Photonics – series 6 (Discrete Fourier Transform (DFT), Fraunhofer and Fresnel diffraction)

Total: 16 points (12 points + 4 „beauty” points) + 25 BONUS POINTS

The Discrete Fourier Transform and Inverse Fourier Transform of a vector $f \in \mathbb{C}^N$ are defined as $\hat{f}_{m+1} = \sum_{l=0}^{N-1} f_{l+1} \cdot e^{-2\pi i m l / N}$ and $f_{l+1} = N^{-1} \sum_{m=0}^{N-1} \hat{f}_{m+1} \cdot e^{2\pi i m l / N}$ respectively, where $l, m = 0 ... (N-1)$. DFT can be calculated by using very efficient Fast Fourier Transform (FFT) algorithm, which is especially fast when the number of points $N = 2^p$, where *p* is integer number. DFT is used as an approximation of continuous FT, which has many important applications in optics. For example FT is essential for the derivation of far-field diffraction.
Useful functions: fft, ifft, fftshift, fft2, ifft2, conv2

Problem 1 (3 points):
(Fraunhofer diffraction – single aperture)
Calculate light intensity distribution in the far field for the following apertures:

1. Circle of diameter *d*.
2. Rectangle $d \times 2d$.
3. Equilateral triangle with side *d*.

How does the image change if the aperture is shifted? Interpret the result – does the far-field image actually shift? Why is it possible to achieve the same intensity distribution for different initial apertures, even though FT and IFT are uniquely defined?
Interpret the dependence of far field distribution on the size of aperture. Is it possible to achieve an arbitrarily narrow diffraction image?
How does the image change if the aperture is substituted by its negative?

Problem 2 (3 points):
(Fraunhofer diffraction – multiple apertures)
Calculate the light intensity in the far field for apertures consisting of several apertures of similar size distributed symmetrically or randomly (without point symmetry).
Is the far field image symmetric in both cases?
Do we always observe non-zero intensity for the zero-order frequency?
Which properties of apertures influence the answers to the previous questions and is it possible to construct a system, for which the answer will be different?

Problem 3 (3 points):
(Fraunhofer diffraction – diffraction gratings)
Calculate light intensity in far field for two-dimensional diffraction grating with period *L* placed inside a circular aperture of radius *5L*.

1. Binary grating with periodically placed lines of constant width
2. Sinusoidal grating with real non-negative transmittance
3. Phase grating (constant transmission with periodic change of phase)
4. Binary phase grating

Which diffraction orders are observed in these cases?

Problem 4 (BONUS – 10 points to any series):

(Image pixelization)

Assume that sinusoidal diffraction grating is constructed from finite size pixels. How does far field diffraction image look for pixelised grating? How does the result depend on pixel size? How does the image look for single pixel? Hint: pixelization means that aperture is divided into smaller rectangular clusters, and values of all transmission coefficients in one cluster are averaged.

Problem 5 (3 points):

(Designing diffractive elements)

Calculate a phase diffractive element and a binary amplitude diffractive element, which transforms a plane wave into several bright round spots in far field image. Calculate far field image from this element.

Hint: The simplest algorithm to calculate transmittance of a diffractive element consists of the following steps:

1. Use far field intensity and random phase to calculate the complex amplitude.
2. Use IFT to calculate the complex transmittance of diffractive element.
3. Use phase-only or binary amplitude (based e.g. on sign of amplitude) coding for the diffractive element.

Problem 6 (BONUS – 5 points to any series):

(Partial coverage of diffractive elements)

Calculate and interpret the far field image of a diffractive element from problem 5 which is partially covered. How does coverage area influence far field image?

Problem 7 (BONUS – 10 points to any series):

(Fresnel lensing)

Calculate the image of an object achieved through phase Fresnel lens:

1. Fix the lens at a constant non-zero distance from the input plane and change the output screen position.
2. Fix the screen position and vary the lens-object distance
3. Switch the phase Fresnel lens to a binary amplitude Fresnel lens. Can you observe several focal planes?

Hint: use Fresnel diffraction limit – calculate image as a convolution of input field and impulse response function of free space.