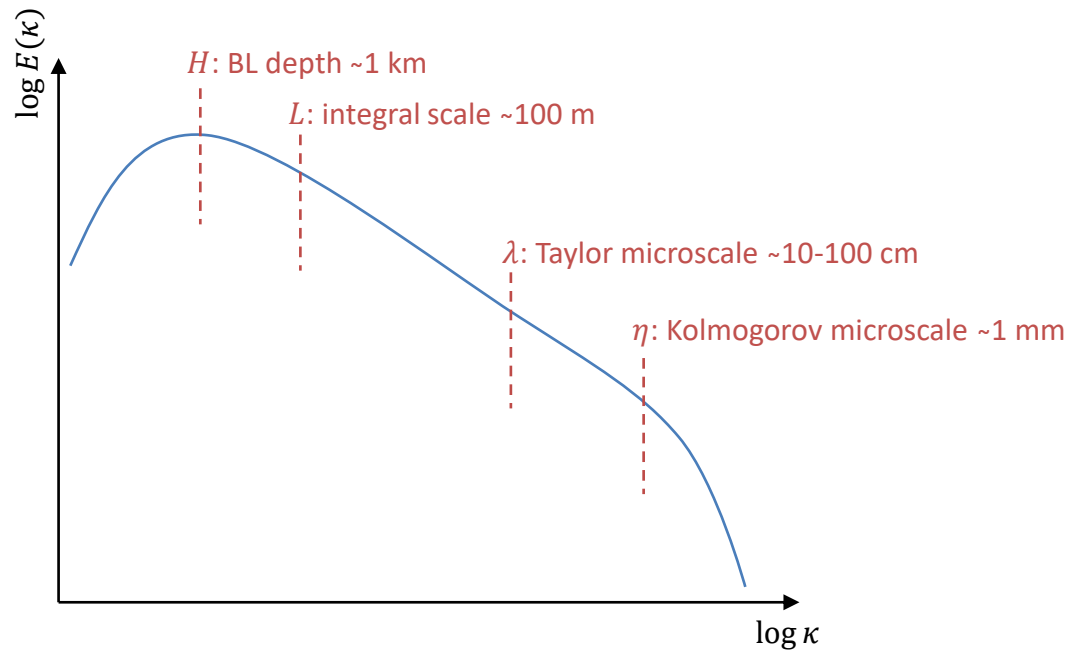
Jakub. L. Nowak¹, Marta Waćławczyk¹, John C. Vassilicos²,
Stanisław Król¹, Szymon P. Malinowski¹

¹Institute of Geophysics, Faculty of Physics, University of Warsaw, Poland

²Univ. Lille, CNRS, ONERA, Arts et Metiers Institute of Technology,
Centrale Lille, UMR 9014 –LMFL – Laboratoire de Mécanique des Fluides de Lille – Kampé de Fériet, Lille, France

LTP 2023 collaboration

Turbulence cascade in the atmosphere

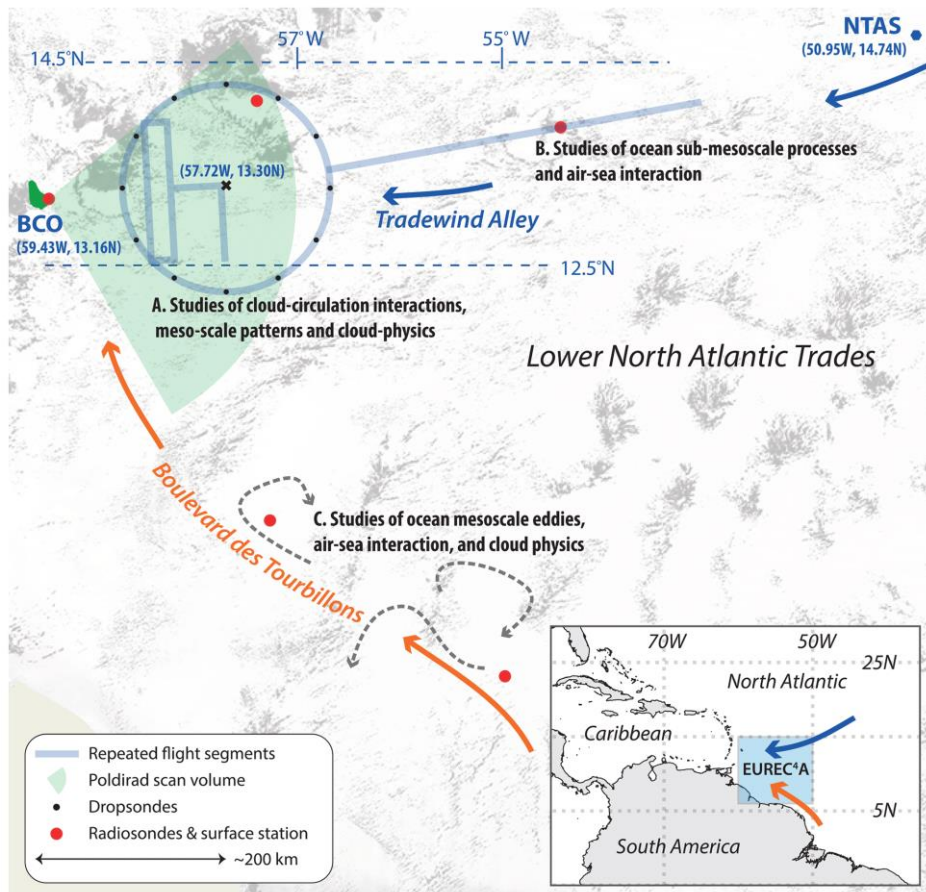


$$\overline{\delta u^2} \sim (\epsilon r)^{\frac{2}{3}}$$

$$\overline{\delta u^3} = -\frac{4}{5}\epsilon r \quad ?$$

$$\overline{\delta u \delta T} = 0 \quad ?$$

Field campaign EUREC4A 2020



Elucidating the role of clouds–circulation coupling in climate

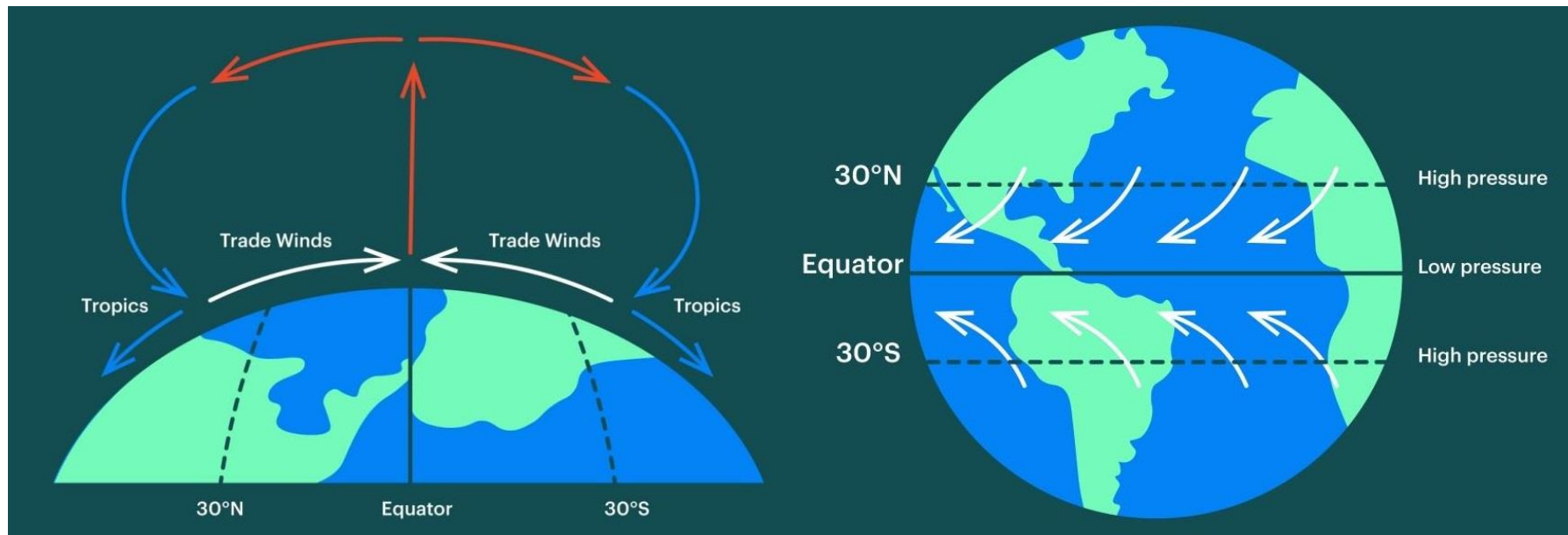
Jan-Feb 2020 in western Atlantic



French research aircraft SAFIRE ATR-42 (19 flights x 4 h)

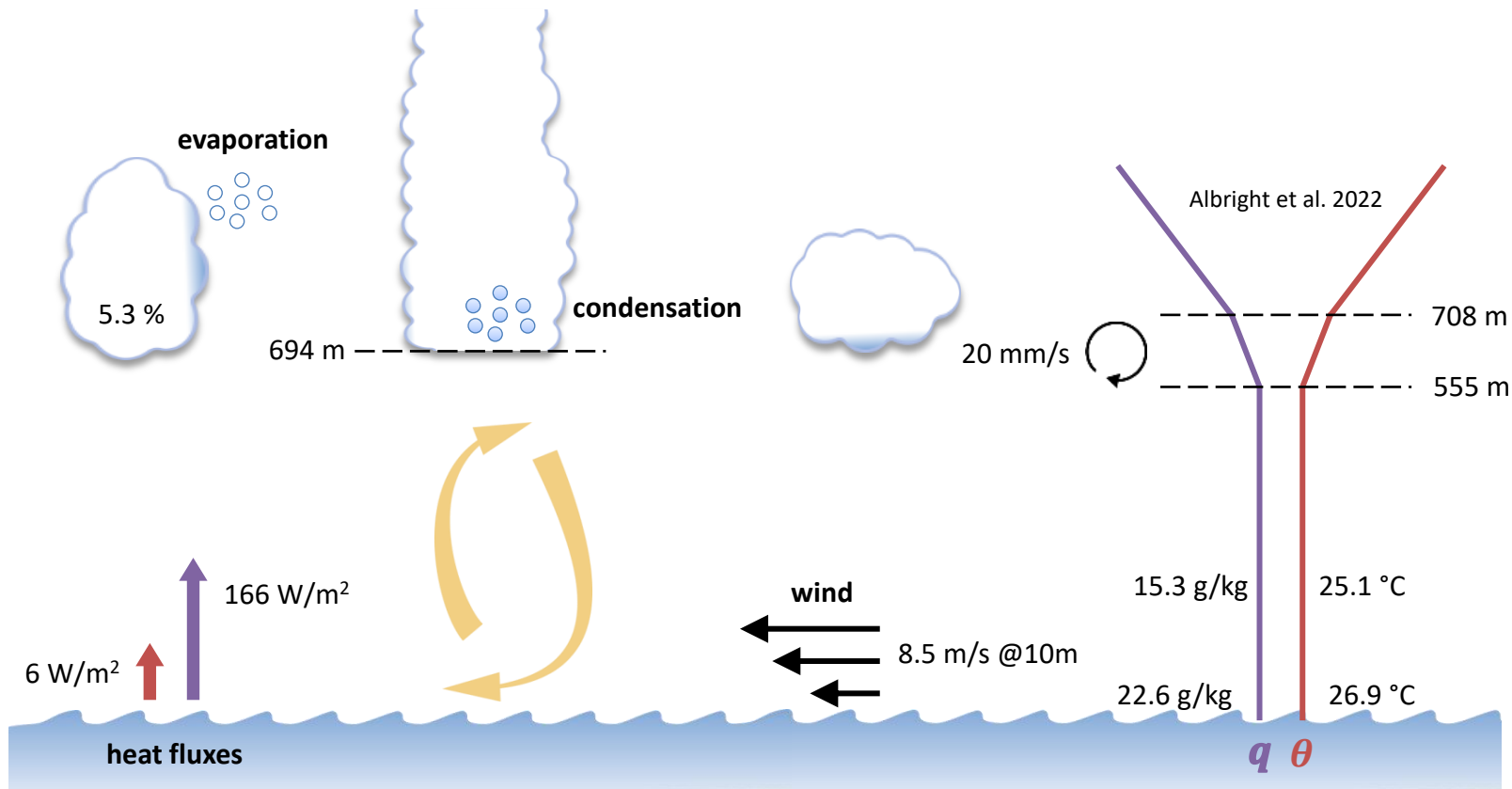
Stevens et al. 2021, Bony et al. 2022

Trade winds

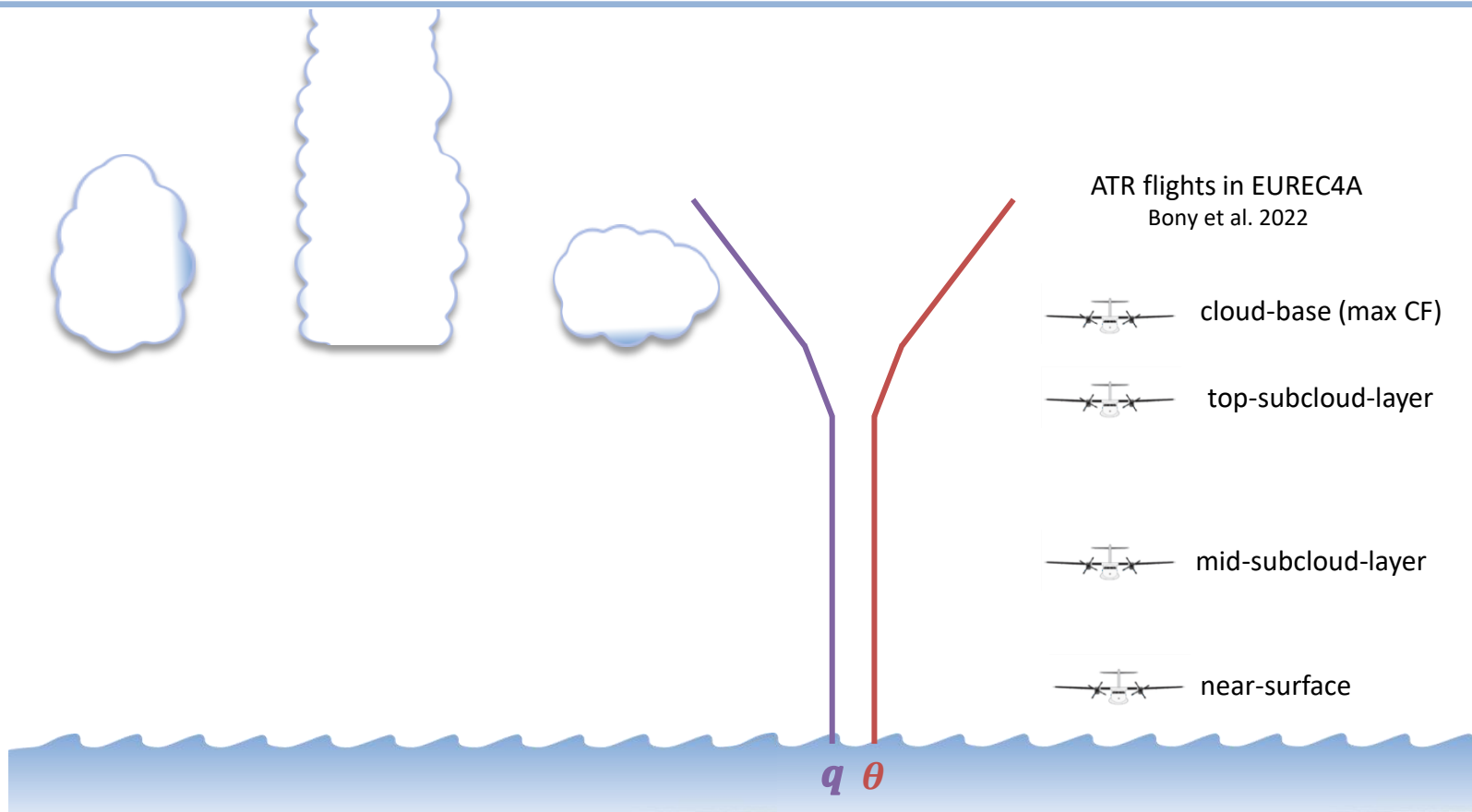


Valerya Milovanova / Windy.app

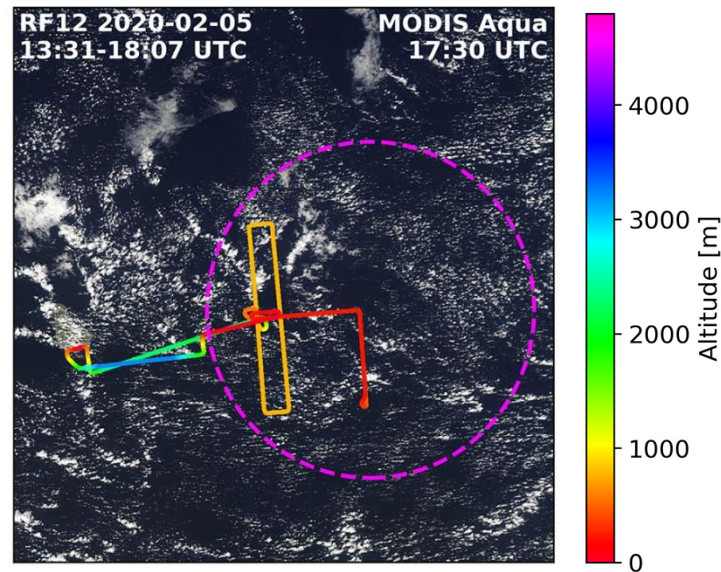
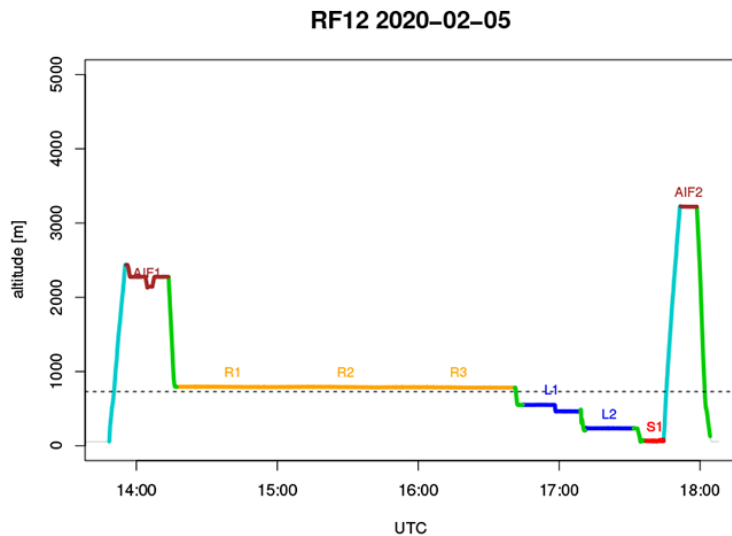
Shallow trade-wind convection



Research flights during EUREC4A



Flight segments



Bony et al. 2022, Brilouet et al. 2021

Level	#	Length [km]	Altitude [m]
cloud-base	114	54 (5)	807 (83)
top-subcloud	18	62 (10)	595 (47)
mid-subcloud	16	54 (7)	292 (27)
near-surface	9	40 (6)	64 (2)

Average values with std among segments given in parentheses.

Selected instrumentation

Instrument	Brief description	Position on ATR
5-hole radome	For measuring the differential pressure around the nose of the aircraft	radome
Pitot probes	Rosemount and Thales transducers connected to Pitot probes measuring static and dynamic pressure	fuselage
Fine wire	fine wire resistance for measuring fast temperature fluctuations	nose (left-hand side)
Licor 7500A	Near-infrared gas analyzer for measuring rapid humidity fluctuations	window (left-hand side)
KH20	Campbell krypton hygrometer for measuring rapid humidity fluctuations	nose (left-hand side)



$$\frac{100 \text{ m/s}}{25 \text{ Hz}} = 4 \text{ m}$$

$$\frac{50 \text{ km} \cdot 25 \text{ Hz}}{100 \text{ m/s}} = 12\,500 \text{ samples}$$

Scale-by-scale budget derivation (1)

$$B = g \frac{\theta_v - \widetilde{\theta}_v}{\widetilde{\theta}_v} \quad \text{buoyancy}$$

$$\theta_v = \theta(1 + 0.61q_v) \quad \text{virtual potential temperature}$$

$$\theta = T \left(\frac{p_0}{p} \right)^{R_d/c_p} \quad \text{potential temperature}$$

T : temperature, q_v : water vapor mass fraction, p : pressure, $p_0 = 1000$ hPa

Incompressible Boussinesq approximation

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = \nabla \pi + \nu \nabla^2 \mathbf{u} + \mathbf{k} B$$

$$\partial_t B + \mathbf{u} \cdot \nabla B = \nu \nabla^2 B$$

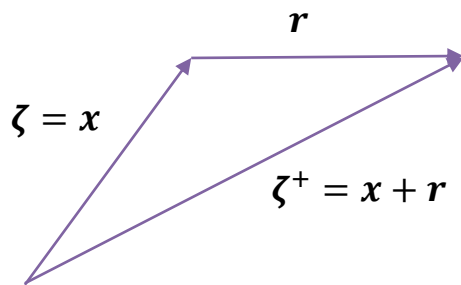
$$\nabla \cdot \mathbf{u} = 0$$

$$\pi = \frac{p}{\rho}, \rho: \text{reference density}, \mathbf{u} = (u, v, w)$$

$$B = g \frac{\theta - \tilde{\theta}}{\widetilde{\theta}_v} + 0.61g \frac{\theta q_v - \tilde{\theta} \widetilde{q}_v}{\widetilde{\theta}_v}$$

$B_\theta \qquad B_q$

Scale-by-scale budget derivation (2)



$$\mathbf{X} = \frac{1}{2}(\zeta + \zeta^+)$$

$$\delta \mathbf{u}(\mathbf{X}, \mathbf{r}) = \mathbf{u}(\zeta^+) - \mathbf{u}(\zeta)$$

$$\delta B(\mathbf{X}, \mathbf{r}) = B(\zeta^+) - B(\zeta)$$

$$\delta \pi(\mathbf{X}, \mathbf{r}) = \pi(\zeta^+) - \pi(\zeta)$$

$$\frac{\partial |\delta \mathbf{u}|^2}{\partial t} + \nabla_{\mathbf{X}} \cdot \mathbf{u}_{\mathbf{X}} |\delta \mathbf{u}|^2 + \nabla_{\mathbf{r}} \cdot \delta \mathbf{u} |\delta \mathbf{u}|^2 = -2 \nabla_{\mathbf{X}} \cdot \delta \mathbf{u} \delta \pi + \frac{\nu}{2} \nabla_{\mathbf{X}}^2 |\delta \mathbf{u}|^2 + 2\nu \nabla_{\mathbf{r}}^2 |\delta \mathbf{u}|^2 - 2\epsilon^+ - 2\epsilon + 2\delta B \delta w$$

horizontal homogeneity + stationarity

$$\frac{\partial}{\partial z} \overline{w_{\mathbf{X}} |\delta \mathbf{u}|^2} + \nabla_{\mathbf{r}} \cdot \overline{\delta \mathbf{u} |\delta \mathbf{u}|^2} = -2 \frac{\partial}{\partial z} \overline{\delta w \delta \pi} + 2\nu \nabla_{\mathbf{r}}^2 \overline{|\delta \mathbf{u}|^2} - 4\bar{\epsilon} + 2\overline{\delta B \delta w}$$

Valente and Vassilicos 2015

Scale-by-scale budget derivation (3)

$$2\overline{\delta B \delta w} - \nabla_r \cdot \overline{\delta \mathbf{u} |\delta \mathbf{u}|^2} - \frac{\partial}{\partial z} \overline{w_X |\delta \mathbf{u}|^2} - 2 \frac{\partial}{\partial z} \overline{\delta w \delta \pi} = 4\bar{\epsilon}$$

local averaging over r-sphere

$$W(r) = \frac{6}{4\pi r^3} \int_{|\rho| \leq r} d^3 \rho \overline{\delta B \delta w} \quad S_3(r) = \frac{3}{4\pi} \int d\Omega_r \overline{\hat{\mathbf{r}} \cdot \delta \mathbf{u} |\delta \mathbf{u}|^2}$$

$$T(r) = \frac{3}{4\pi r^3} \frac{\partial}{\partial z} \int_{|\rho| \leq r} d^3 \rho \left(\overline{w_X |\delta \mathbf{u}|^2} + 2 \overline{\delta w \delta \pi} \right)$$

$$\boxed{W - \frac{S_3}{r} - T = 4\bar{\epsilon}}$$

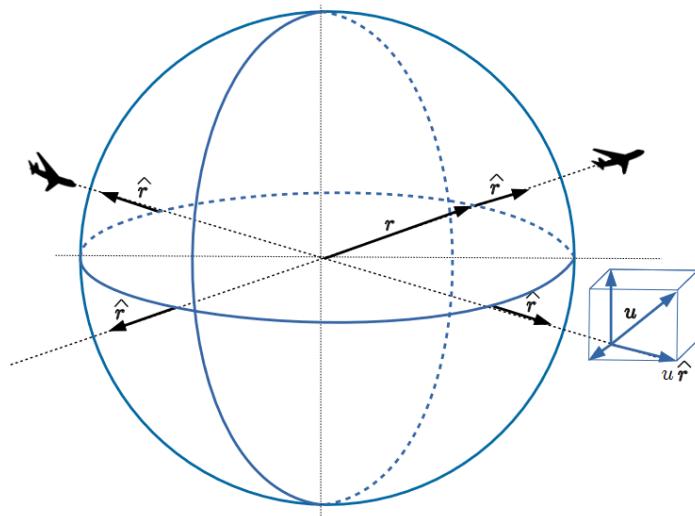
Scale-by-scale budget equation

$$W - \frac{S_3}{r} - T = 4\bar{\epsilon}$$

$$W(r) = \frac{6}{4\pi r^3} \int_{|\rho| \leq r} d^3\rho \overline{\delta B \delta w}$$

$$S_3(r) = \frac{3}{4\pi} \int d\Omega_r \hat{r} \cdot \overline{\delta \mathbf{u} |\delta \mathbf{u}|^2}$$

$$T(r) = \frac{3}{4\pi r^3} \frac{\partial}{\partial z} \int_{|\rho| \leq r} d^3\rho \left(\overline{w_X |\delta \mathbf{u}|^2} + 2 \overline{\delta w \delta \pi} \right)$$



Approximation with measured quantities

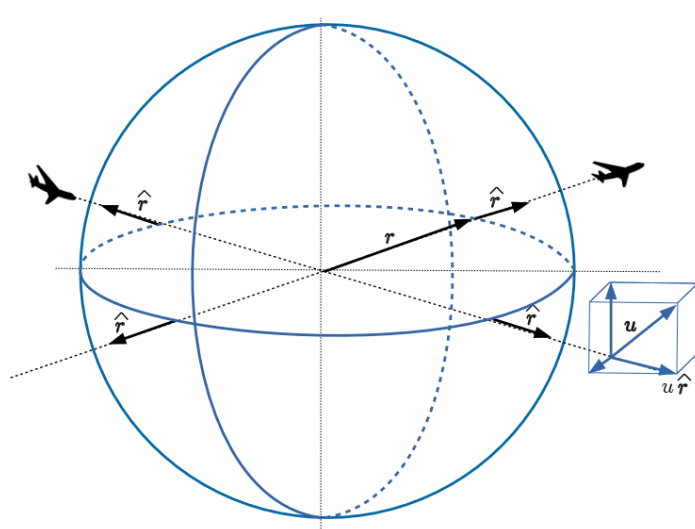
$$W - \frac{S_3}{r} - T = 4\bar{\epsilon}$$

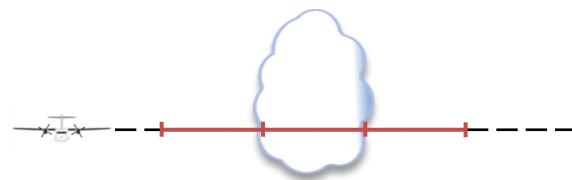
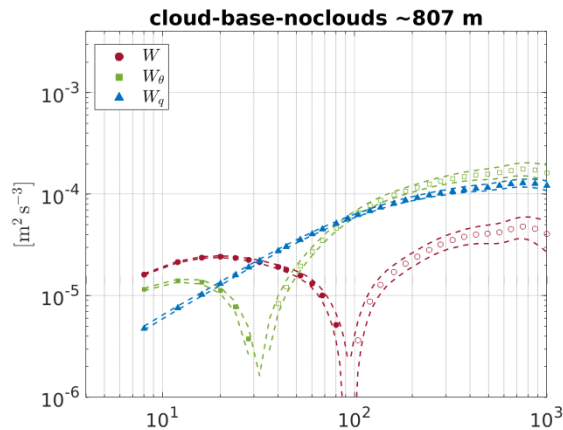
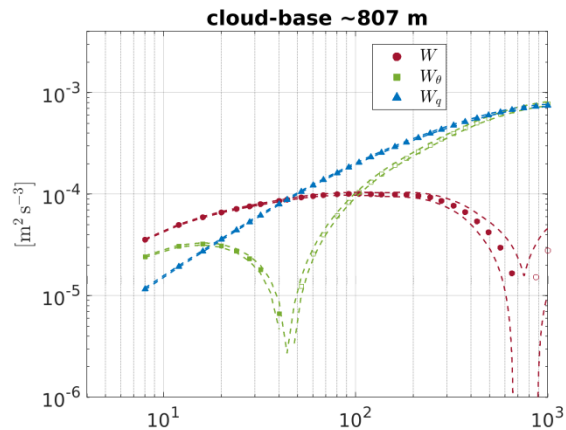
$$W(r) \approx \frac{6}{r^3} \int_0^r d\rho \rho^2 \overline{\delta B \delta w} = W_\theta + W_q$$

$$S_3(r) \approx 3 \overline{\delta u |\delta \mathbf{u}|^2}$$

$$T(r) \approx \frac{3}{r^3} \frac{\partial}{\partial z} \int_0^r d\rho \rho^2 (\overline{w_X |\delta \mathbf{u}|^2} + 2 \overline{\delta w \delta \pi}) = T_u + T_p$$

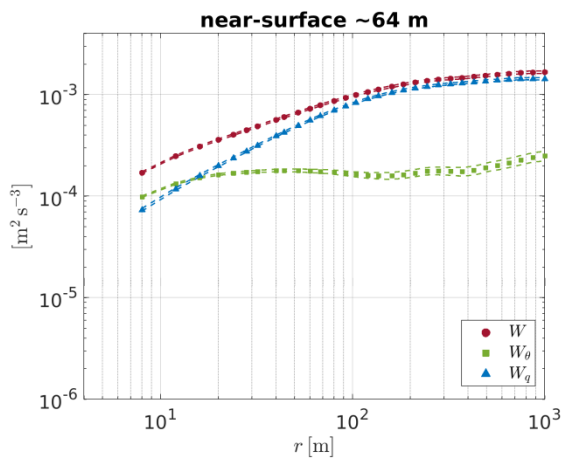
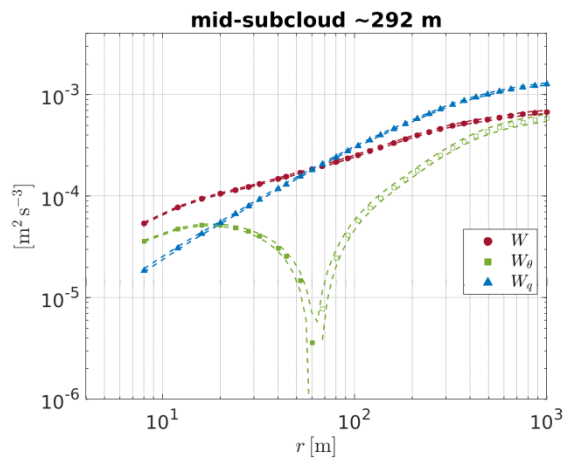
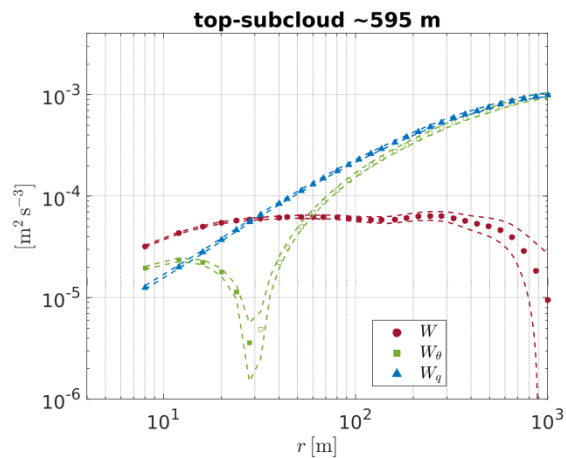
$$S_2(r) = C(\bar{\epsilon} r)^{\frac{2}{3}}$$

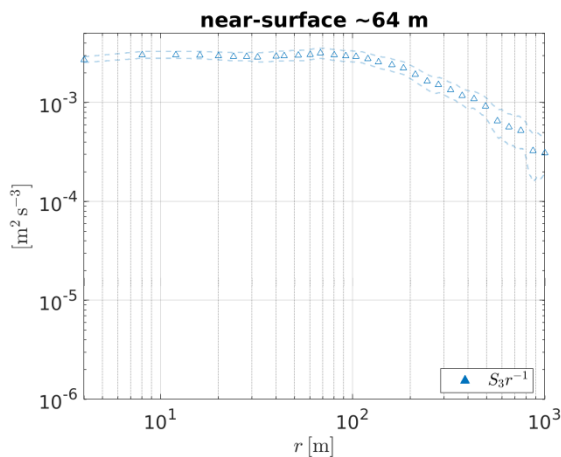
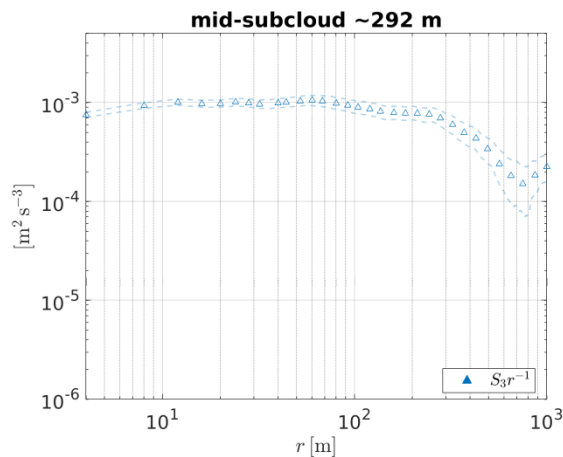
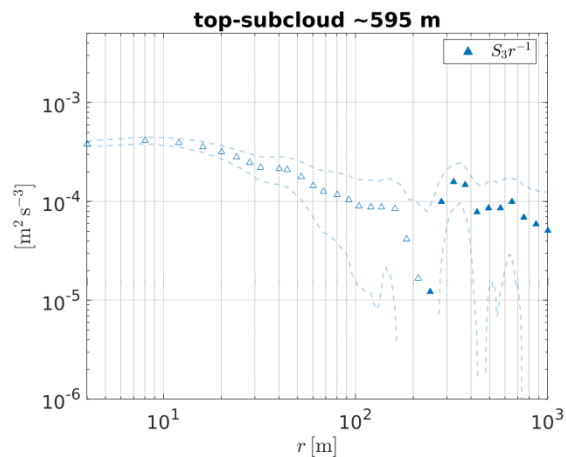
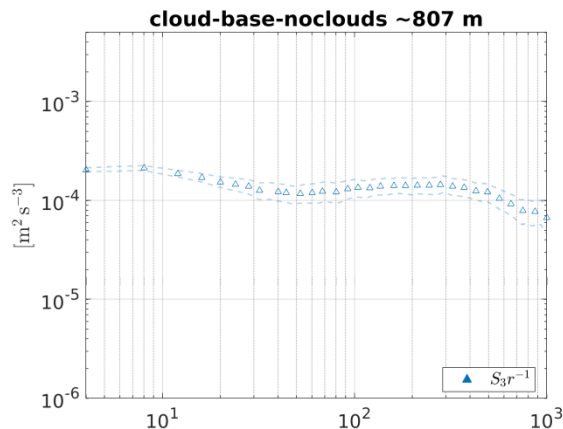
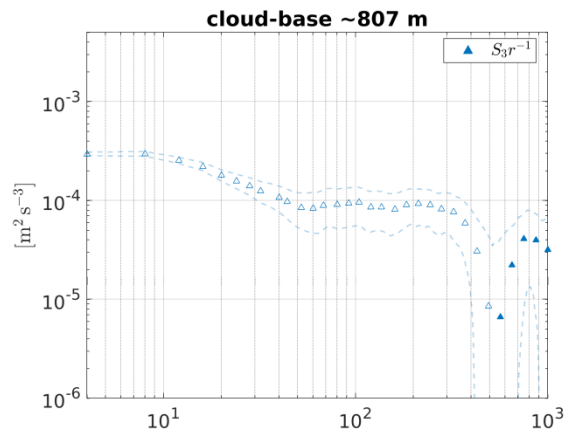




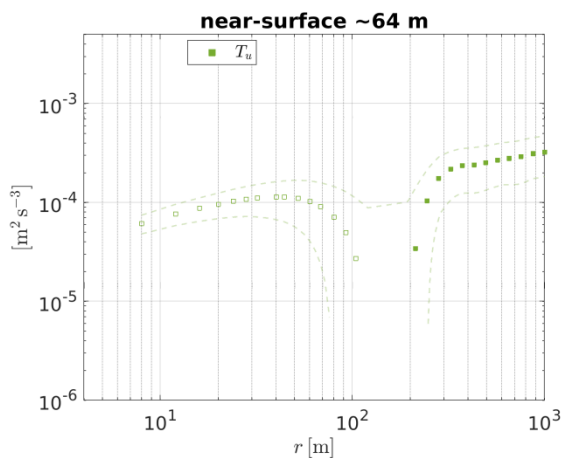
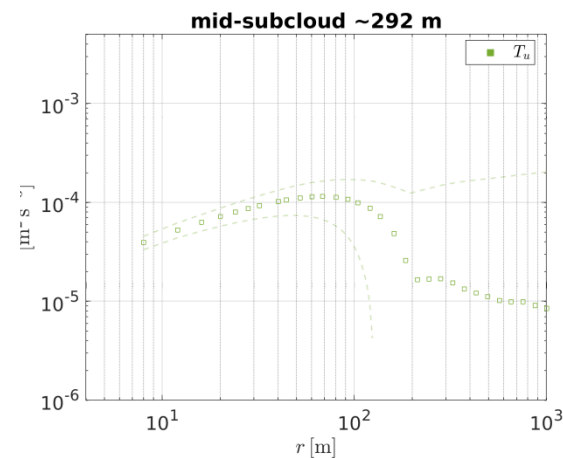
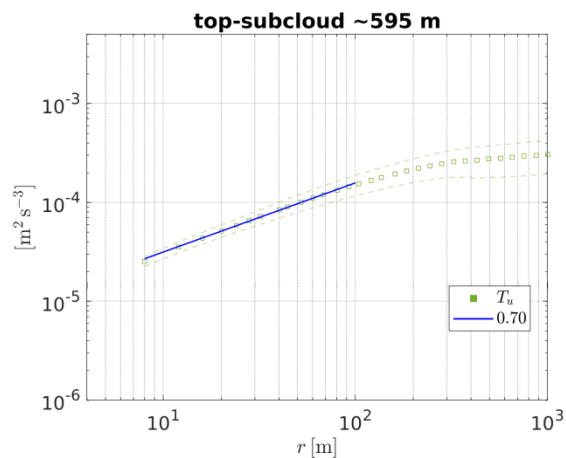
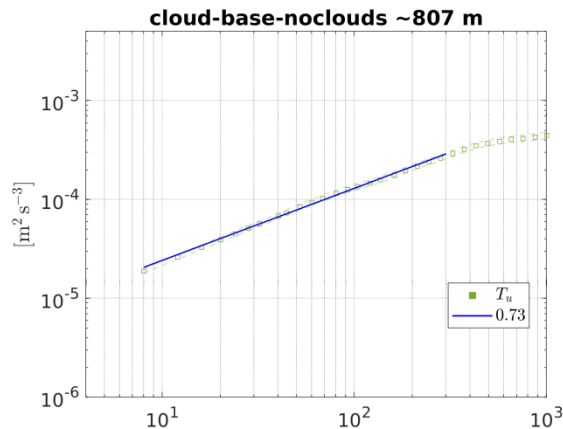
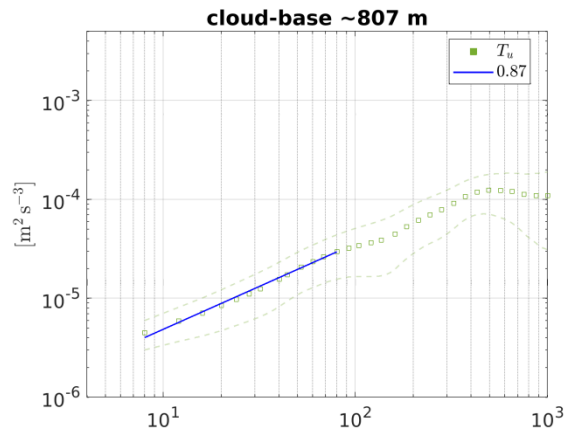
$$LWC > 0.01 \frac{\text{g}}{\text{m}^3} \quad (1 \text{ Hz})$$

$$RH > 98 \% \quad (25 \text{ Hz})$$





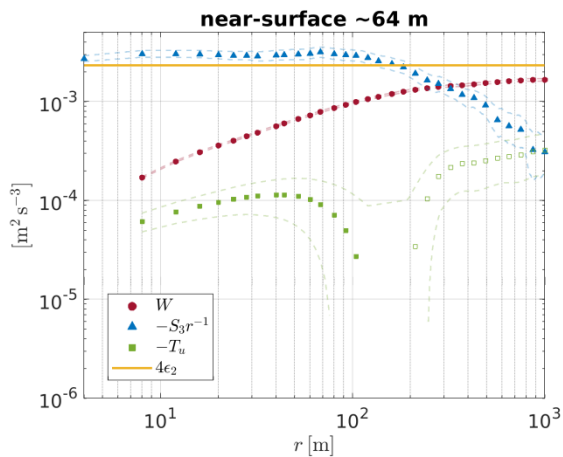
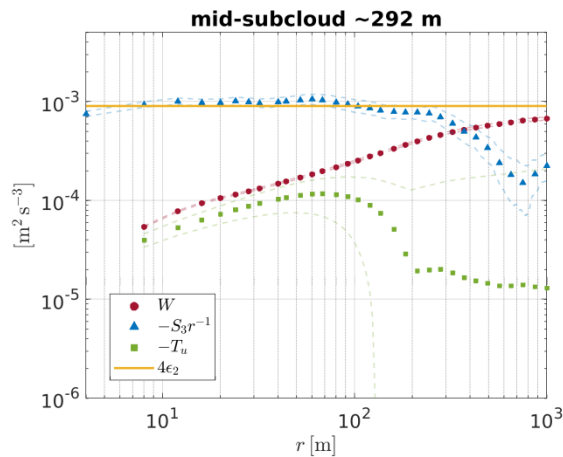
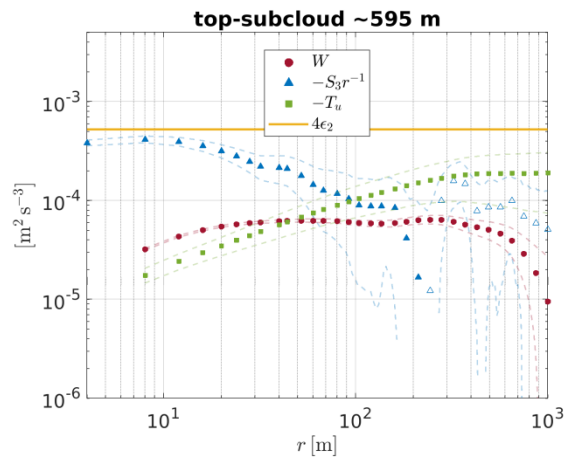
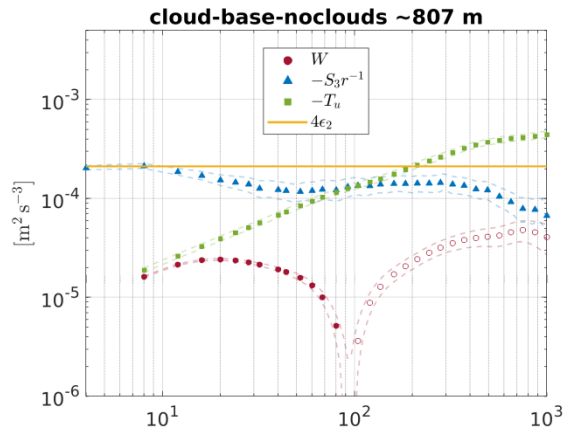
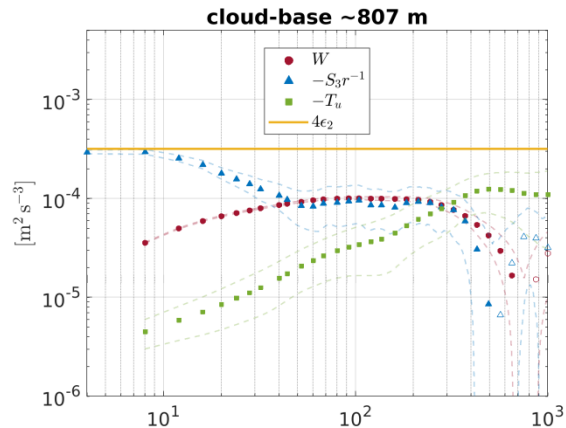
Filled symbols – positive values
Open symbols – negative values
Dashed lines – uncertainty range



Filled symbols – positive values
 Open symbols – negative values
 Dashed lines – uncertainty range

Budget terms

$$W - S_3/r - T = 4\bar{\epsilon}$$



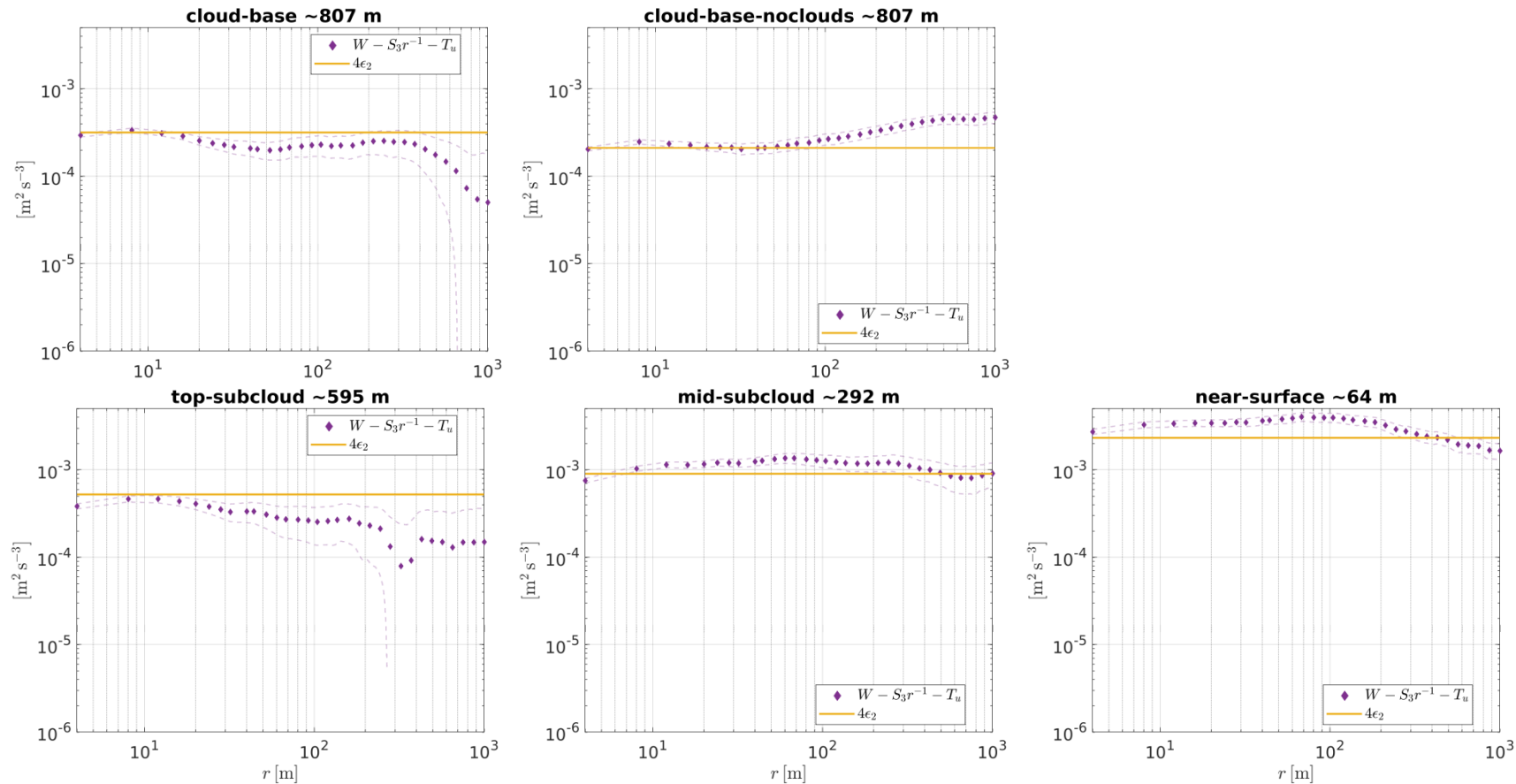
Filled symbols – positive values

Open symbols – negative values

Dashed lines – uncertainty range

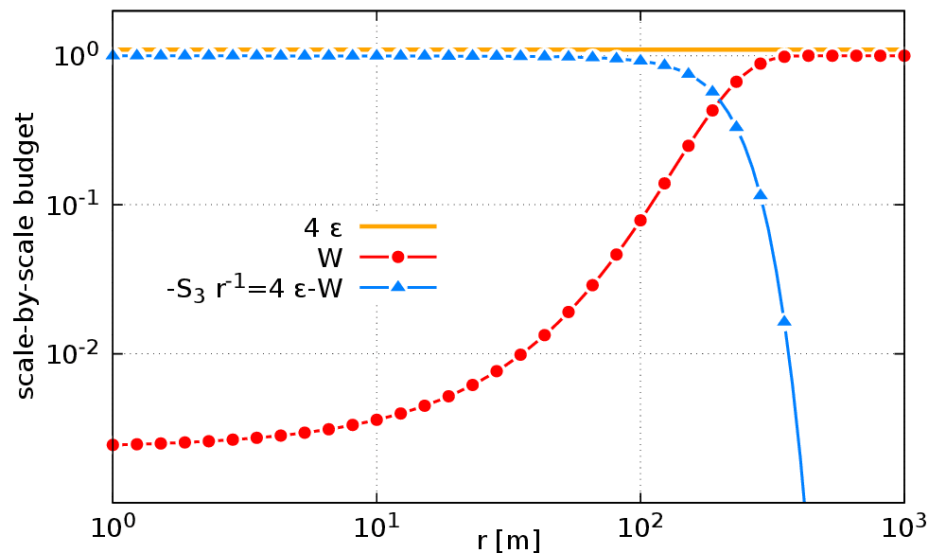
Total budget

$$W - S_3/r - T = 4\bar{\epsilon}$$

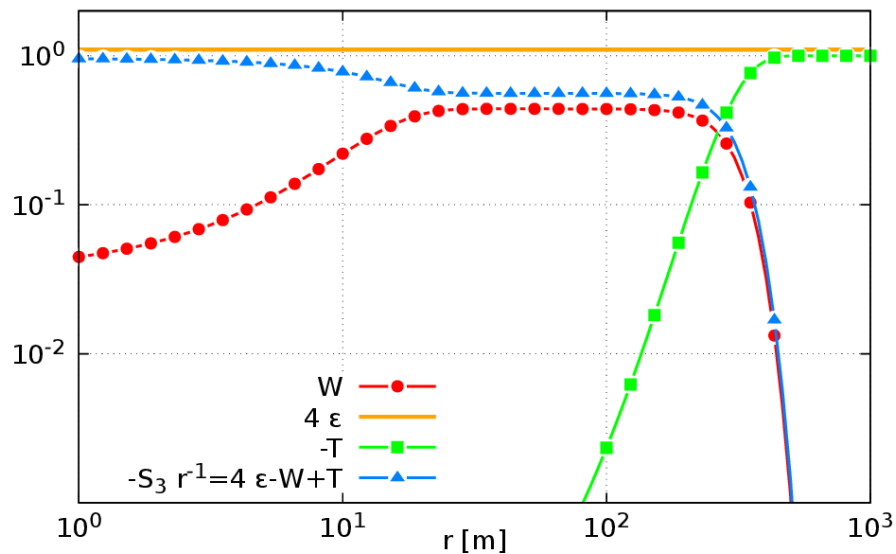


Idealized budgets

Equilibrium cascade
(near-surface and mid-subcloud)



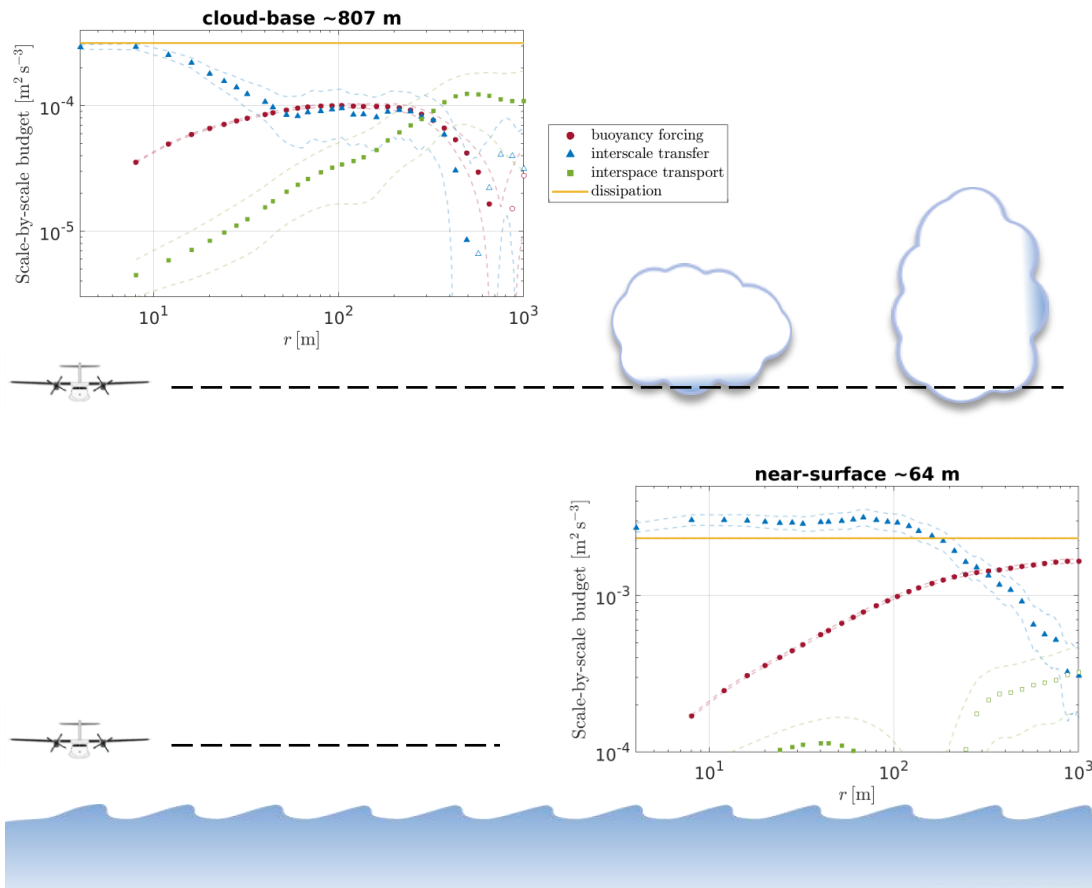
Non-equilibrium cascade
(top-subcloud and cloud-base)



Conclusions

- Airborne in-situ measurements in shallow trade-wind regime of atmospheric boundary layer over ocean are used to investigate the scale-by-scale budget of turbulence kinetic energy.
- The budget involving buoyancy forcing, nonlinear interscale transfer, interspace transport and dissipation is approximately closed up to ~ 200 m.
- At the near-surface and mid-subcloud levels, Kolmogorov equilibrium is observed over a range of scales from a few to above 100 m.
- At the top-subcloud and cloud-base levels, the budget is far from Kolmogorov equilibrium at scales above 10 m, with significant contribution of buoyancy forcing and turbulent transport. Buoyancy produces turbulence at intermediate scales while at large scales the stable stratification consumes part of the energy which is supposedly supplied by the transport from below.
- The contribution to the buoyancy forcing related to humidity variations is substantial and strictly positive at all levels. Almost always, it dominates over the contribution related to temperature only.

Energy cascades in the convective atmospheric boundary layer - summary



Jakub. L. Nowak¹, Marta Wacławczyk¹, John C. Vassilicos², Stanisław Król¹, Szymon P. Malinowski¹

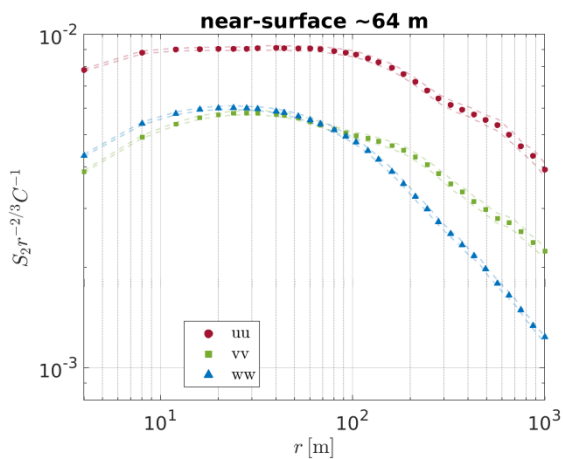
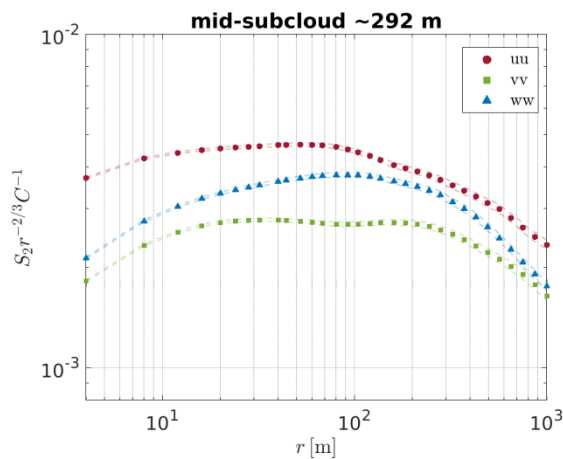
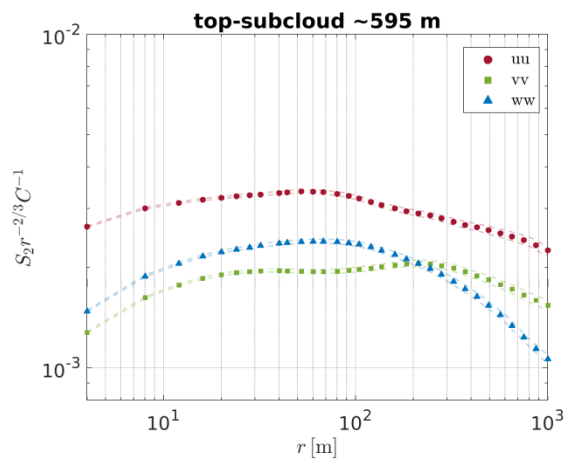
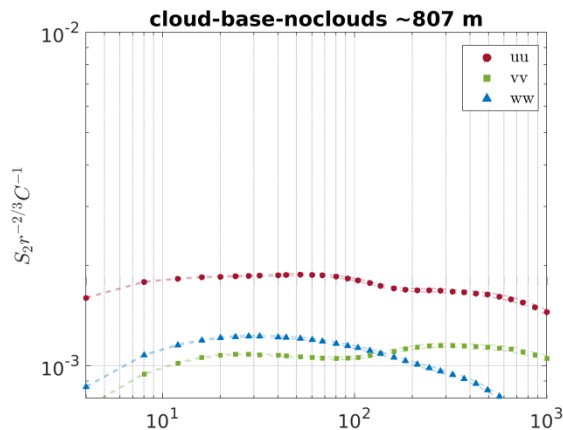
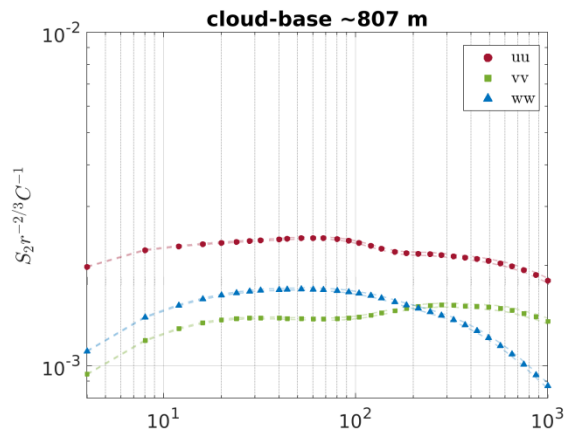
¹Institute of Geophysics, Faculty of Physics, University of Warsaw, Poland

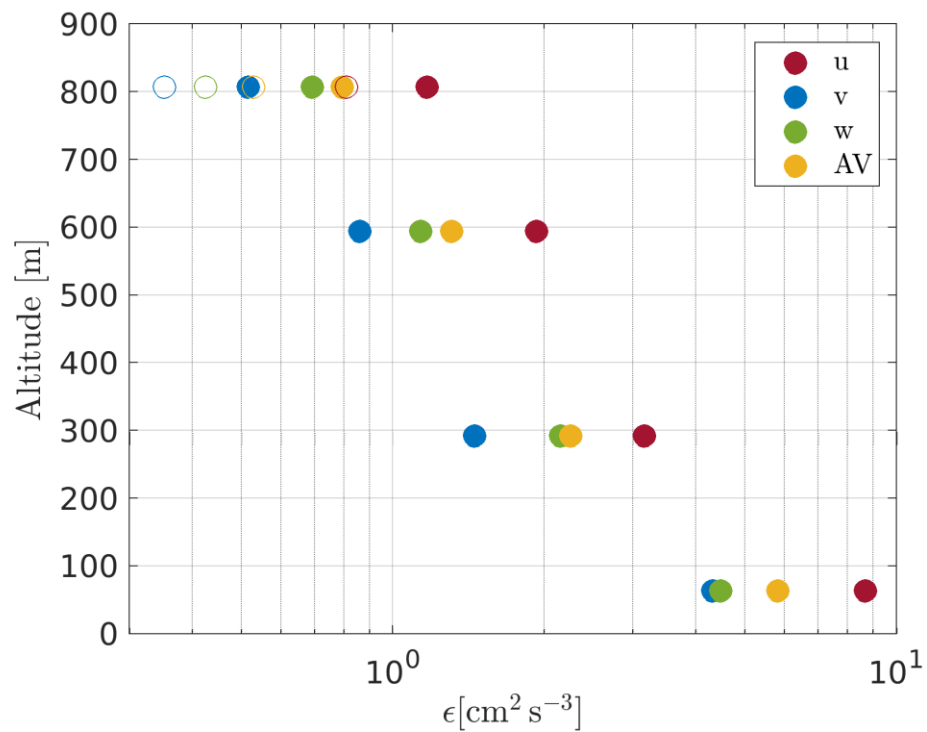
²Univ. Lille, CNRS, ONERA, Arts et Metiers Institute of Technology, Centrale Lille, UMR 9014 –LMFL – Laboratoire de Mécanique des Fluides de Lille – Kampé de Fériet, Lille, France

LTP 2023 collaboration

2nd order structure functions

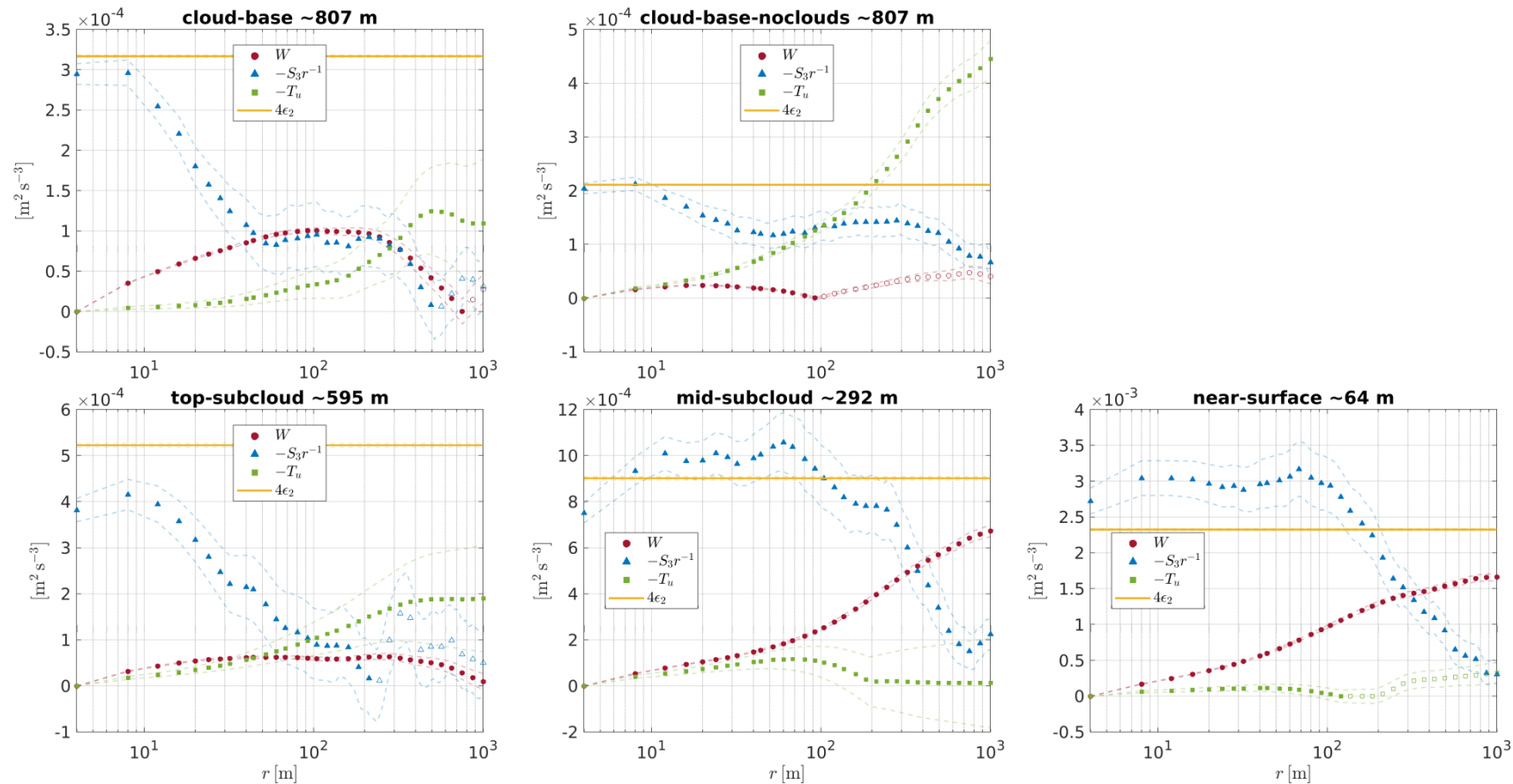
$$S_2/Cr^{2/3}$$





Open symbols - clouds excluded

Budget terms in linear scale



Total budget in linear scale

