Lagrangian stochastic microphysics at unresolved scales in turbulent cloud simulations

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EGU2020: 4-8 May 2020

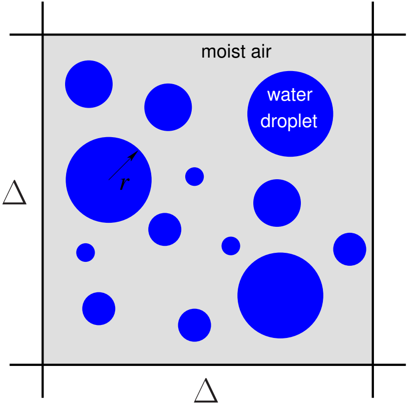
Diffusional growth

Droplet growth

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{r} D \langle S \rangle$$

$\langle S \rangle$ - mean-field supersaturation

LES grid box



$\Delta \in$ inertial range

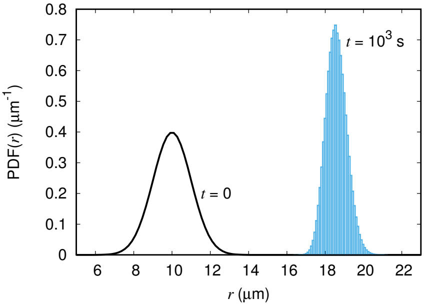
Diffusional growth

driven by mean-field supersaturation

- Droplets exposed to the same $\langle S \rangle$

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{r} D \langle S \rangle$$

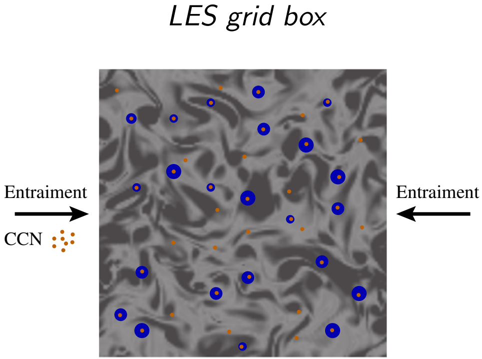
- Narrow size distribution!



Microphysical variability

at sub-grid scales (SGS)

- $S = \langle S \rangle + S'$
- Mixing
- Activation/deactivation
- Superdroplets



Stochastic activation

Köhler potential plus fluctuations

Growth equation:

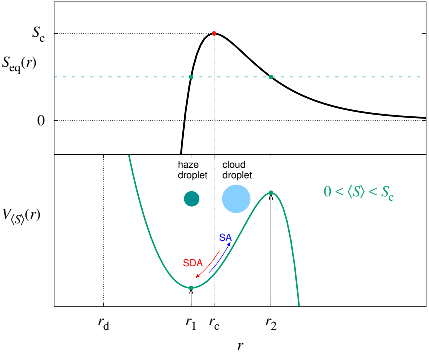
$$r\frac{\mathrm{d}r}{\mathrm{d}t} = D\left[\langle S\rangle + S' - \frac{A}{r} + \frac{B}{r^3}\right]$$

Define:

$$x \equiv r^2$$

"Brownian" x:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{\partial V}{\partial x} + 2DS'$$



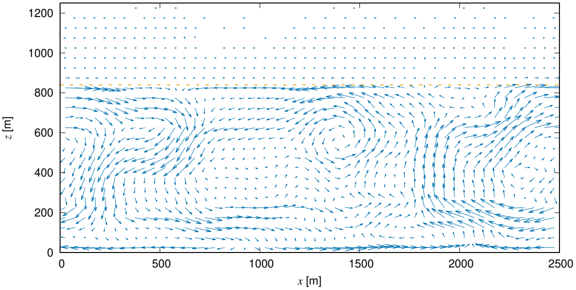
Abade, Grabowski and Pawlowska, JAS, **75** (2018)

Kinematic framework

Synthetic turbulent-like ABL flow

Turbulent-like ABL flow

$$\mathbf{u} = (u, w) = \left(\frac{\partial \psi}{\partial z}, -\frac{\partial \psi}{\partial x}\right) \qquad \psi(\mathbf{r}, t) = \sum \text{random harmonics}$$

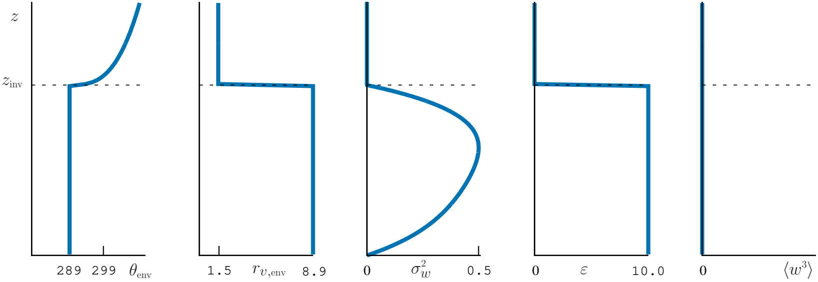


$$\langle w^2 \rangle = \sigma_w^2(z) \qquad \langle w(x', z) w(x' + x, z) \rangle = \hat{C}_w(x)\, \sigma_w^2(z)$$

Pinsky *et al.*, JAS, **65** (2008), ..., Magaritz-Ronen *et al.*, ACP, **16** (2016)

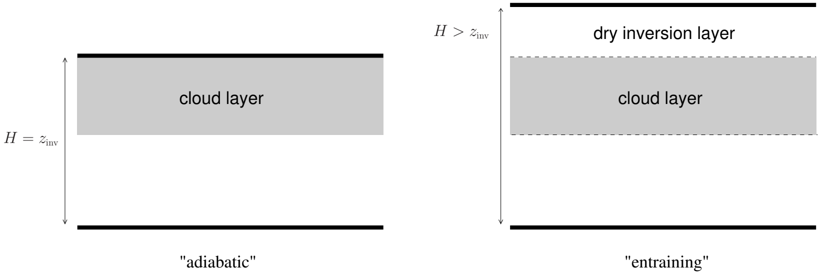
Turbulent-like ABL flow

Vertical structure



Turbulent-like ABL flow

Adiabatic and entraining configurations



Stochastic Lagrangian microphysical schemes

Differ in the stochastic variables used in the superdroplets state vector to describe the SGS variability.

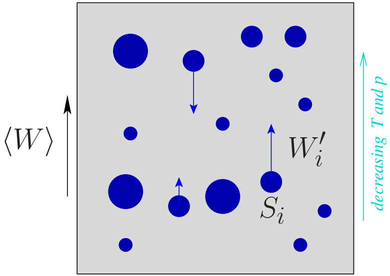
- (w', S') model

  Fluctuations in vertical velocity and supersaturation

- (Θ, Q) model

  Lagrangian potential temperature and vapor mixing ratio

Supersaturation and velocity fluctuations



$$\frac{\mathrm{d}S'_i}{\mathrm{d}t} = -\frac{S'_i}{\tau_\mathrm{c}} - \frac{S'_i}{\tau_\mathrm{m}} + aW'_i(t)$$

$$\tau_\mathrm{c} \sim \frac{1}{N\langle r \rangle} \quad \text{(condensation)}$$

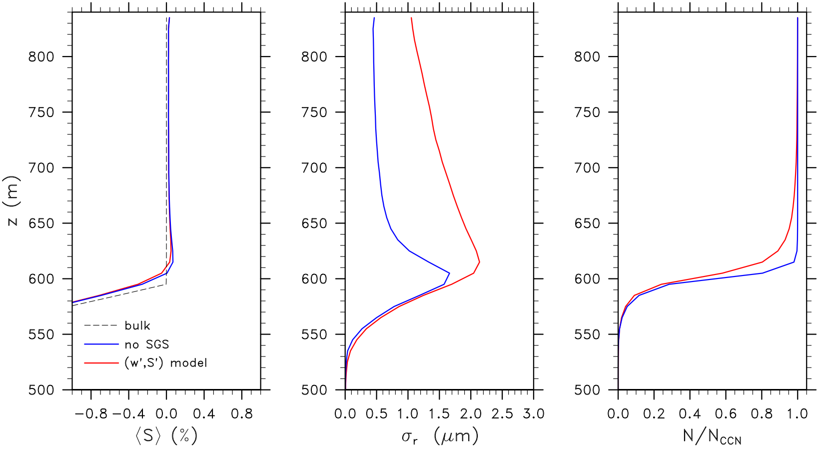
$\tau_\mathrm{m} \sim$ eddy turnover time (mixing)

- $W'(t)$: O-U process with parameters $\sigma^2_{W'}$ and τ_m.

Celani *et al.*, EPL, **70** (2005); Grabowski and Abade, JAS, **74** (2017)

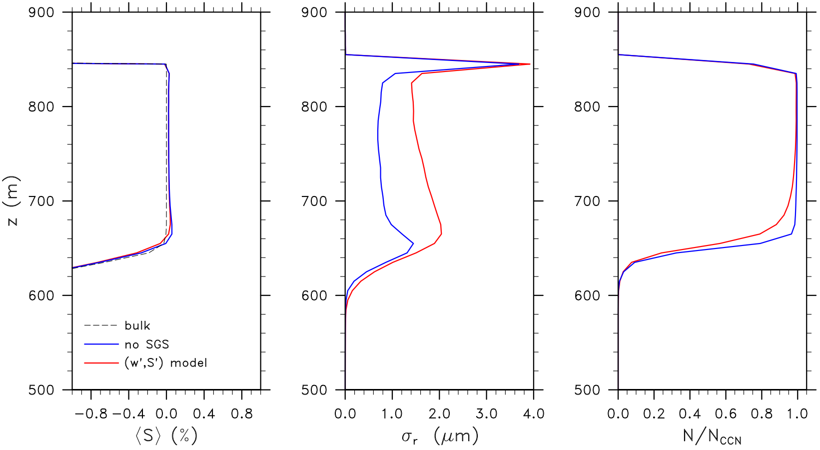
Microphysical profiles

Adiabatic configuration



Microphysical profiles

entraining configuration



(Θ, Q) model: Eulerian description [1]

Thermodynamic scalar $\theta = \langle \theta \rangle + \theta'$

$$\frac{\partial \theta}{\partial t} + \mathbf{u} \cdot \nabla \theta = S_\theta$$

$$\frac{\partial \langle \theta \rangle}{\partial t} + \langle \mathbf{u} \rangle \cdot \nabla \langle \theta \rangle = -\nabla \cdot \mathbf{J} + \langle S_\theta \rangle$$

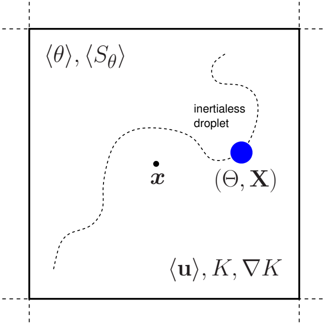
SGS turbulent flux:

$$\mathbf{J} = \langle \mathbf{u}' \theta \rangle \approx -K \, \nabla \langle \theta \rangle$$

[1] Described here only for θ for simplicity.

Lagrangian description

Stochastic variables $(\Theta, \mathbf{X})$

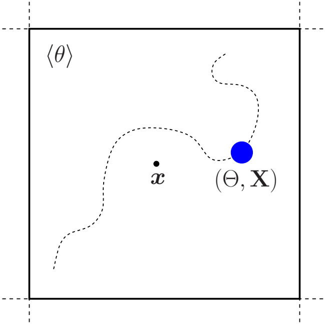


- Langevin equations:

$$\mathrm{d}\Theta = \underbrace{-\frac{\Theta - \langle\theta\rangle}{\tau_m}\,\mathrm{d}t}_{\text{SGS mixing}} + S_\theta\,\mathrm{d}t,$$

$$\mathrm{d}\mathbf{X} = [\langle\mathbf{u}\rangle + \nabla K]\,\mathrm{d}t + \sqrt{2K}\,\mathrm{d}\mathbf{W}$$

Probability description



- Probability that $\theta < \Theta < \theta + \mathrm{d}\theta$

$$f(\theta; \boldsymbol{x}, t)\, \mathrm{d}\theta$$

- f - probability density function
- Average

$$\langle\theta\rangle = \int \theta\, f(\theta; \boldsymbol{x}, t)\, \mathrm{d}\theta$$

Fokker-Planck equation for $f(\theta; \mathbf{x}, t)$:

$$\frac{\partial f}{\partial t} = - \frac{\partial}{\partial \theta}\left[\left(-\frac{\theta - \langle\theta\rangle}{\tau_m} + S_\theta\right) f\right]$$

$$- \frac{\partial}{\partial \mathbf{x}} \cdot \left[\left(\langle \mathbf{u} \rangle + \nabla K\right) f\right] + \frac{\partial}{\partial \mathbf{x}} \cdot \frac{\partial}{\partial \mathbf{x}} \left(K f\right) \qquad (1)$$

Performing

$$\int \theta \, [\text{Eq. } (1)] \, \mathrm{d}\theta \, ...$$

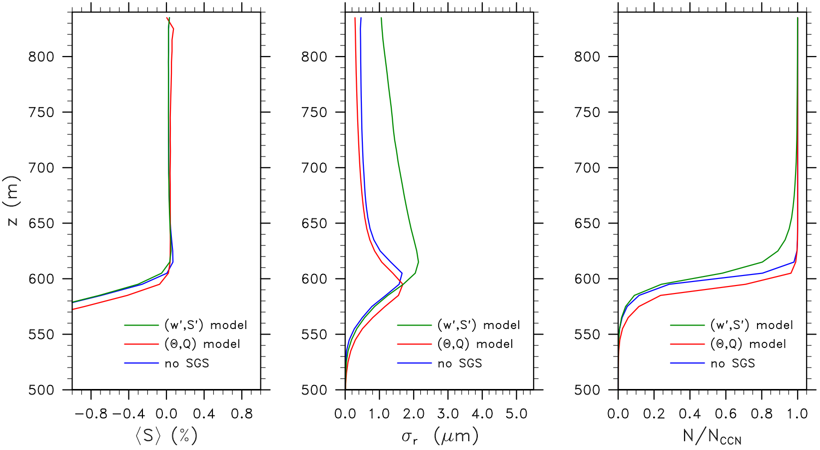
... one recovers the Eulerian equation for the average:

$$\frac{\partial\langle\theta\rangle}{\partial t} + \langle\mathbf{u}\rangle \cdot \nabla\langle\theta\rangle = -\nabla \cdot \mathbf{J} + \langle S_\theta\rangle$$

$$\mathbf{J} = -K\nabla\langle\theta\rangle$$

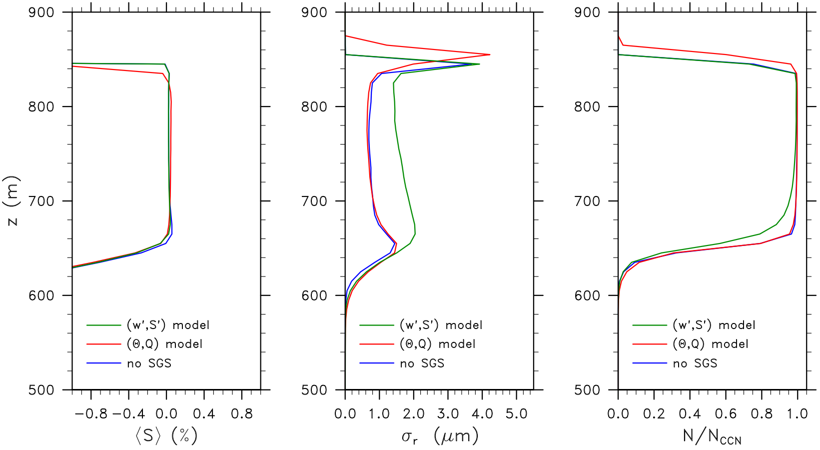
Microphysical profiles

Adiabatic configuration



Microphysical profiles

entraining configuration



Summary

- Simple models to mimic SGS variability
- Broadening of the droplet-size distribution
- Thermodynamic feedback: extends the distance of activation
- Statistical equivalence: Eulerian $\times$ Lagrangian

Acknowledgements

UNIWERSYTET
WARSZAWSKI

NATIONAL SCIENCE CENTRE
POLAND