Effects of forcing time scale on the simulated turbulent flows and turbulent collision statistics of cloud droplets

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Motivation and objectives

- Homogeneous and isotropic turbulence (HIT) simulated by the pseudo-spectral algorithm has been widely used to study statistics, structure, and dynamics of small-scale turbulence and dynamics of suspended inertial particles.
- The simulations address a number of applications ranging from turbulent collision of cloud droplets (Grabowski & Wang 2013, Wang et al. 2009), turbulent dispersion of dust particles, combustion processes, scalar transport…
- The simplest turbulence system is 3D isotropic homogeneous turbulence. The flow is driven by the forcing scheme.

- Evaluate the effects of forcing time scale in the large-scale stochastic forcing scheme of Eswaran and Pope [Comput. Fluids. 1998], on the simulated flow structures and statistics of forced turbulence.
- Evaluate the effects of forcing time scale on the kinematic collision statistics of inertial particles.

Turbulent flow – Eulerian approach

- Incompressible turbulence

$$\frac{\partial \mathbf{U}}{\partial t} = \mathbf{U}\times\omega - \nabla\left(\frac{P}{\rho}+\frac{1}{2}\mathbf{U}^2\right) + \nu\nabla^2\mathbf{U} + f(\mathbf{x},t) \quad \text{where} \quad \nu = \frac{0.45}{(N/32)^{4/3}}$$

$$\nabla\cdot\mathbf{U} = 0$$

$$\hat{\mathbf{f}}(\mathbf{k},t) = \sum_{j=1}^{3} A_j(\mathbf{k},t)e^{2\pi i r(\mathbf{k},t,j)}, \quad A_j(\mathbf{k},t) = \sqrt{\frac{-4\sigma_f^2 \ln\theta(\mathbf{k},t,j)\Delta t}{t_f}}$$

  - acceleration variance σ_f^2
  - time scale t_f

Average rate of energy input

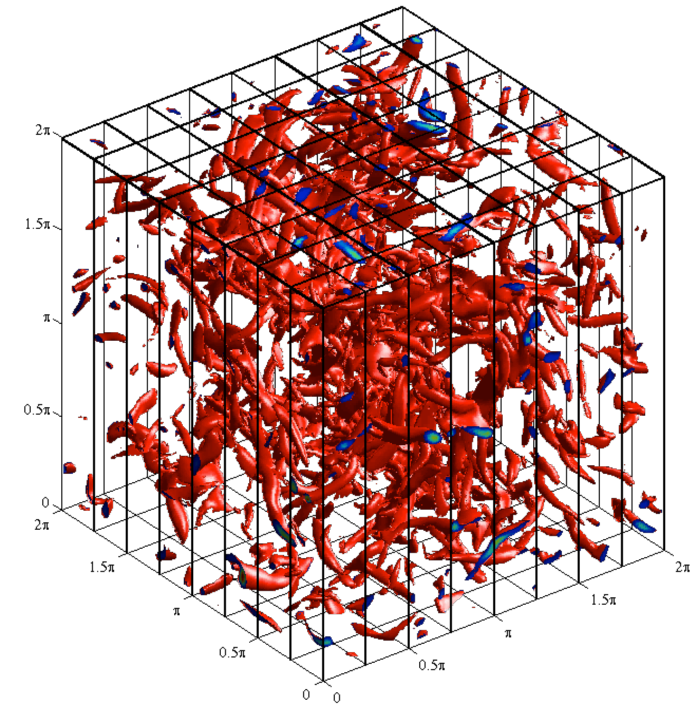
$$\varepsilon = \varepsilon_0 \times \frac{1}{1+t_f\left(\sigma_f^2 t_f N_f k_0^2\right)^{1/3}/\beta} \qquad \varepsilon_0 = 4\sigma_f^2 t_f N_f$$

$$\varepsilon_0 = 3600$$

$$t_f = t^*\sqrt{\frac{\nu}{\varepsilon_0}} \qquad \sigma_f^2 = \frac{\varepsilon_0}{4N_f t_f}$$

- In simulations, t^* is in the range of 0.01 to 10,000.

Parallel MPI implementation based on 2D domain decomposition

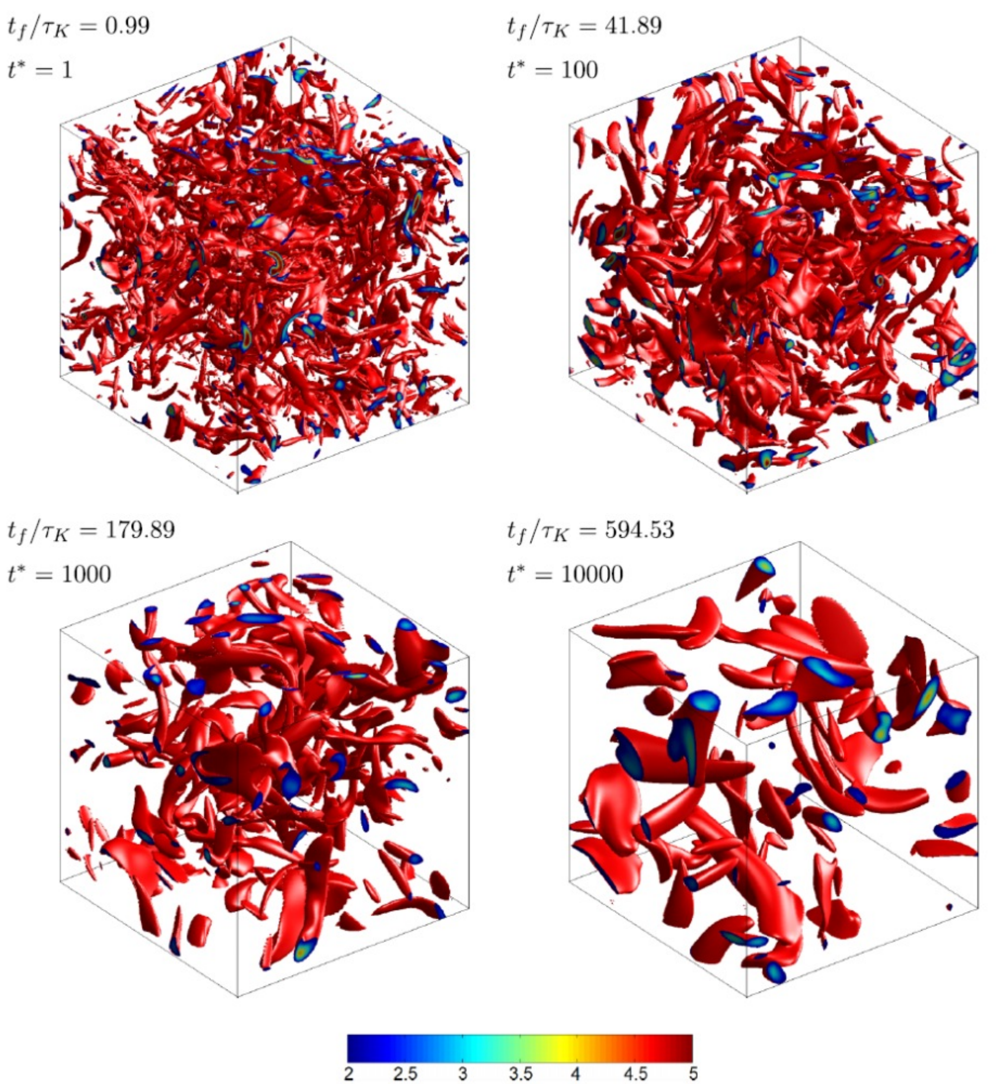


- Pseudo-spectral method (DNS)

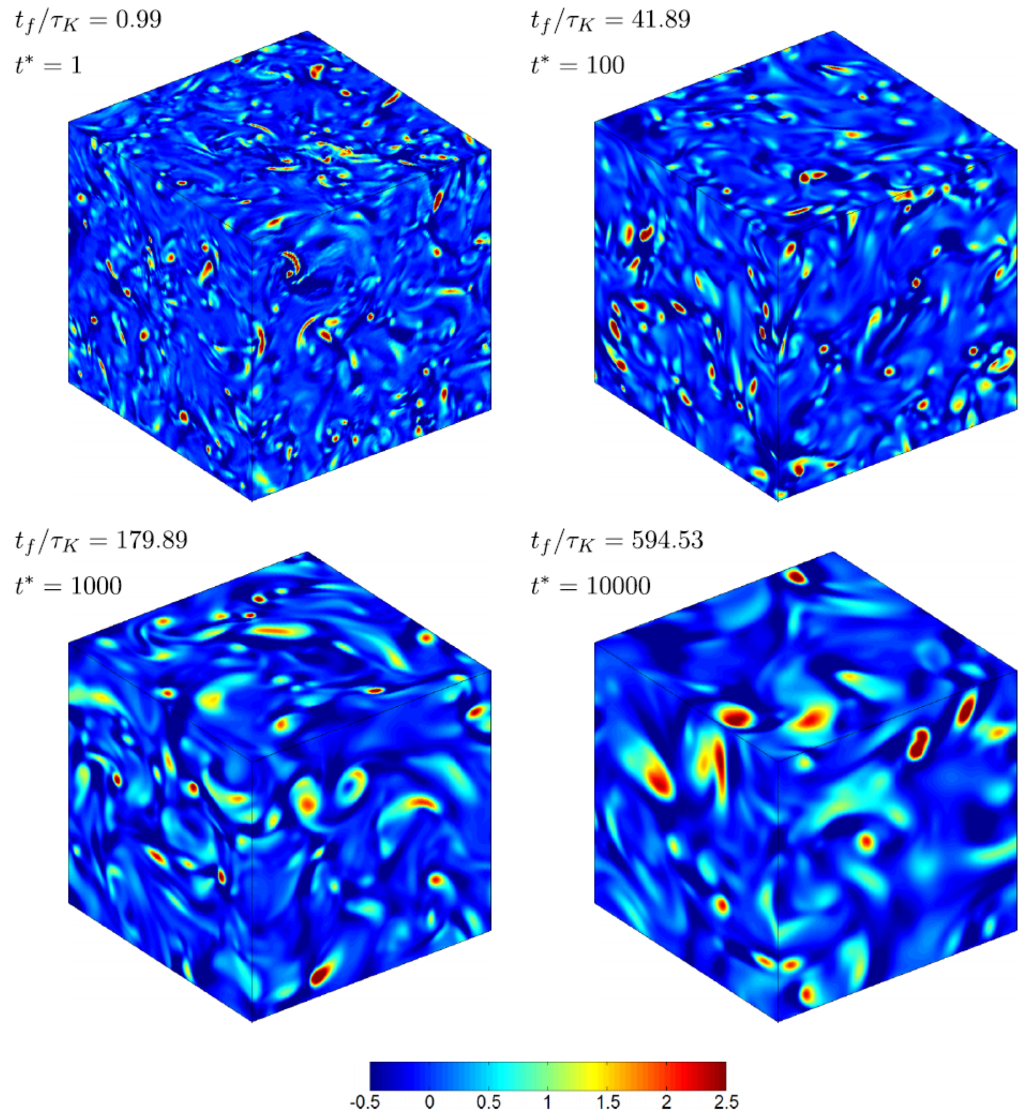
  3 Dimensional Parallel FFT:

  Ayala and Wang *Parallel Computing*, (2013)
- Periodic BC in a cube

Turbulent flows forced with different time scales



Vorticity isosurfaces (in red) and vorticity magnitude inside vortex structures at the boundaries of the domain (shown with color mapping) for the simulated flows with different forcing time scales ($N = 128^3$).

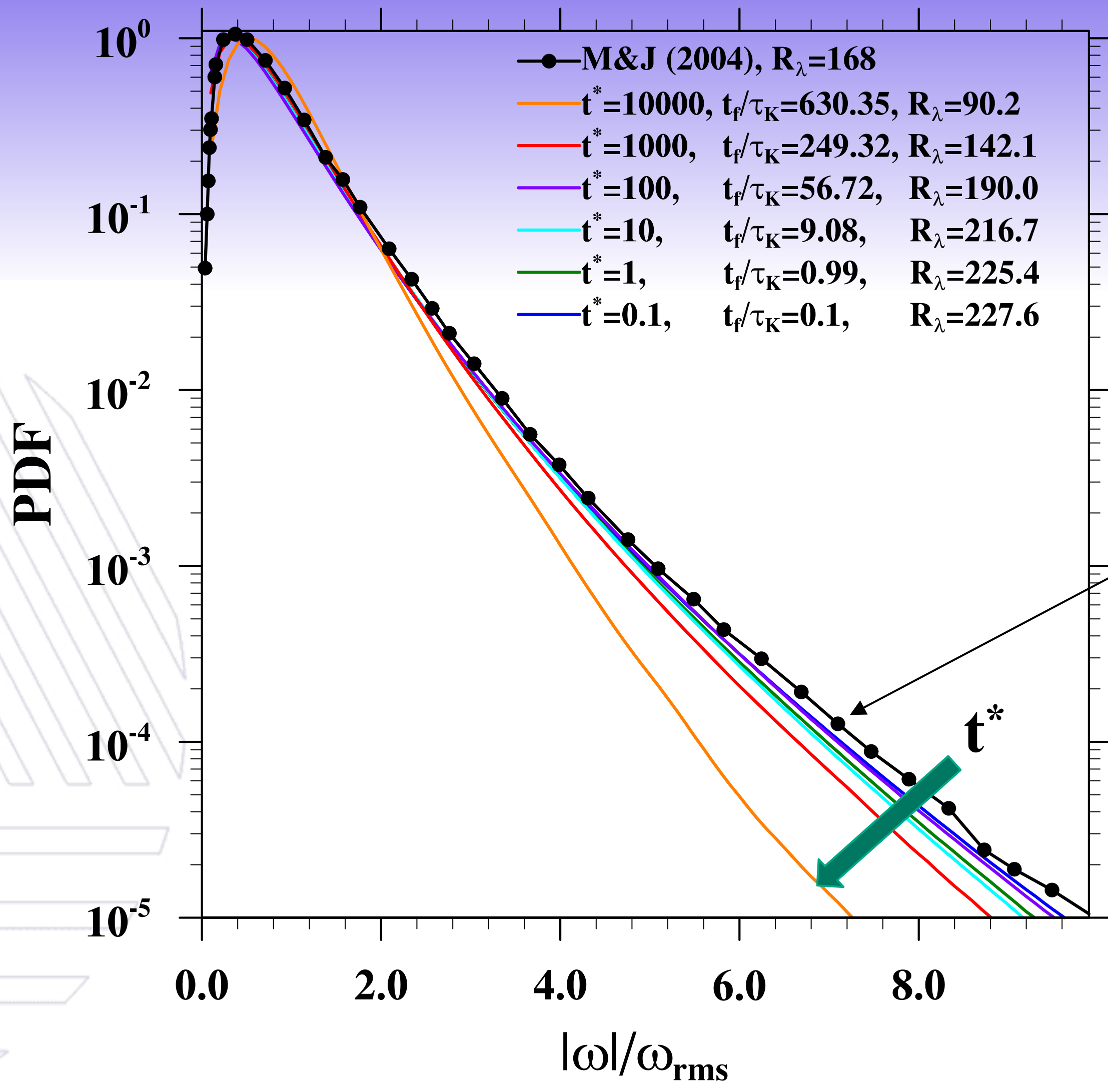


The second invariant of the deformation tensor II_2 normalized by its r.m.s. value.

$$II_2 = -\frac{1}{2}\frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_i} = \frac{1}{2}\left(\frac{\omega_i\omega_i}{2} - S_{ij}S_{ij}\right)$$

The size of vortical structures at small scales increases with increasing t_f.

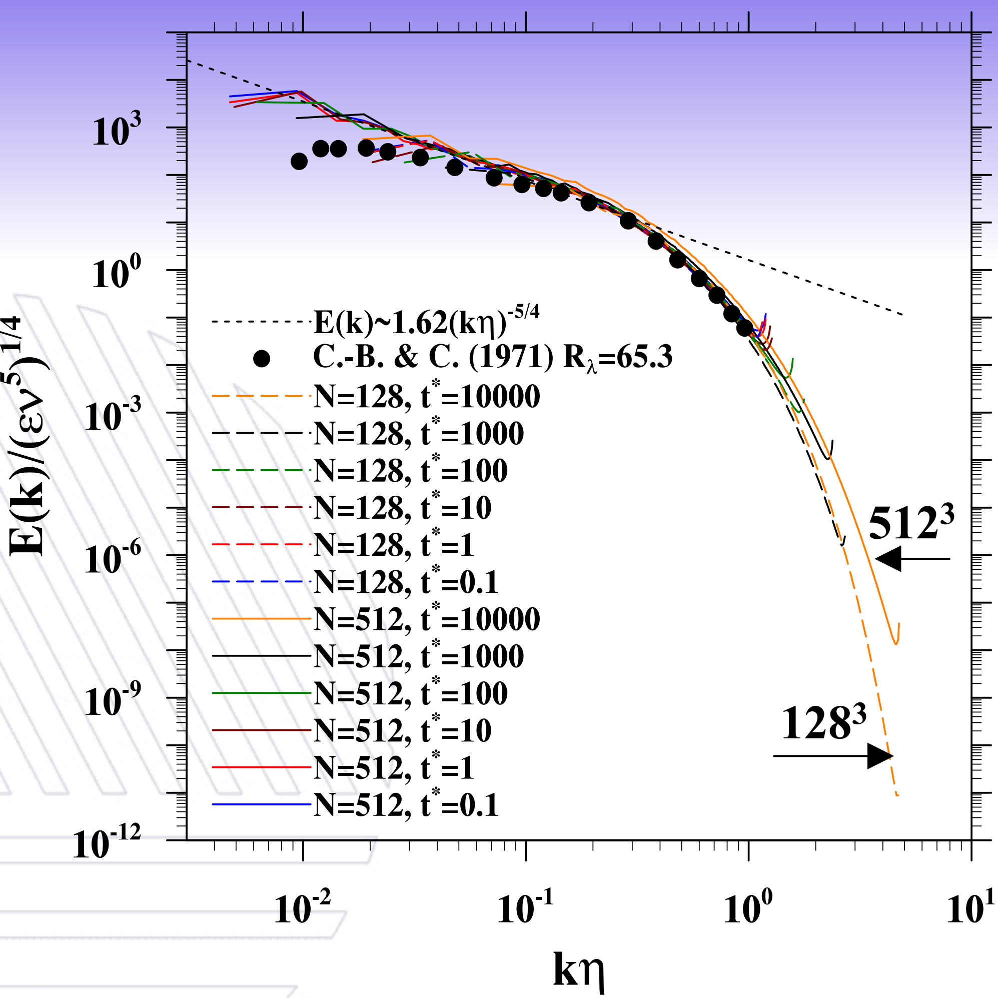
Probability density function of normalized vorticity



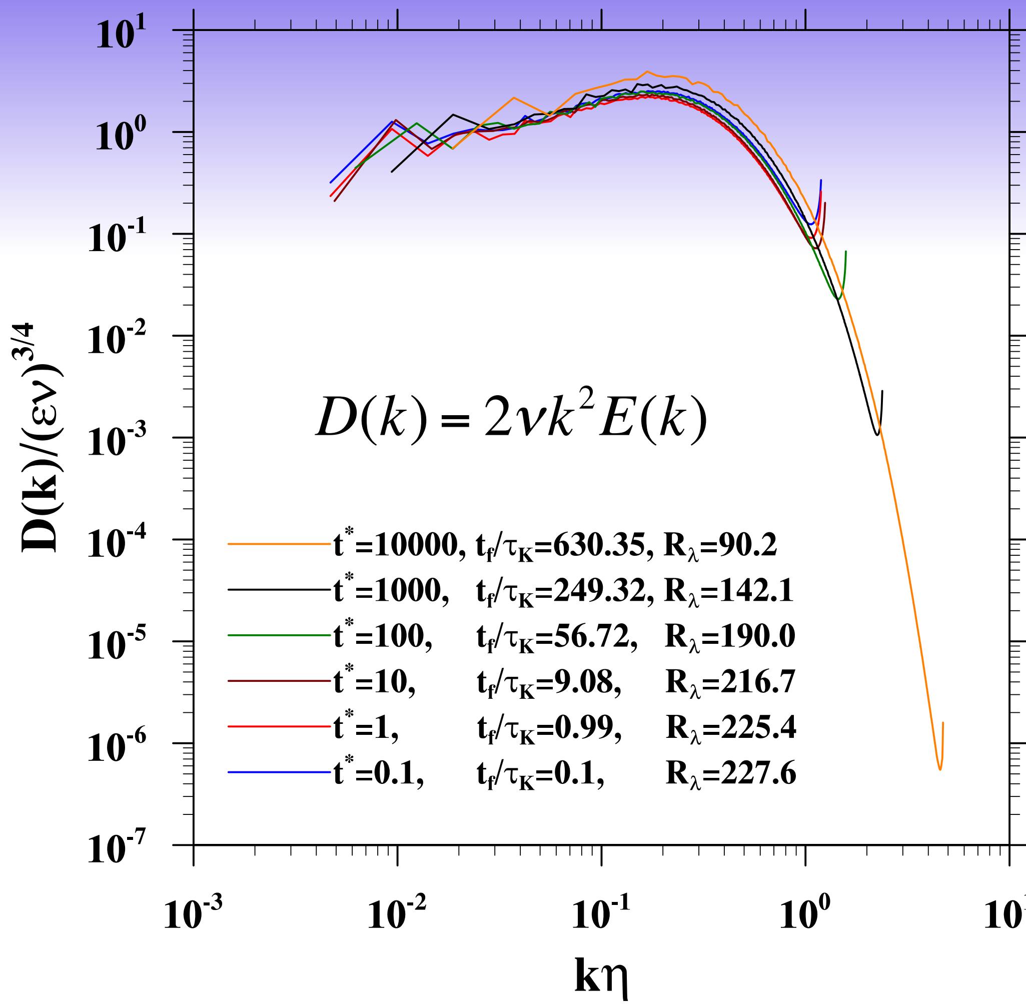
Moisy F. & Jimenez J. *J. Fluid Mech.*, **513**, 111-133 (2004)

PDF of the normalized vorticity from simulations at mesh sizes 512^3 with different forcing time scales

Energy and dissipation rate spectra



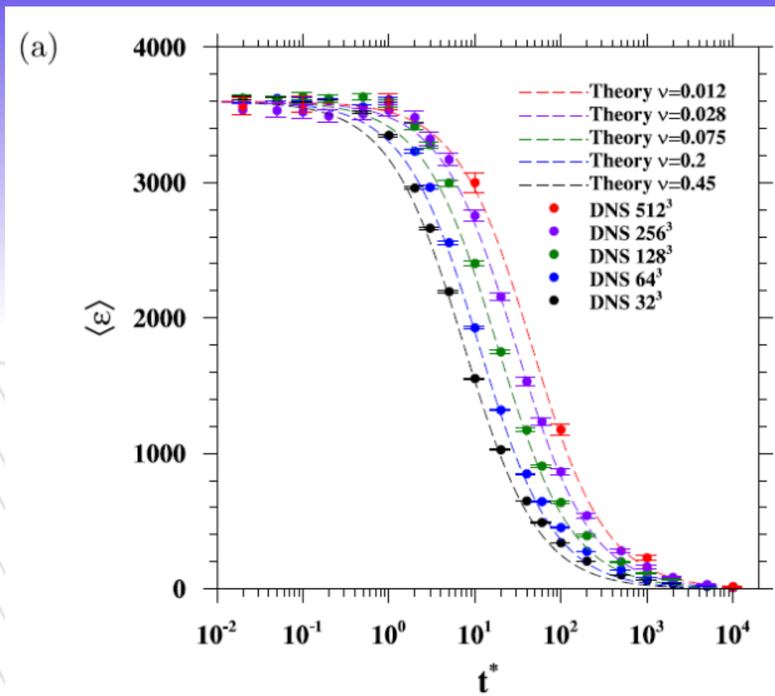
The normalized energy spectra of the simulated flows at two different resolutions 128^3 (dashed lines) and 512^3 (solid lines).



Normalized dissipation rate spectra (512^3).

1. Energy spectra from DNS are in excellent agreement with the experimental spectrum (in the inertial and dissipation ranges).
2. For a given grid resolution, the spectra obtained with different t_f overlap except when t_f is too large leading to a too small R_λ.

Energy dissipation rate

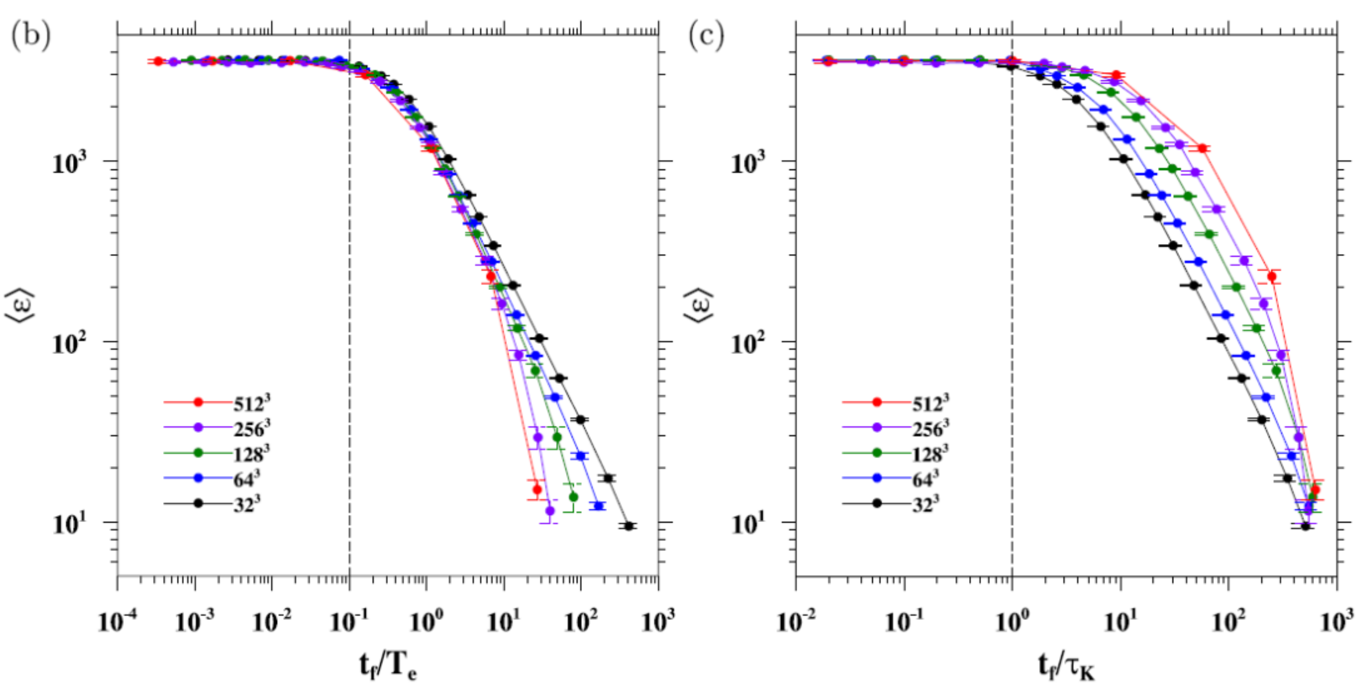


Energy dissipation rates as function of t^*. The model predictions are also shown with $\beta = 0.8$.

$$\varepsilon = \varepsilon_0 \times \frac{1}{1 + t_f \left(\sigma_f^2 t_f N_f k_0^2\right)^{1/3} / \beta}$$

In agreement with the model prediction $< \epsilon >$ is not sensitive to forcing time scale if $t_f \leq \tau_K$ or $t_f \leq 0.1\,Te$.

When $t_f > \tau_K$, the actual energy dissipation rate decreases with the forcing time scale, suggesting that less energy is supplied to the system.

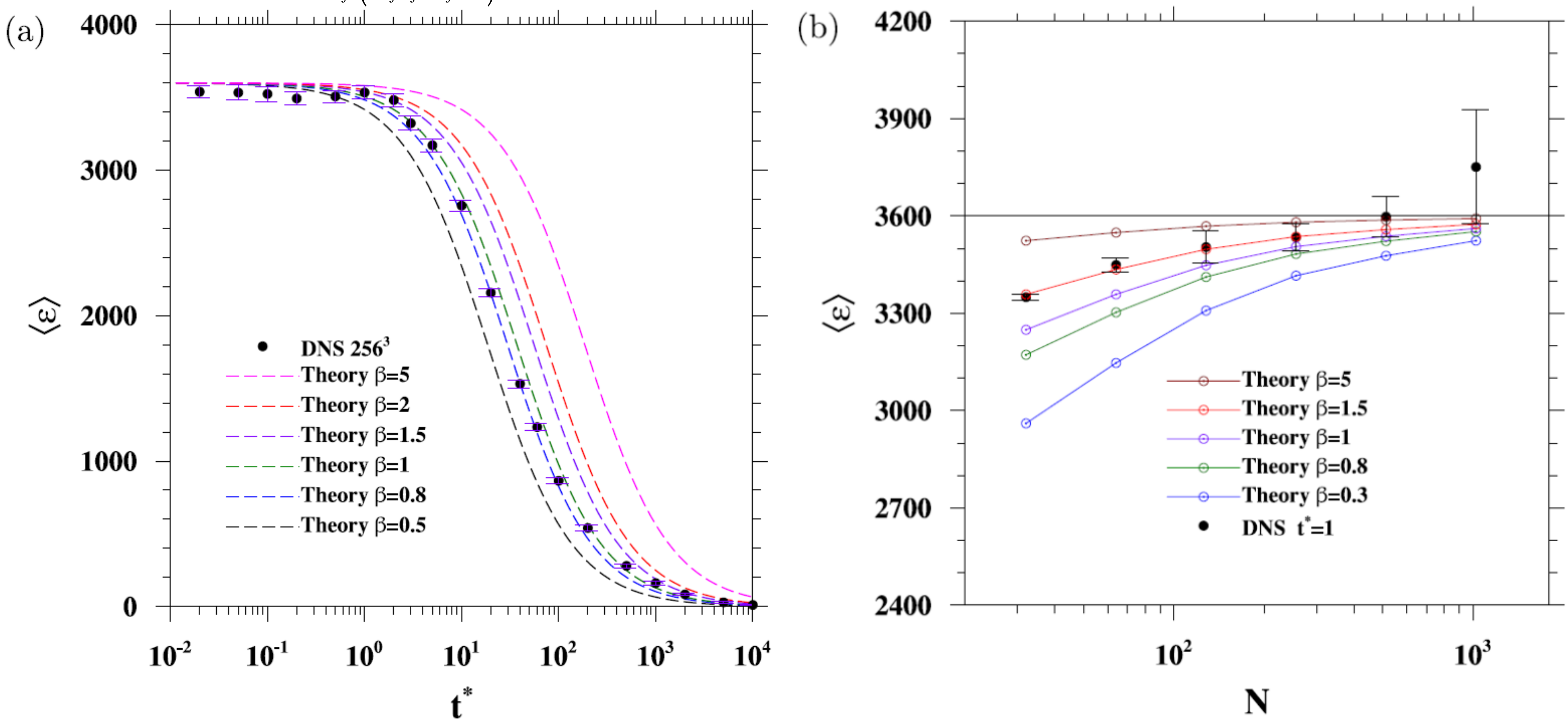


Panel (b) energy dissipation rate from DNS in log-log scale, with the x-axis being the forcing time scale normalized by eddy turnover time.

Panel (c) $< \epsilon >$ as in panel (b), with the x-axis being the forcing time scale normalized by the Kolmogorov time.

Energy dissipation rate

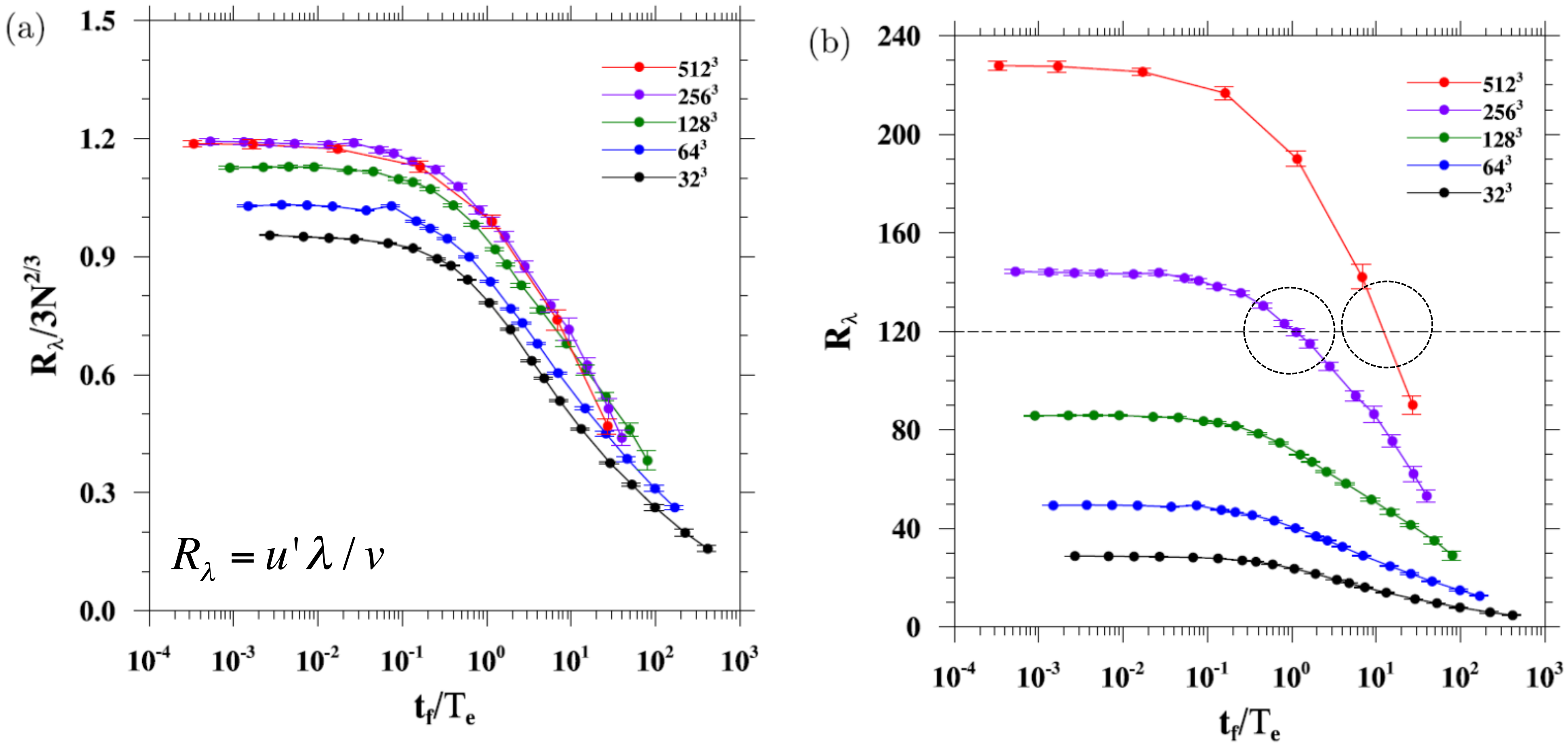
$$\varepsilon = \varepsilon_0 \times \frac{1}{1 + t_f \left(\sigma_f^2 t_f N_f k_0^2\right)^{1/3} / \beta}$$



Panel (a) energy dissipation rates as a function of t^*. Markers represent results from DNS at resolution $N = 256$. Dashed lines - model predictions for different β values. Panel (b) energy dissipation rate from DNS (black markers) for $t^* = 1$ at different resolutions. Color lines represent analytical predictions.

We confirmed that the fitting coefficient $\beta = 0.8$ gives fine prediction of expected energy dissipation rate in a wide range of forcing time scales.

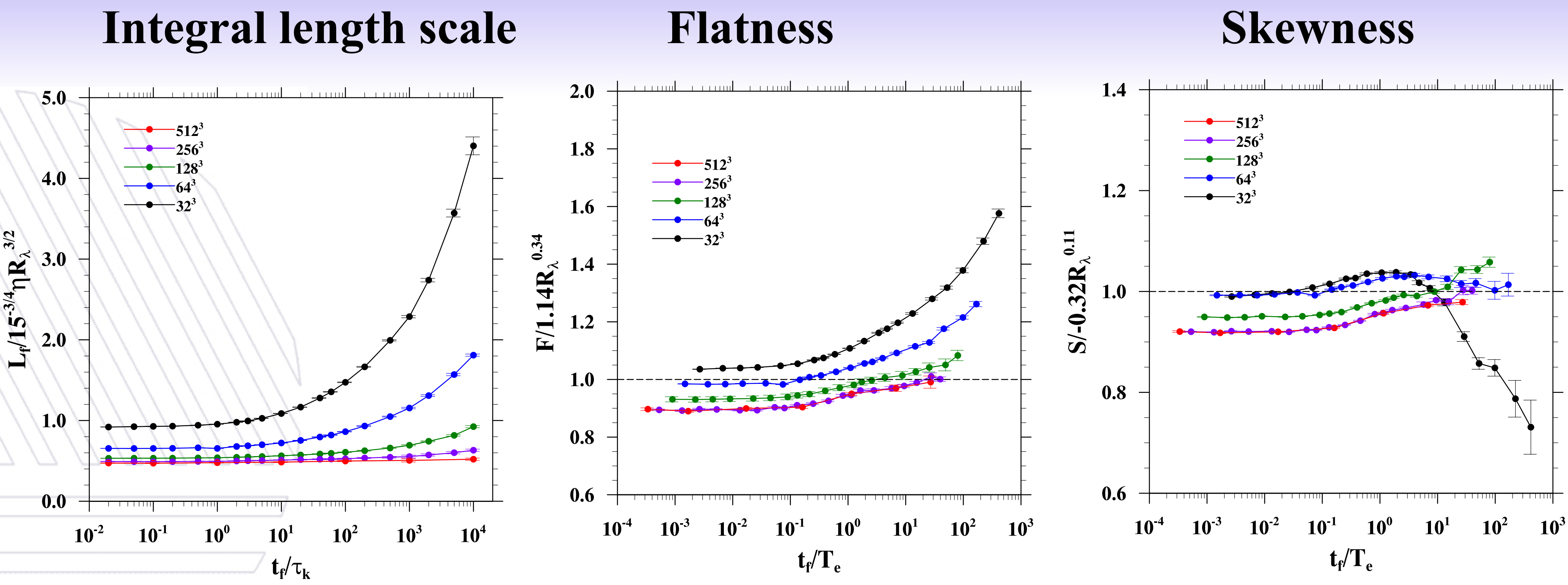
Taylor microscale Reynolds number



(a) Taylor microscale Reynolds number normalized by $3N^{2/3}$ and (b) Taylor microscale Reynolds number as a function of forcing time scale.

Similarly to ϵ, the Taylor microscale Reynolds number is also not sensitive to forcing time scale if $t_f \leq \tau_K$.
For larger t_f, R_λ decreases with the forcing time scale.

Other flow statistics



The integral length scale, flatness and skewness show a strong dependence on the forcing time scale when the scale is large, which could be a result of very low flow Reynolds numbers.

Lagrangian particle tracking

$$\frac{d\mathbf{V}^{(k)}(t)}{dt} = -\frac{\mathbf{V}^{(k)}(t) - \mathbf{U}(Y^{(k)}(t),t)}{\tau_p^{(k)}} + \mathbf{g}$$

$$\frac{d\mathbf{Y}^{(k)}(t)}{dt} = \mathbf{V}^{(k)}(t)$$

- Particles are governed by Stokes drag, gravity & inertia
- Periodic BC
- 2D domain decomposition
- Droplets modeled as “solid particles”

a (μm)	St	Sv	a/η	τ_p/T_e^*
10	0.063	0.446	0.017	0.0029
15	0.143	1.004	0.025	0.0065
20	0.254	1.784	0.034	0.0115
22.5	0.321	2.258	0.038	0.0146
27.5	0.480	3.374	0.046	0.0218
30	0.571	4.015	0.051	0.0260
40	1.015	7.138	0.068	0.0462
50	1.585	11.153	0.084	0.0722
60	2.283	16.060	0.101	0.1039

Parallel MPI implementation based on 2D domain decomposition

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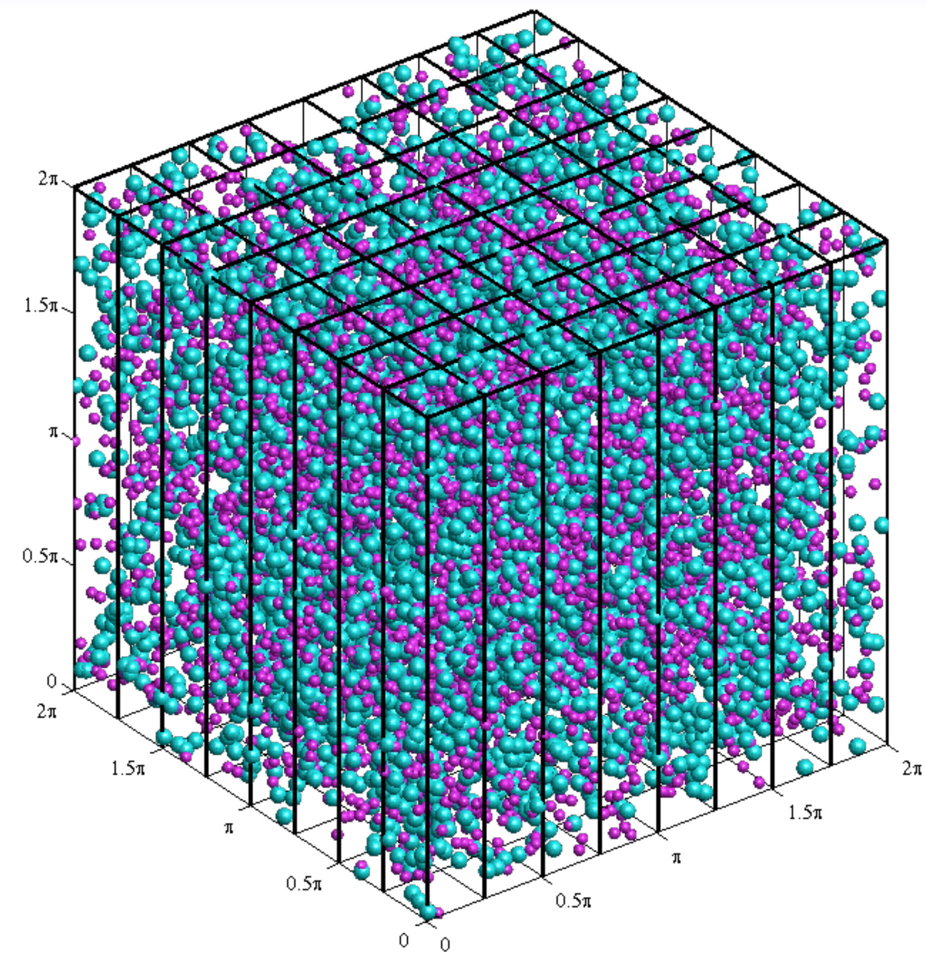
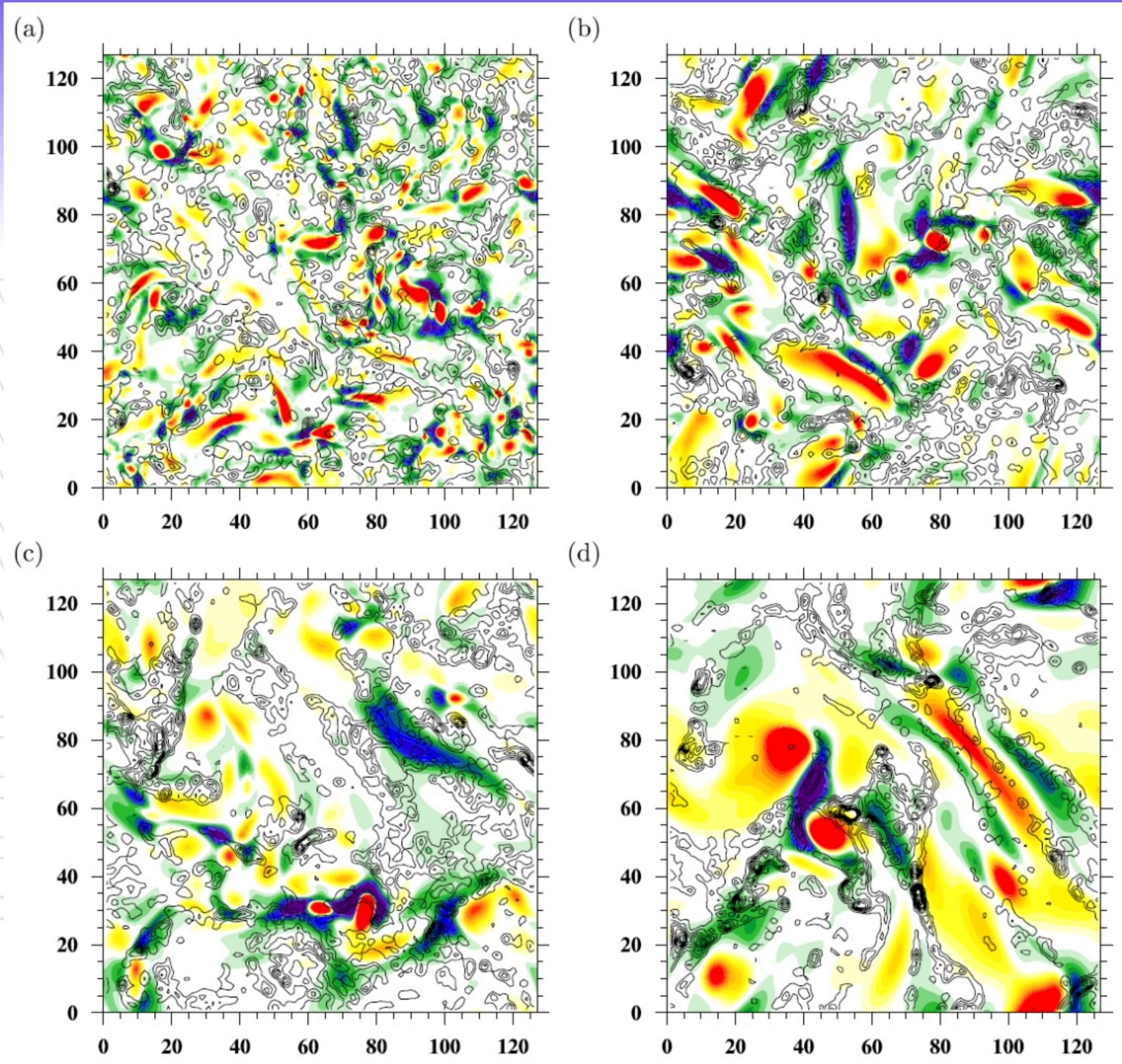
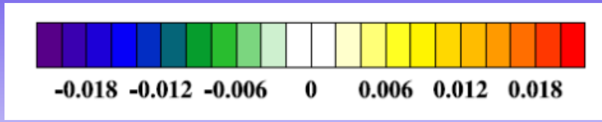


Preferential concentration of cloud droplets



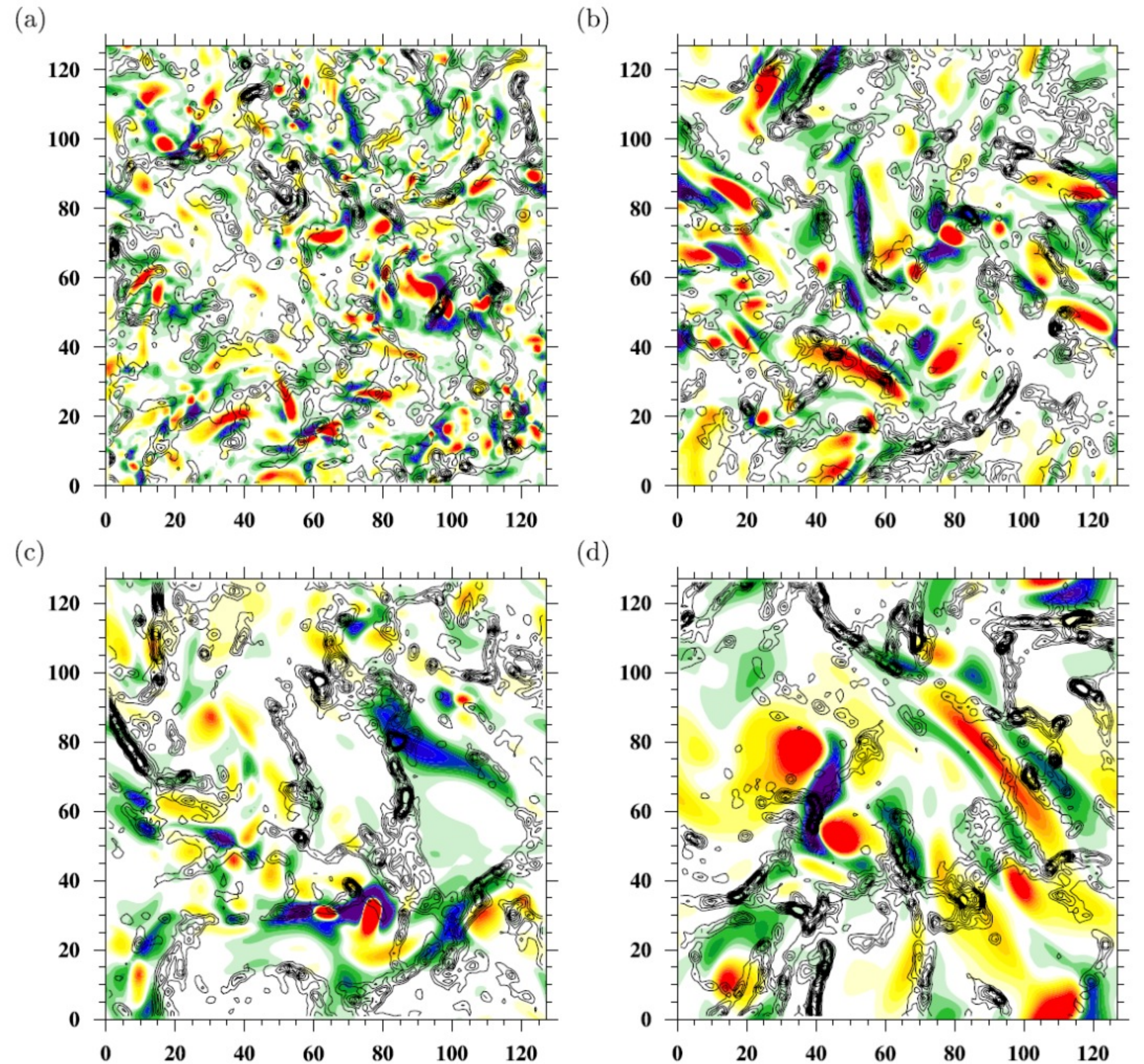


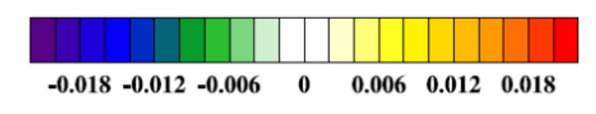
Black contour - distributions of particles with Stokes number of 0.254 (droplet radius a = 20 μm)

Colour - the second invariant in a two-dimensional horizontal cross-section of the computational domain.

Different panels correspond to simulations performed with different forcing time scales
(a) $t* = 1$, (b) $t* = 100$,
(c) $t* = 1000$, and (d) $t* = 10000$.

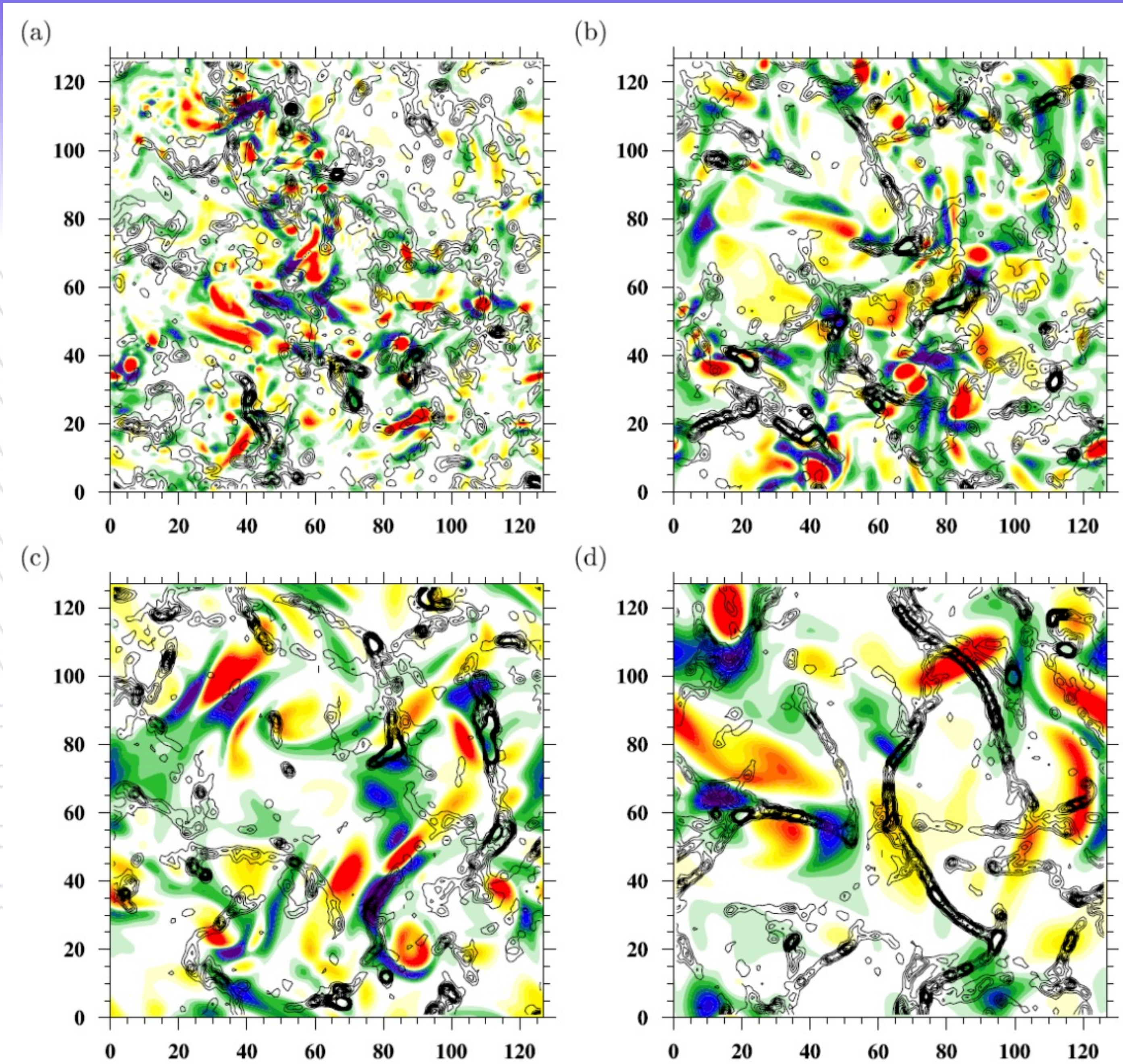
Preferential concentration of cloud droplets

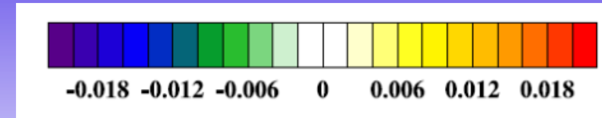




Black contour - distributions of particles with Stokes number of 0.571 (droplet radius a = 30 μm)

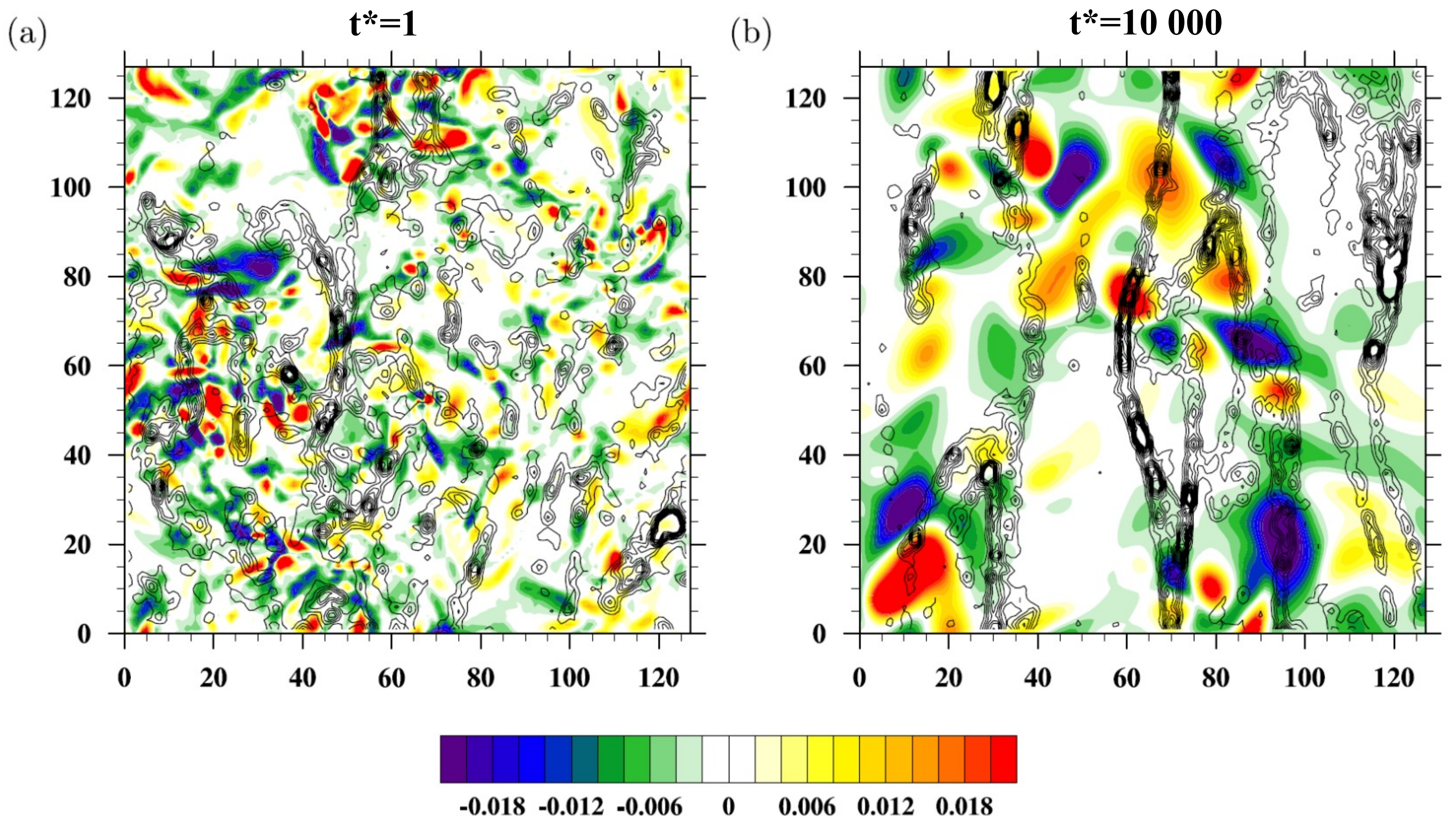
Preferential concentration of cloud droplets





Black contour - distributions of particles with Stokes number of 1.585 (droplet radius a = 50 μm)

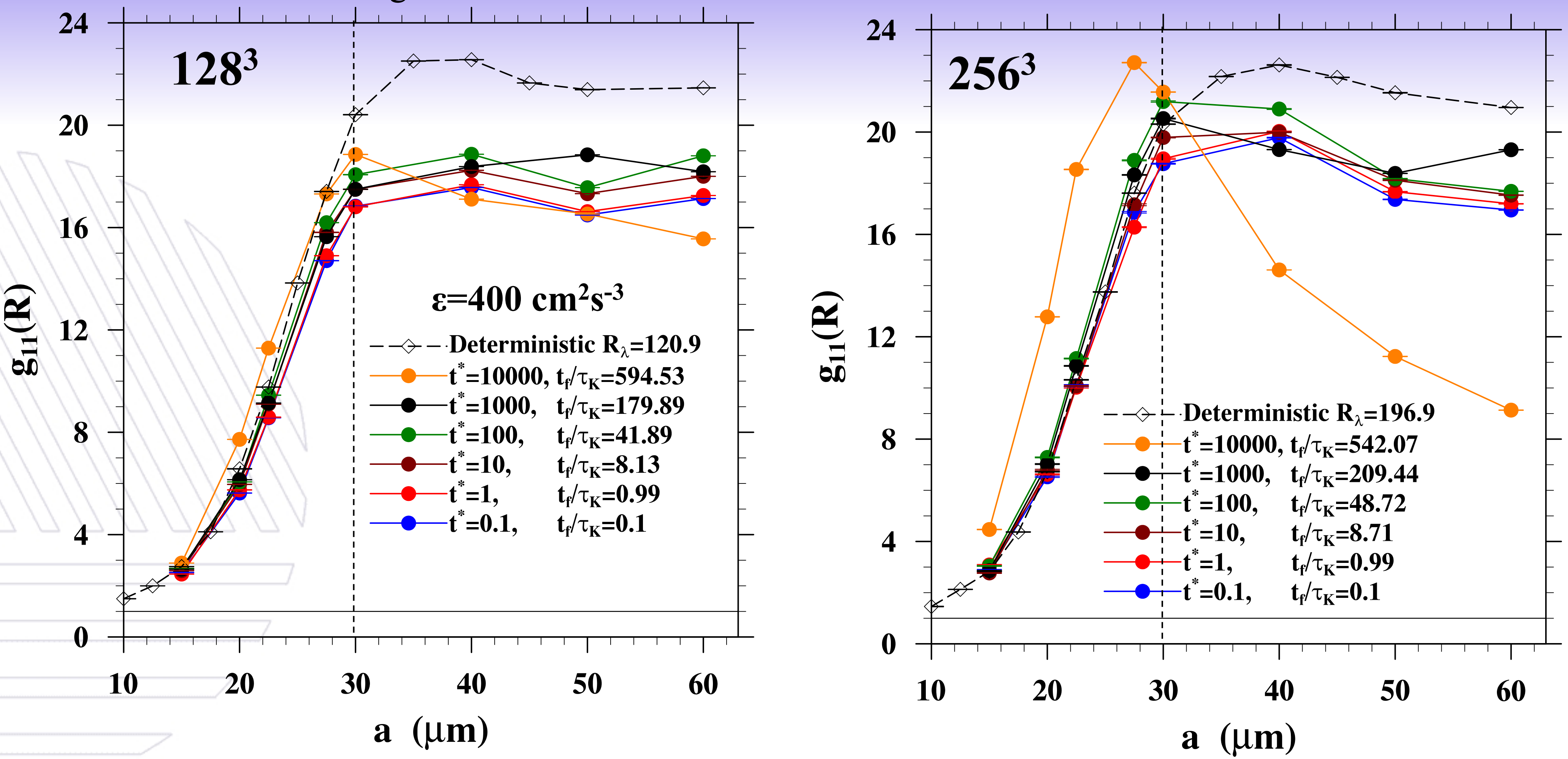
Preferential concentration of cloud droplets



Vertical cross-section. St = 1.585, droplet radius a = 50 μm

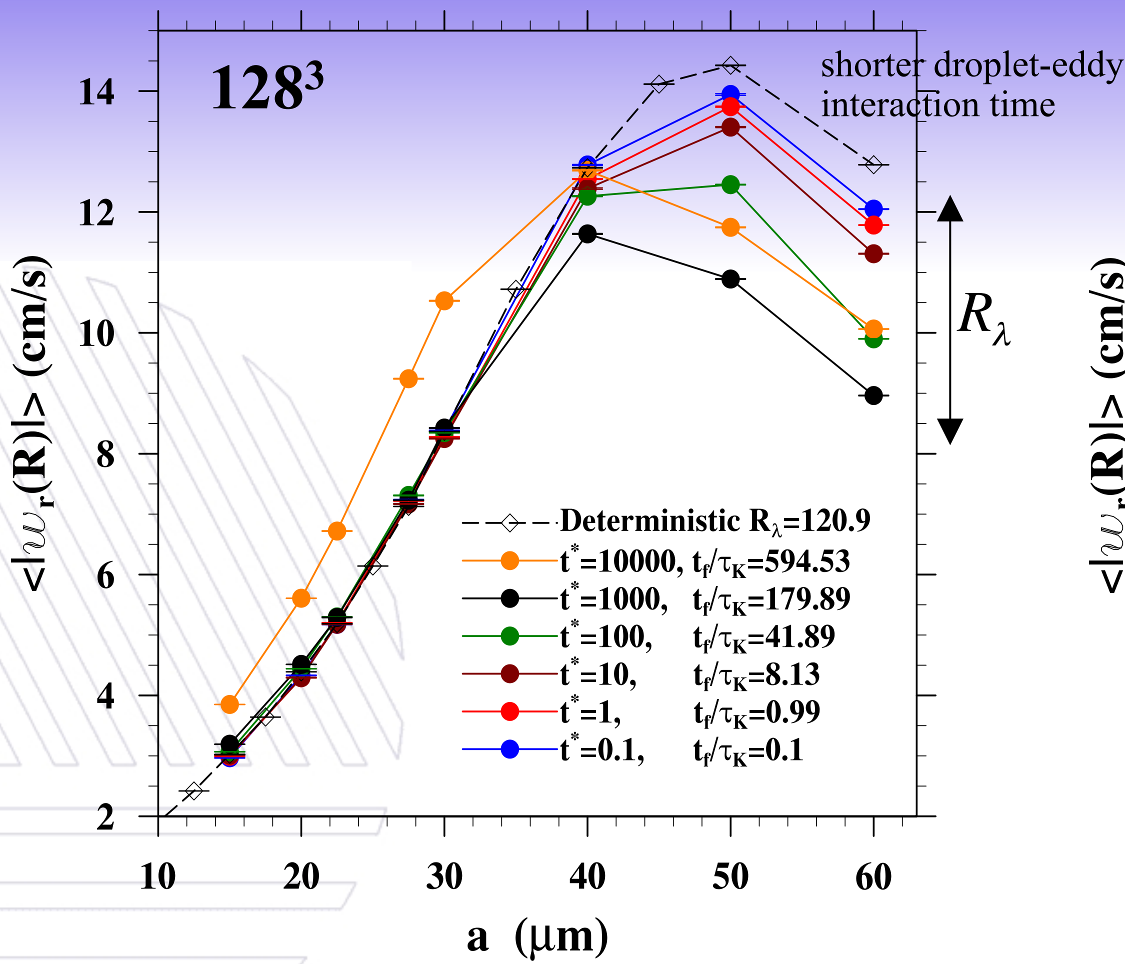
Monodisperse radial distribution function at contact

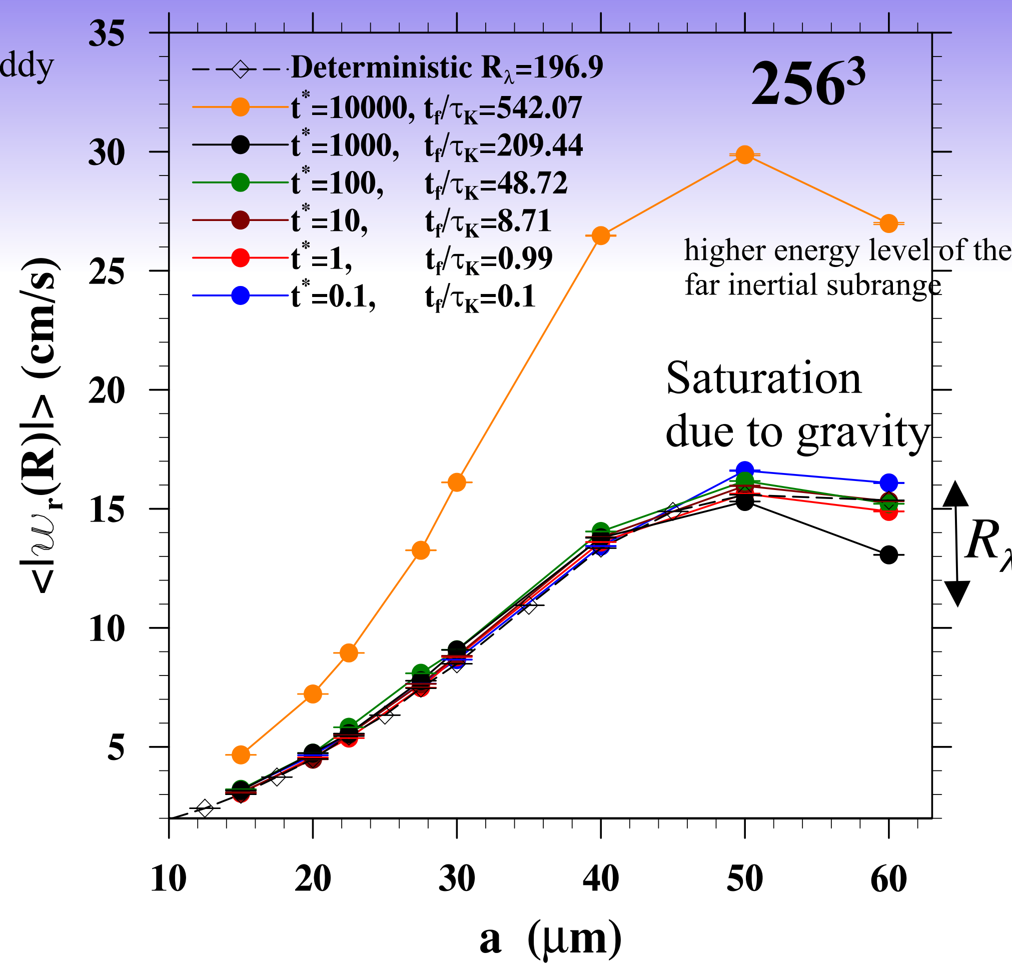
The saturation is a combined result
of gravitational sedimentation and inertia



When the gravity is included, the settling droplets accumulate in the downward flow regions forming elongated (filament-like) structures, but these structures have a length scale larger than the collision radius thus do not contribute significantly to the RDF here.

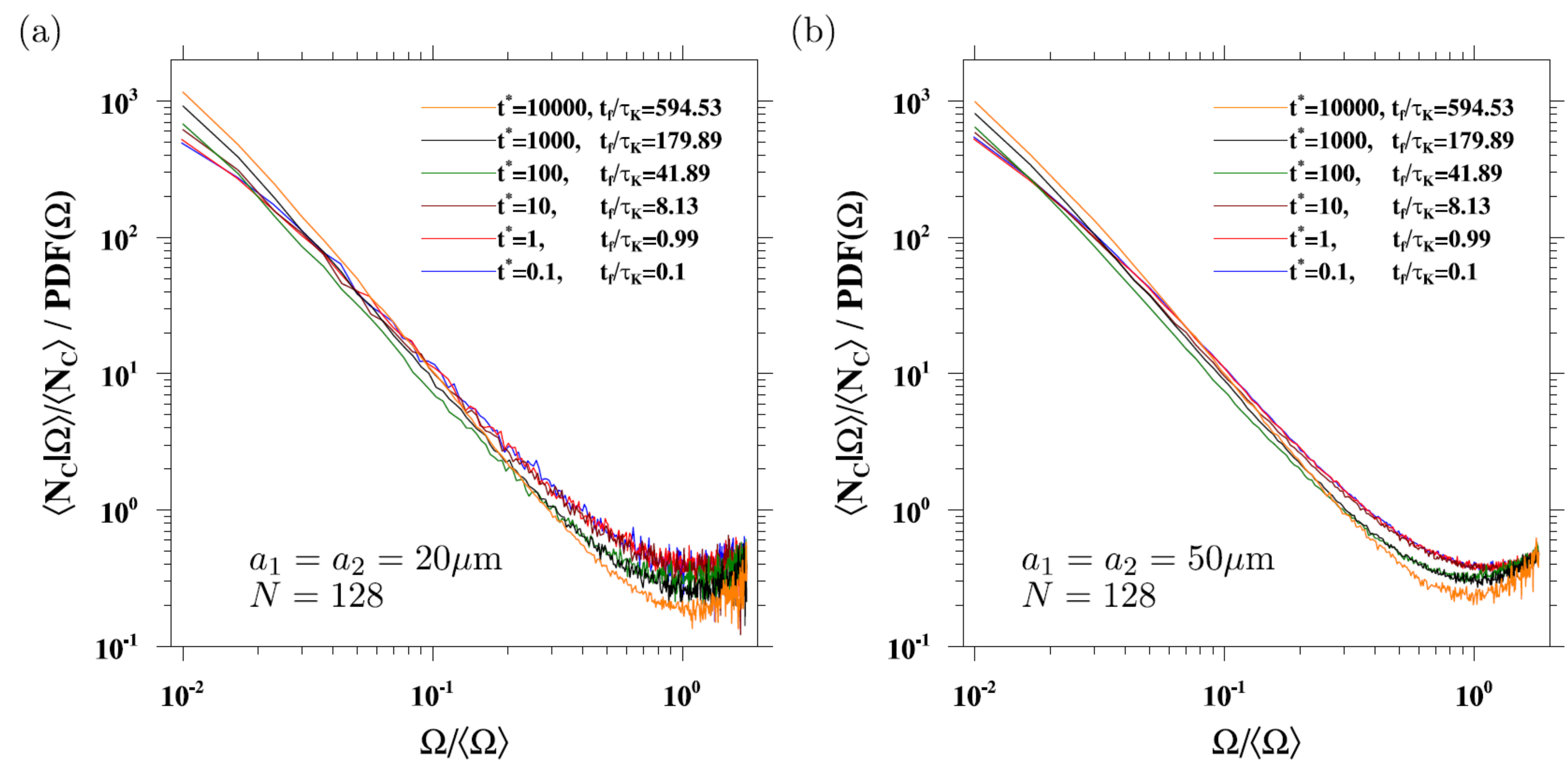
The radial relative velocity at contact





- For small particles relative velocity increases monotonically with the particle size. It is due to increasing contributions from larger scale of turbulent motion and increasing nonlocal effects (e.g. caustics).
- For larger particles, the gravity gradually diminishes the effect of turbulent motion due to shorter droplet-eddy interaction time, leading to slowly decreasing w_r with increasing droplet size.

Dynamic statistics conditioned on local enstrophy

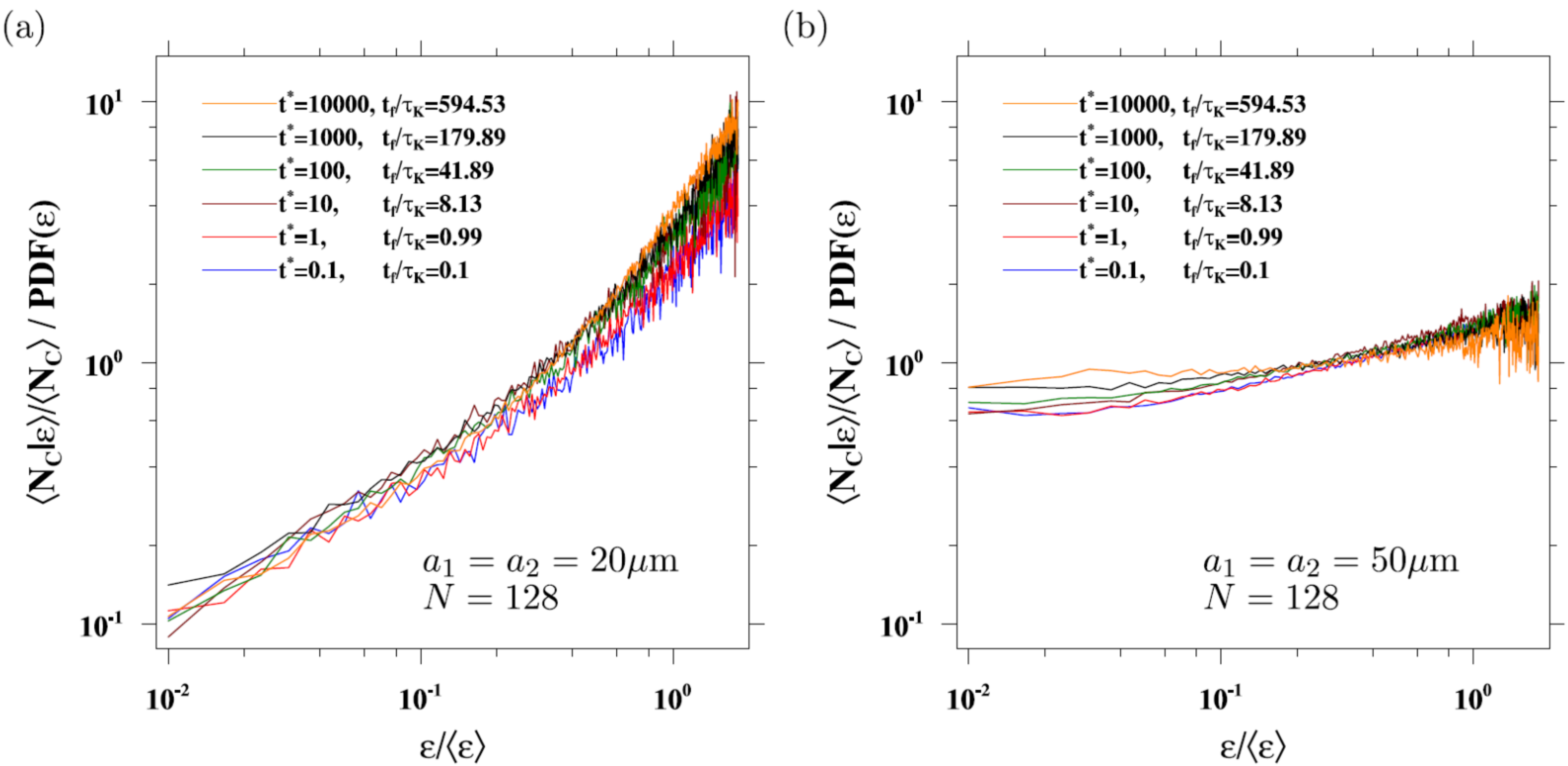


Correlation between the local collision rate and enstrophy as a function of normalized enstrophy.

$\langle N_C|\Omega\rangle$ defines conditional expectation of the number density (collisions).
$\langle N_C\rangle$ is the average collision rate.
PDF(Ω) refers to probability density function of the fluid enstrophy. Normalization by PDF is required since different enstrophy occupy different amount of physical space.

Collision statistics computed with different forcing time scales are actually quite similar. Figure indicates that there is no qualitative difference between the simulations performed with different droplet sizes

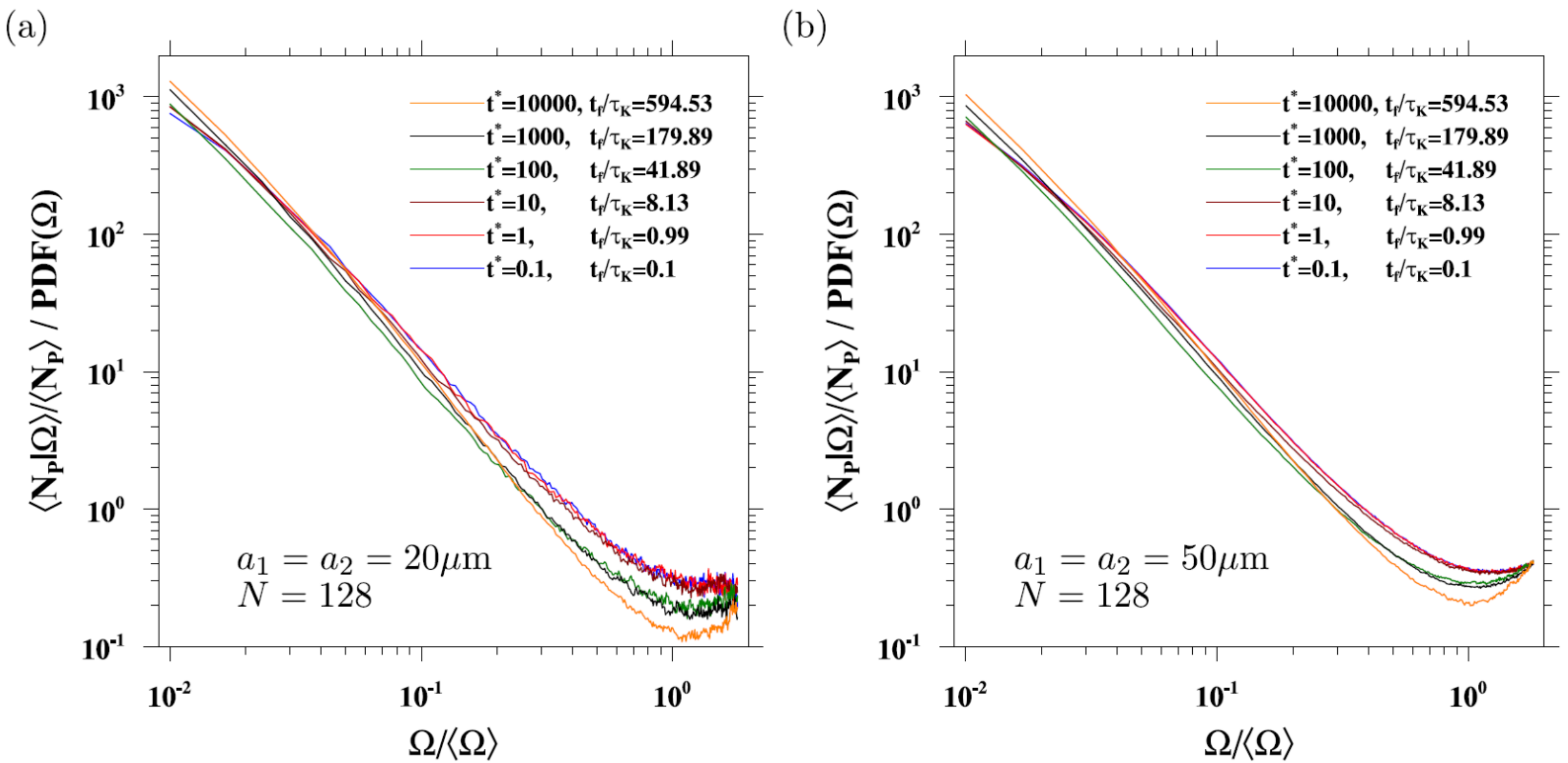
Dynamic statistics conditioned on local energy dissipation rate



Correlation between the local collision rate and energy dissipation rate as a function of normalized energy dissipation rate. Panel (a) monodisperse systems with 20 μm droplets; (b) monodisperse systems with 50 μm droplets.

For 20 μm droplets the regions with higher collision rate are correlated with the region of high energy dissipation rate. However, for larger droplets (50 μm), the spatial distribution of collision events corresponds closely to PDF(ϵ) of the flow.

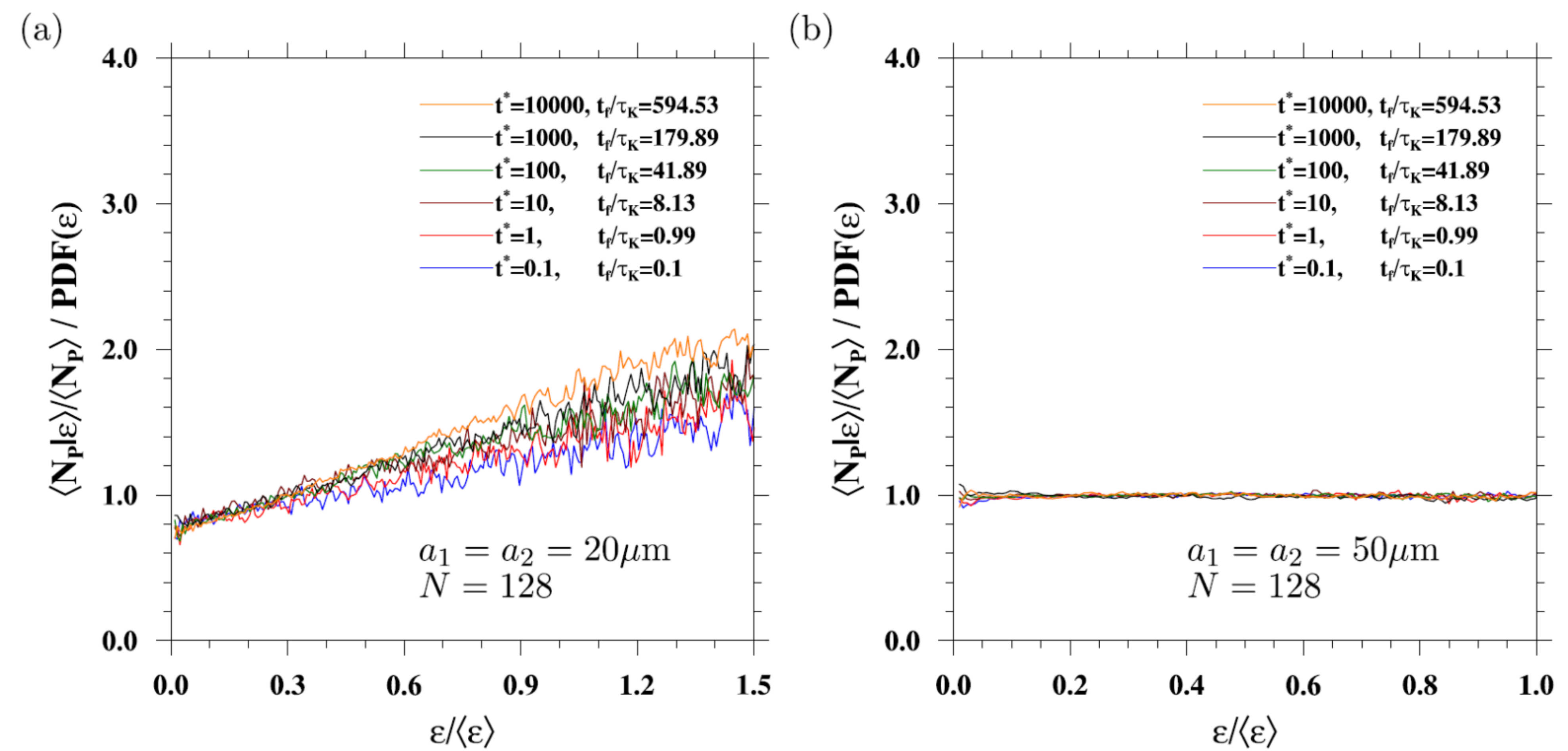
Kinematic statistics conditioned on local enstrophy



Statistics for nearly touching droplets. Panel (a) monodisperse systems with 20 μm droplets; (b) monodisperse systems with 50 μm droplets.

Figures demonstrate a good agreement of simulation results with different forcing time scales.
We conclude that the forcing time scale does not significantly affect these conditional statistics.

Kinematic statistics conditioned on local energy dissipation rate



The correlation is less sensitive to ϵ than it was observed for colliding pairs

Summary and conclusions

- Using DNS, we have examined the effects of forcing time scale on the characteristics of the forced turbulent flows.
- We find that the size of vortical structures at small scales increases with increasing time scale.
- The simulation results lead to the conclusion that the flow properties do not change if $t_f < t_K$
- Both the flow dissipation rate and Taylor microscale flow Reynolds number depend on the forcing time scale.
- The skewness and flatness show a strong dependence on the forcing time scale when the forcing time scale is large, which could be a result of very low flow Reynolds numbers.
- We investigated the effects of forcing time scale on the kinematic collision statistics. The results show that the RDF and w_r may depend on the forcing time scale if it becomes large. This dependence, however, can be largely explained in terms of the altered flow Reynolds number and the changing range of flow length scales present in the turbulent flow.