Photonics – series 7 (Stratified structures, transfer and scattering matrices)

Total: 44 points (33 points + 11 „beauty" points) + 10 BONUS POINTS

Before solving problems prepare the following functions, as they will be massively helpful:
- Calculate the transfer matrix of an interface between two media. Input arguments are the dielectric constants or refractive indices of both media, polarization (TE\TM) and incident angle or wavevector ratio k_y/k_0 . Hint: Note that incident angle in problems 1,2,4-9 is equal to 0, as this fact simplifies calculations.
- Calculate the propagation matrix of a chosen medium. Input arguments are the dielectric constant or refractive index of the medium, medium thickness or normalized thickness ($d \cdot k_0$) and incident angle or wavevector ratio k_y/k_0 .
- Transform given scattering matrix into the transfer matrix and vice versa

Problem 1 (3 points):

(Anti-reflective coating)

A thin layer consisting of a material with the refractive index $n_2=\sqrt{n_1 \cdot n_3}$ and thickness $d=0.25 \cdot \lambda / n_2$ works as an anti-reflective coating between air $n_1=1$ and silica glass $n_3=1.46$. Show the dependence of intensity reflection coefficient $R=|r_{13}|^2$ on the refractive index of the coating n_2 and its thickness d .

Show the transmission coefficient for anti-reflective coating made of MgF_2 ($n_2=1.37$)?

Problem 2 (3 points):

(Interference on thin film – symmetric Fabry-Perot interferometer)

Plane wave with wavelength λ and linear polarization (TE or TM) propagates from medium with refractive index n_1 through a layer of thickness d and refractive index n_2 back into the medium n_1 . All interfaces are parallel to each other and perpendicular to the propagation direction of the incident wave. Calculate the intensity transmission coefficient $T=|t_{13}|^2$ and intensity reflection coefficient $R=|r_{13}|^2$ as a function of wavelength. Assume propagation through a water film in the air for several different values of *d*.

Problem 3 (3 points):

(Optical tunnelling)

Consider the system from the previous problem, with $n_1=1.45, n_2=1$ and incident angle $\theta>\theta_{crit}=asin(n_2/n_1)$. Calculate the ransmission and reflection coefficients $T=|t_{13}|^2$ and $R=|r_{13}|^2$ as a function of layer thickness for a selected polarization.

Problem 4 (3 points):

(Laser action resonator)

Consider a resonator system consisting of two DBR mirrors with reflection coefficients $r_1, r_2 \sim \pm 0.9958$ located parallel to each other at a distance d . Assume that refractive index of the medium between the mirrors is equal to $n=1-i\alpha$, where $\alpha>0$ is responsible for amplification of the time-dependent field $\propto \exp(-i\omega t)$. Prove that the total transmission coefficient of the system can be higher than one and even infinite.

Infinite transmission in experimental system means that infinitesimally small perturbation can produce very high output intensity. In practice output intensity is limited by factors not included in this model (for example limited energy transfer into the medium).

Hint: amplification during a full cycle should be higher than radiation of energy $|r_1 \cdot r_2| = e^{-2\alpha d k_0}$, but it requires certain resonator length.

Problem 5 (3 points):

(Bragg mirror)

A Bragg mirror consists of interchanging layers of two materials with dielectric indices n_i and thicknesses d_i selected so that $d_i \cdot n_i = \lambda/4$ in every layer. Calculate the reflection coefficient as a function of the number of layers. How many layers should be included in a Bragg mirror so that the reflection coefficient exceeds $R = |r|^2 \geq 0.995$. Assume that $n_1 = 3.5$, $n_2 = 3$.

Problem 6 (3 points):

(Resonant tunnelling – transparent metal)

Tunnelling through a system of two barriers (e.g. two thin metal layers or two thin dielectric layers with incident angle higher than the critical angle) may take place without any reflection. In the case of negligible absorption this leads to the total transmission $T = 1$. Multiplication of this system can lead to significantly thicker structures, such as „transparent metals" - transparent materials constructed mainly from metal.

Calculate the transmission through two thin layers of silver as a function of the distance between layers. Assume that the layer thickness is $d_{Ag} = 15\text{nm}$, in case of wavelength $\lambda = 633\text{nm}$ refractive index $n_{Ag} = 0.059 + 4.281\text{i}$. How does the system change if layers consist of a perfect conductor (e.g. $n_{Ag} = 4.281\text{i}$)?

Problem 7 (3 points):

(Interference filter – photonic crystal with a defect)

Consider a Bragg mirror made of 22.5 pairs of layers with refractive indices $n_1 = 3.5$ and $n_2 = 3$ in the air. Calculate the transmission coefficient as a function of wavelength for a perpendicular wave propagation. How does the transmission spectrum change if the centrally located layer had a different thickness? How to interpret the appearance of narrow spectral line inside the photonic band-gap? Can you think of a possible application for such a structure?

Problem 8 (BONUS - 5 points):

(Wide-band anti-reflective coating)

Assume that you can use materials with an arbitrary refractive index $n_1 < n_i < n_2$ to construct an anti-reflective coating between materials with refractive indices n_1 and n_2. To construct it, assume a structure made of N layers with dielectric permittivities changing via geometric mean between $\epsilon_1 = n_1^2$ and $\epsilon_2 = n_2^2$. Layers thicknesses $d_i = 0.25 \cdot \lambda_0 / n_i$. Calculate the reflection spectrum $R(\lambda)$ for $0.5\lambda_0 < \lambda < 5\lambda$.

Hint: geometric mean $n_2 = \sqrt{n_1 \cdot n_3}$

Problem 9 (3 points):

(Electromagnetic absorber)

Consider a structure constructed from thin (several nanometers) silver layer suspended in the air at a distance $d = \lambda/4$ from a thick layer of the same metal. Calculate the reflection coefficient as a function of the thin layer thickness for chrome ($n_{Cr} = 3.34 + 4.27i$ at $\lambda = 633$). Repeat the calculations for slightly higher distance between layers.

Problem 10 (3 points):

(Modal structure of a planar waveguide)

Calculate the reflection coefficient of a thin dielectric layer with refractive index *n* in the air as a function of $k_0 \cdot d$ and β / k_0 , where k_0 is wavenumber in free space, β is wavevector component parallel to the interface, and d is layer thickness. Prepare a plot for a selected polarization and for $1 \leq \beta / k_0 \leq n$. Locate waveguide modes in the plot. Can you approximate the effective refractive index of these modes based on the image?

Problem 11 (3 points):

(Modal structure of a double-core waveguide)

Repeat calculations from problem 10 for two close thin dielectric layers with refractive index *n* suspended in the air. Discuss the difference between a single- and double-layered structures.

Problem 12 (3 points):

(Photonic band gap)

Repeat calculations from problem 10 in the range $0 \leq \beta / k_0 \leq 1$ for large but finite number of periodically located layers. Discuss the physical meaning of different areas in the plot.

Problem 13 (BONUS - 5 points):

(Omnidirectional Bragg reflection)

Repeat calculations from problem 12 for a Bragg mirror constructed from layers with refractive indices $n_1 = 1.7$ and $n_2 = 3.4$. Note that if light has Bragg wavelength, photonic band gap appears for every $\beta < k_0$. What may be a possible application for this kind of structure?