Particle-turbulence interactions in atmospheric clouds

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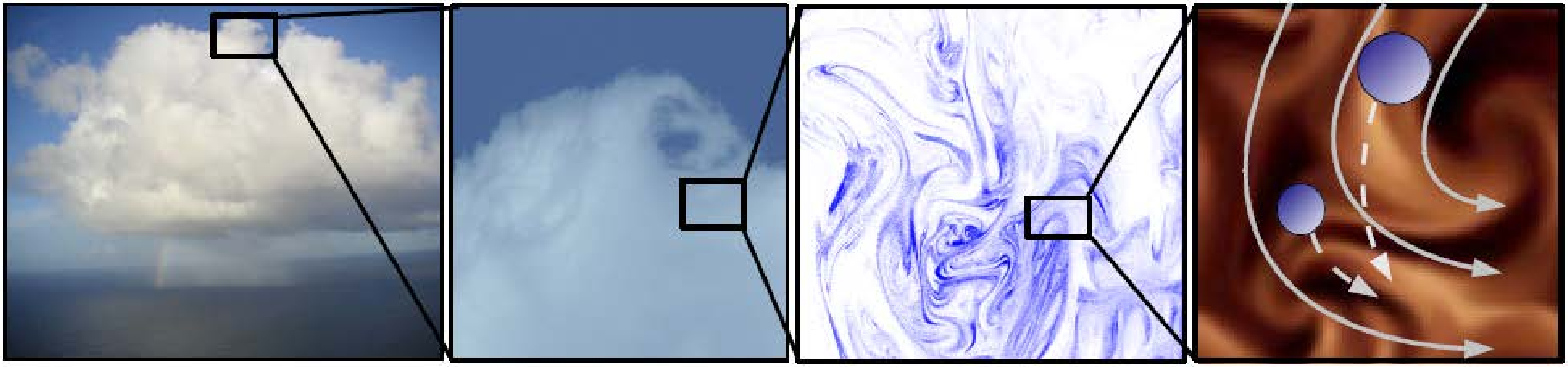
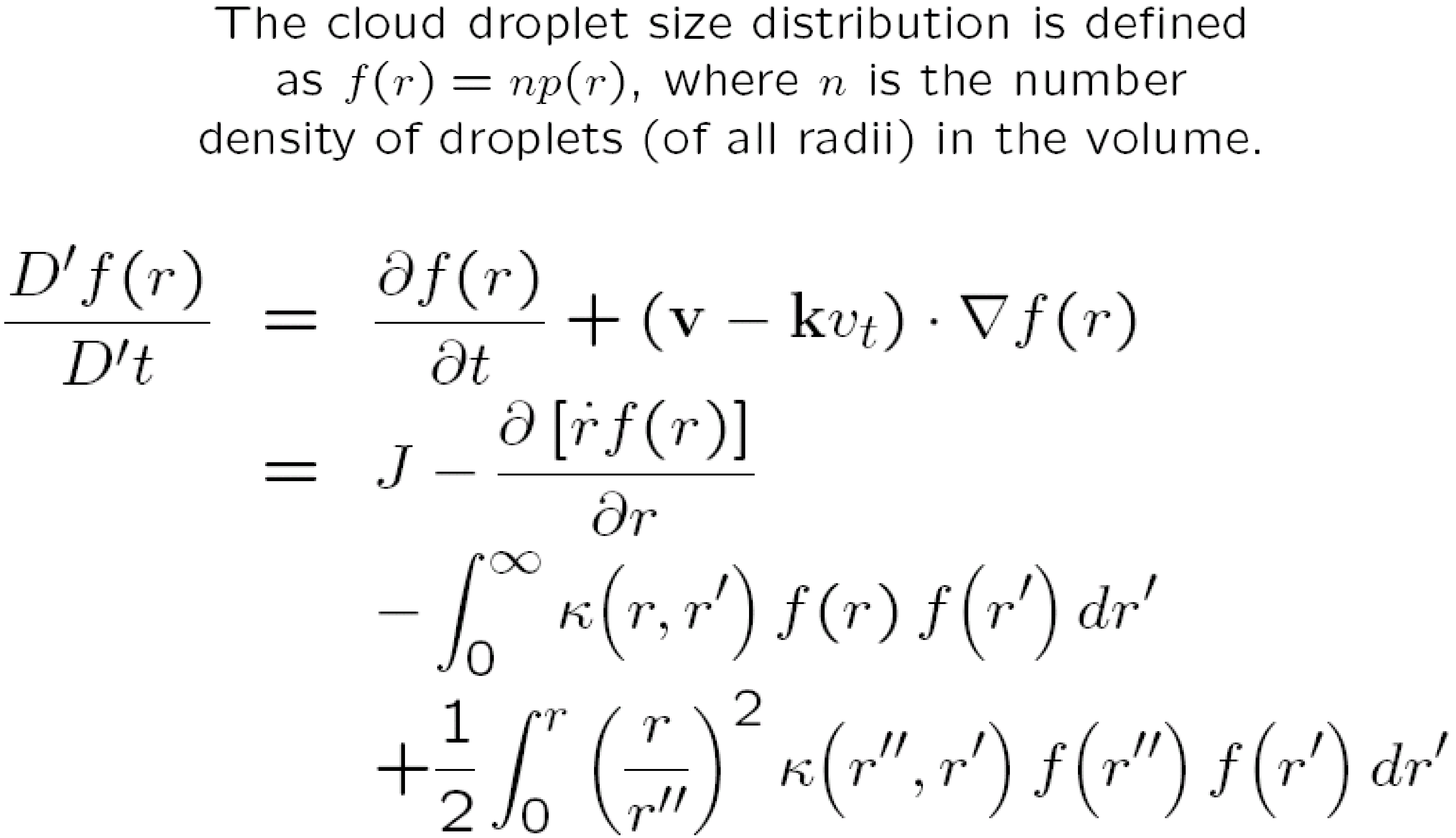


Adapted from Bodenschatz et al. 2010

Size distribution evolution (warm clouds)

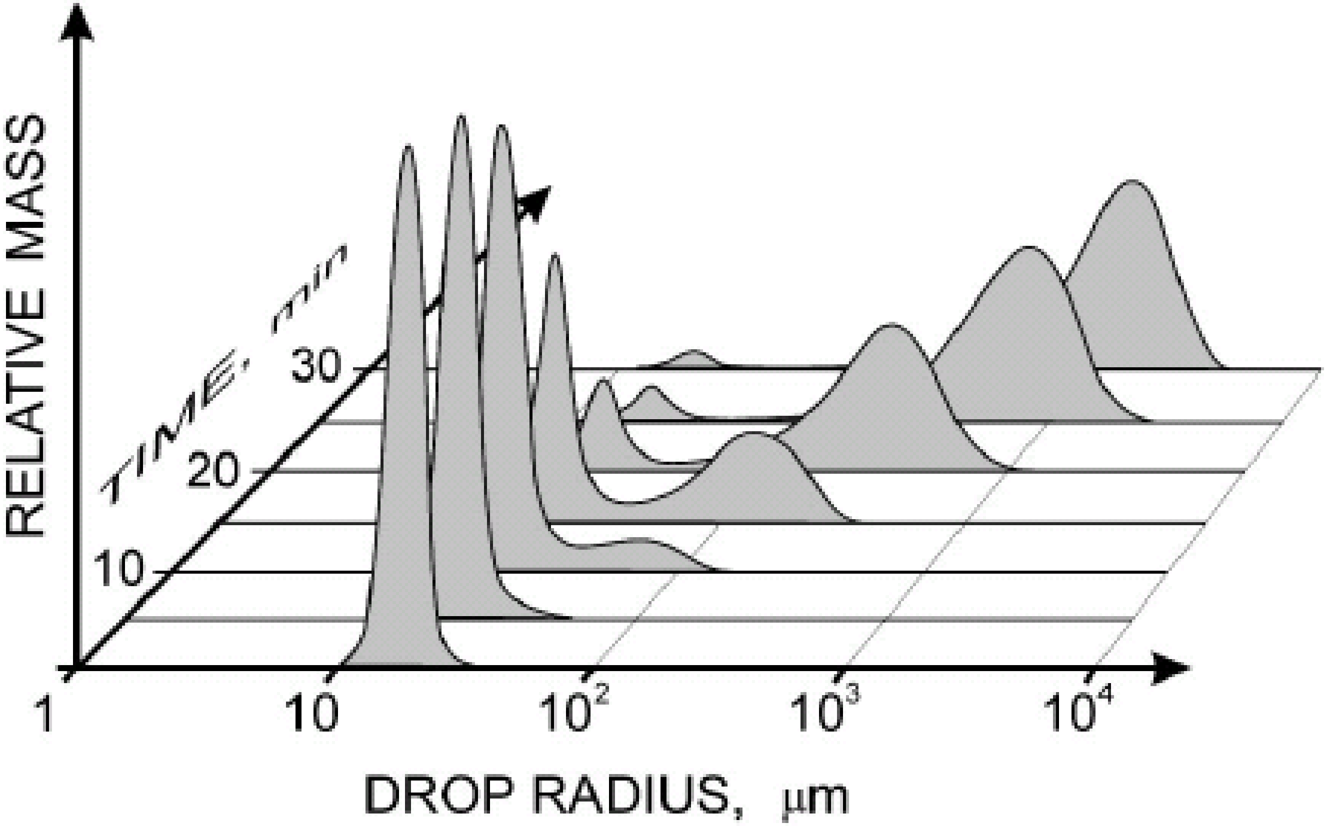


The cloud droplet size distribution is defined as $f(r) = np(r)$, where n is the number density of droplets (of all radii) in the volume.

$$
\begin{aligned}
\frac{D' f(r)}{D' t} &= \frac{\partial f(r)}{\partial t} + (\mathbf{v} - \mathbf{k} v_t) \cdot \nabla f(r) \\
&= J - \frac{\partial \left[\dot{r} f(r) \right]}{\partial r} \\
&\quad - \int_0^\infty \kappa\left(r, r'\right) f(r)\, f\left(r'\right) dr' \\
&\quad + \frac{1}{2} \int_0^r \left(\frac{r}{r''} \right)^2 \kappa\left(r'', r'\right) f\left(r''\right) f\left(r'\right) dr'
\end{aligned}
$$

Shaw 2003

Size distribution evolution (warm clouds)



Shaw 2003

Generalized collision kernel…

Cloud droplet collision rate

$$\dot{N}_{12} = \bar{n}_1 \bar{n}_2 K_{12}$$

Cloud physics … gravity-dominated (cylindrical)

$K_{12}^{g} = \pi R^2 |w_1 - w_2| \epsilon_{12}$ where $R = r_1 + r_2$

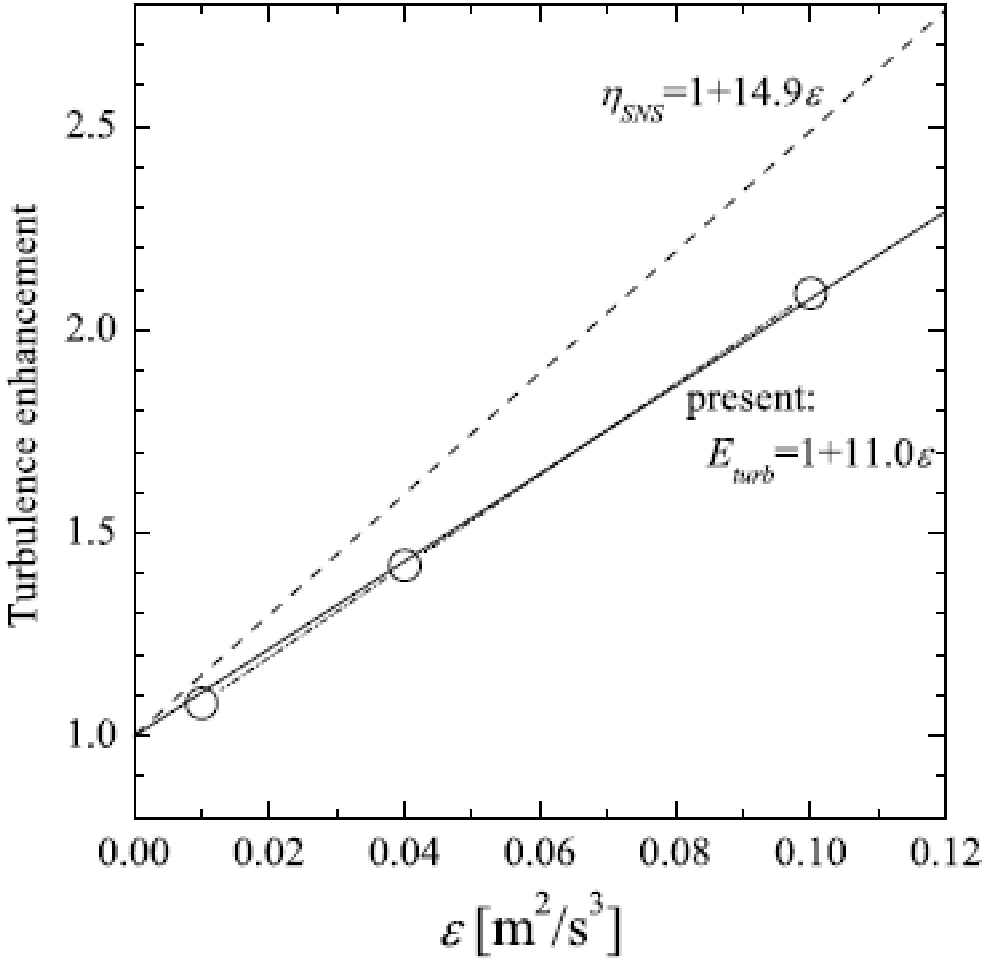
Generalized … allows for turbulence (spherical)

$$K_{12}^{t} = 2\pi R^2 g_{12}(R) \langle w_{12}(R) \rangle_{-}$$

$g_{12}(r)$	radial distribution function (probability of finding droplets of sizes 1 and 2 separated by distance r, relative to that expected for a perfectly random distribution)
$\langle w_{12}(r) \rangle_{-}$	*inward* radial velocity, averaged over spherical surface

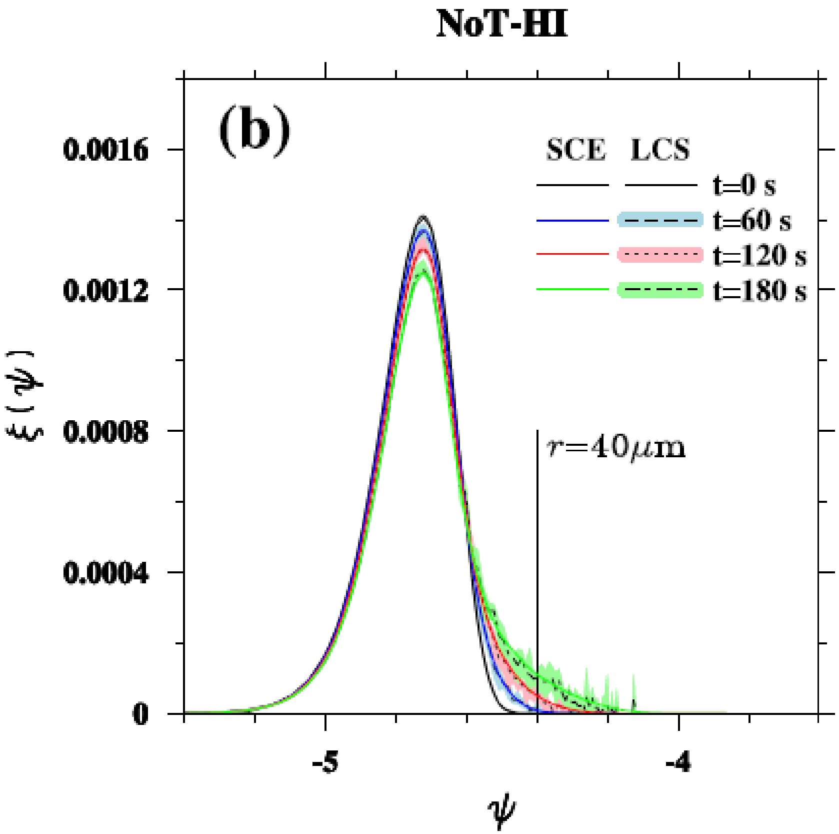
Sundaram and Collins 1997; Wang et al. 1998

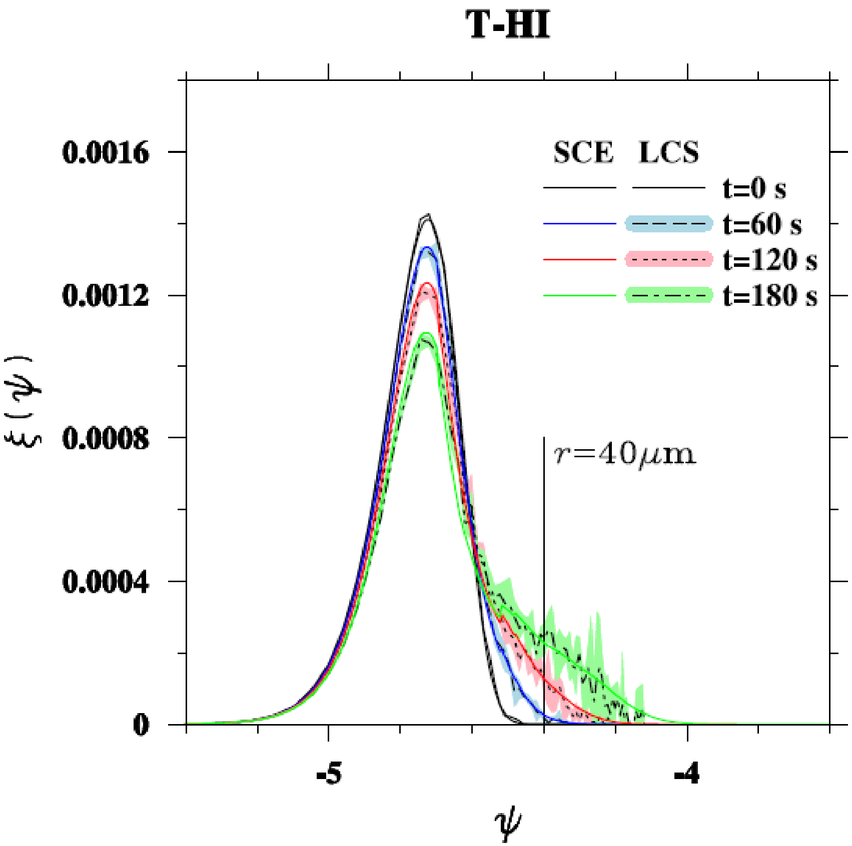
Turbulence enhancement of collision rate



Onishi et al. 2015

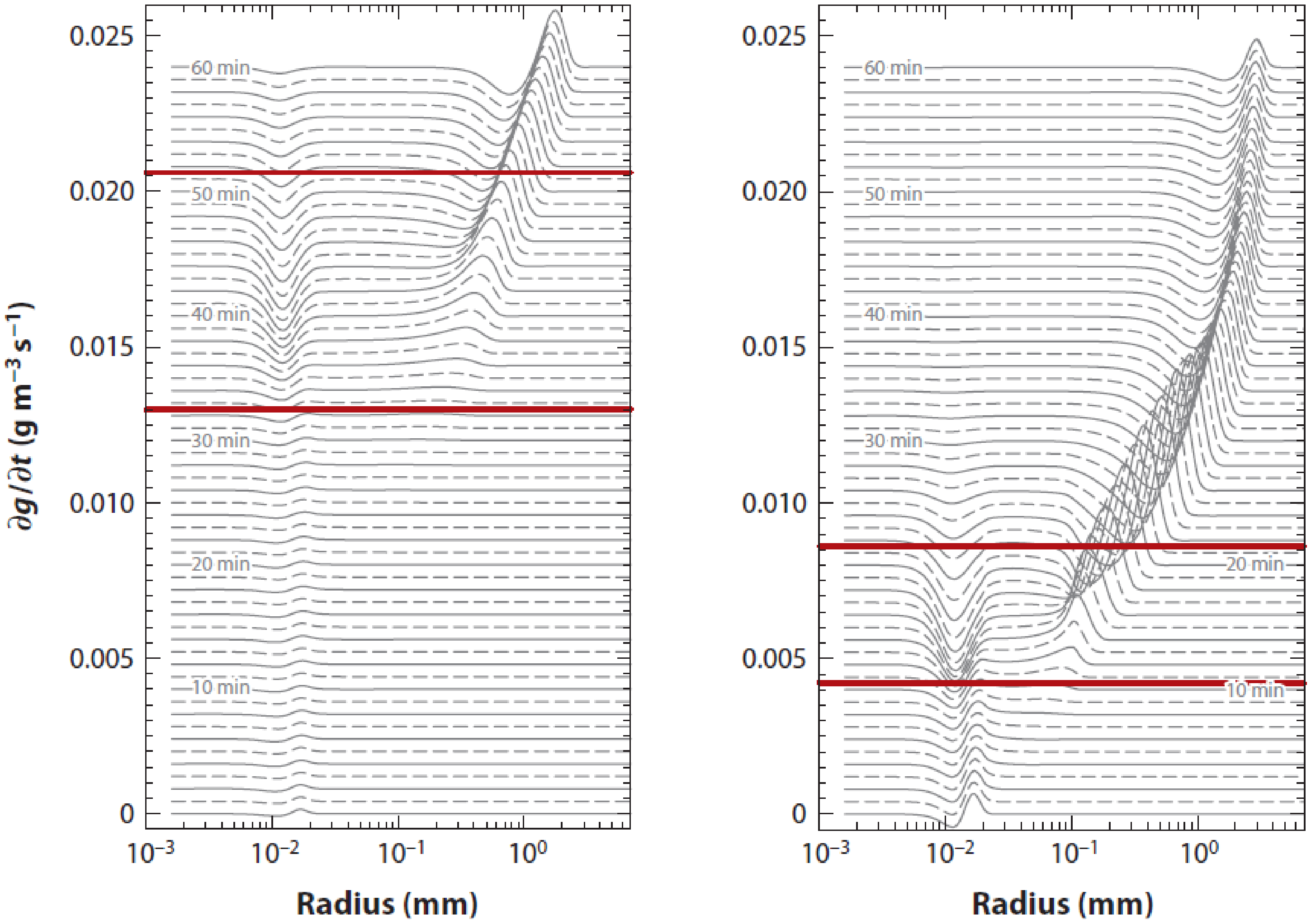
Turbulence enhancement of collision rate





Onishi et al. 2015

Turbulence enhancement of collision rate



Grabowski & Wang 2013

Gravitational settling vs. droplet inertia

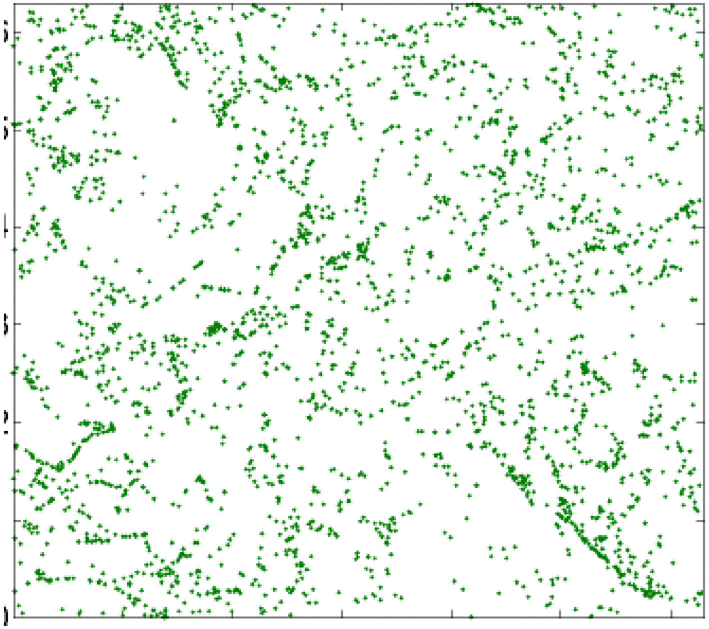
$$\frac{dv_i}{dt} = -\frac{1}{\tau_p}(v_i - u_i) + g_i \qquad w_i \equiv v_i - u_i$$

$$\frac{dw_i}{dt} = -\frac{w_i}{\tau_p} - \frac{du_i}{dt} + g_i$$

$$\tilde{t} \equiv \frac{t}{\tau_\eta} \qquad \tilde{w} \equiv \frac{w}{u_\eta} \qquad \mathrm{St} \equiv \frac{\tau_p}{\tau_\eta} \qquad \mathrm{Ac} \equiv \frac{a_\eta}{g}$$

$$\frac{d\tilde{w}_i}{d\tilde{t}} = -\frac{\tilde{w}_i}{\mathrm{St}} - \frac{d\tilde{u}_i}{d\tilde{t}} + \frac{\hat{g}}{\mathrm{Ac}}$$

$$\mathrm{Sv} \equiv \frac{v_T}{u_\eta} = \frac{\tau_p g}{u_\eta} = \frac{\mathrm{St}}{\mathrm{Ac}}$$



ACTOS: Airborne Cloud Turbulence Observation System

Siebert et al BAMS 2006

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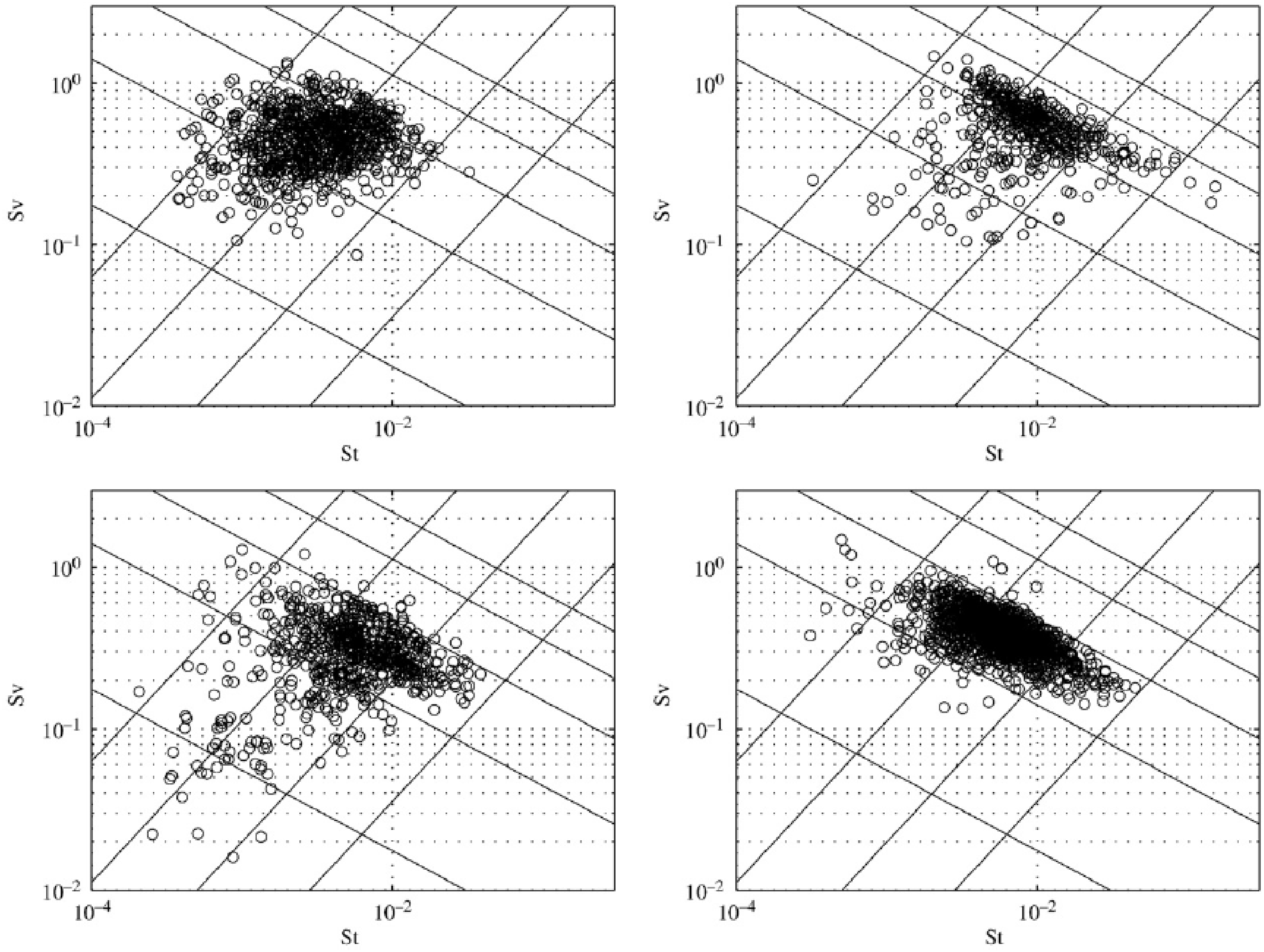
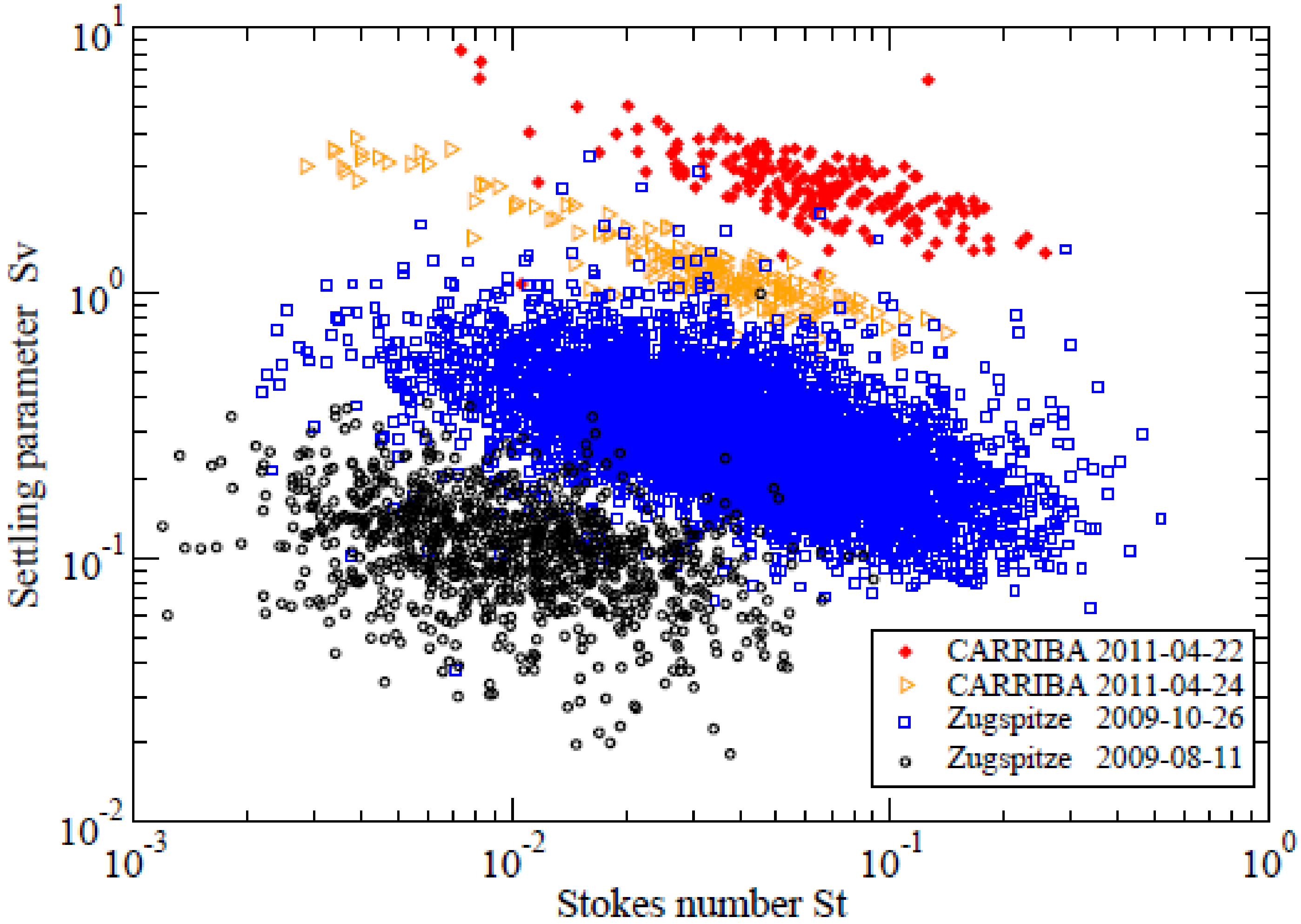
Gravitational settling vs. droplet inertia

432 H. Siebert et al. / Atmospheric Research 97 (2010) 426–437



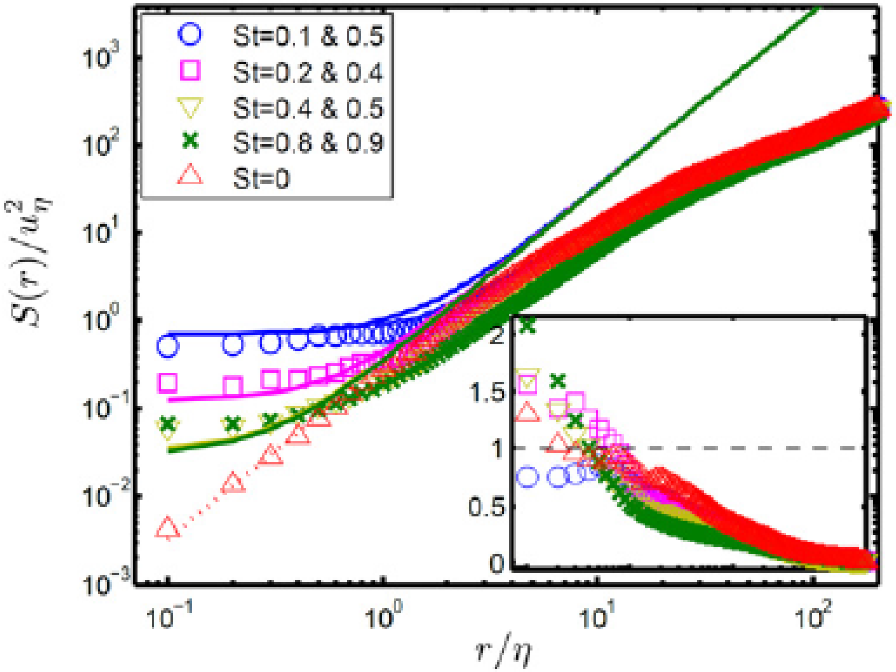
Gravitational settling vs. droplet inertia



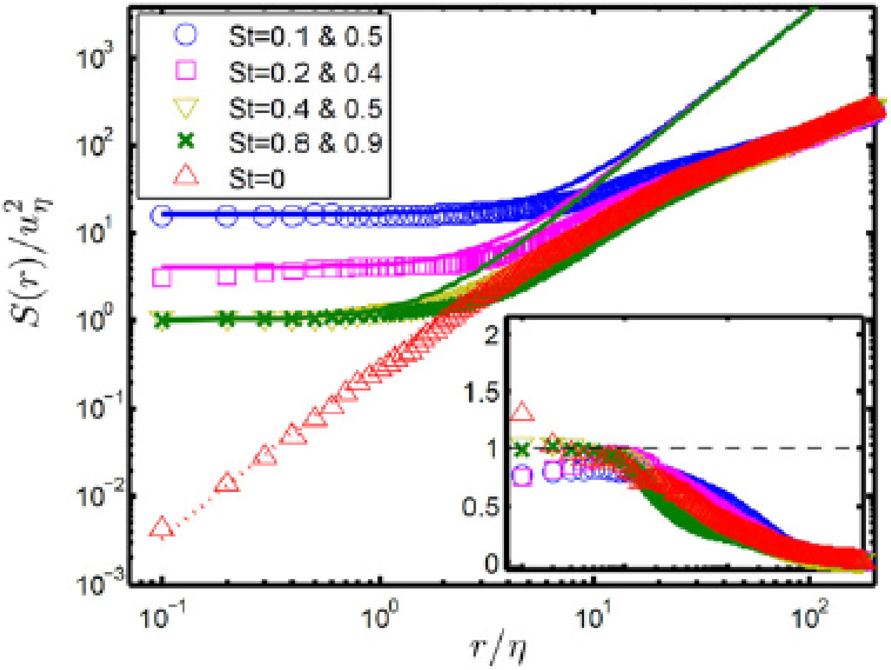
Siebert et al. AMTD 2015

Relative velocity...

no gravity



gravity

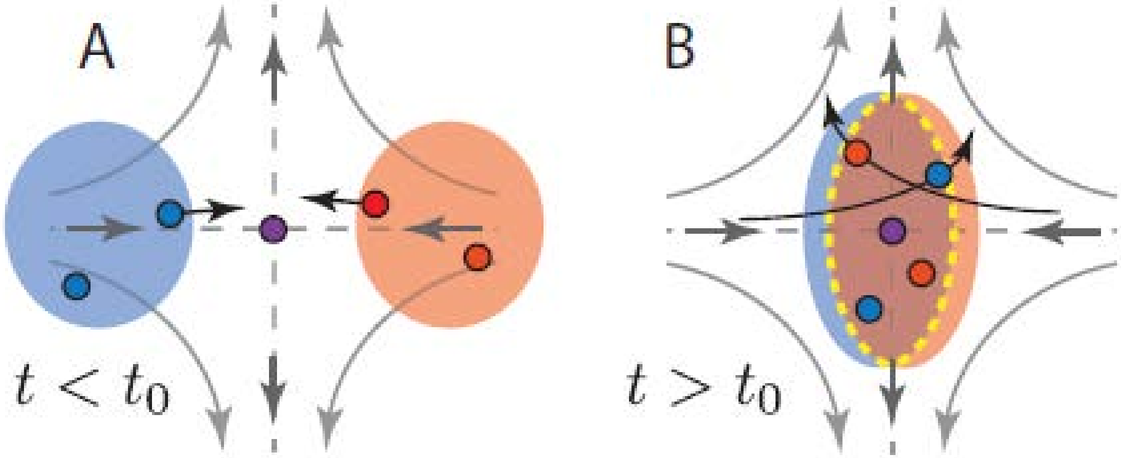


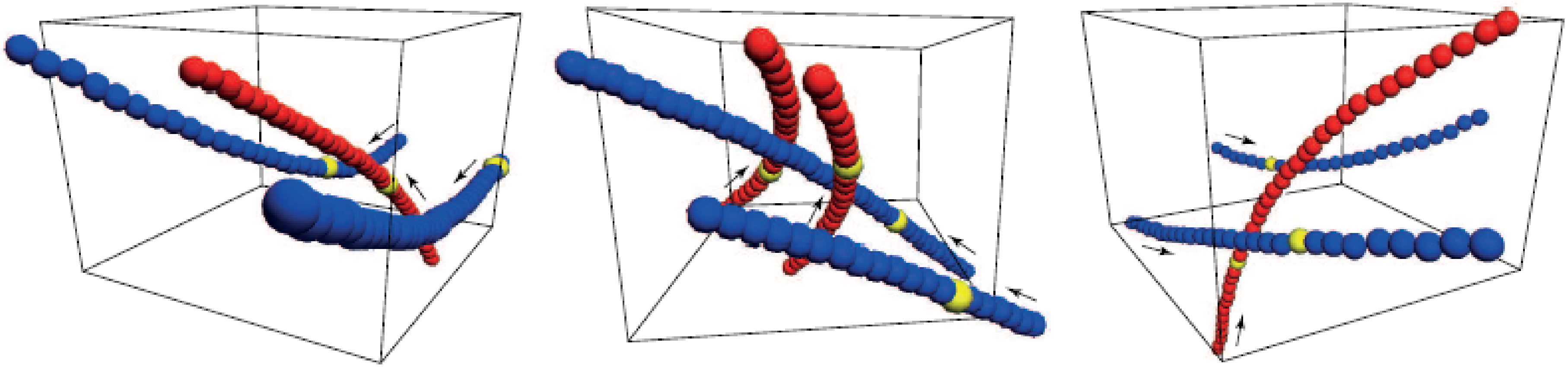
$$\boldsymbol{\Delta v} = \boldsymbol{\Delta u} - \gamma u_\eta \Delta \mathrm{St} \boldsymbol{k} - \tau_\eta \left(\mathrm{St}_1 \boldsymbol{a}_1 - \mathrm{St}_2 \boldsymbol{a}_2 \right), \qquad \gamma = |\boldsymbol{g}|/a_\eta$$

$$\begin{aligned} S(r) = {} & \langle (\boldsymbol{\Delta u})^2 \rangle + \gamma^2 u_\eta^2 (\Delta \mathrm{St})^2 + \tau_\eta^2 (\Delta \mathrm{St})^2 \langle \overline{\boldsymbol{a}}^2 \rangle - 2 \tau_\eta \overline{\mathrm{St}} \langle \boldsymbol{\Delta u} \cdot \boldsymbol{\Delta a} \rangle + \tau_\eta^2 \overline{\mathrm{St}}^2 \langle (\boldsymbol{\Delta a})^2 \rangle \\ & - 2 \gamma u_\eta \Delta \mathrm{St} \langle \boldsymbol{k} \cdot \boldsymbol{\Delta u} \rangle - 2 \tau_\eta \Delta \mathrm{St} \langle \overline{\boldsymbol{a}} \cdot \boldsymbol{\Delta u} \rangle + 2 \gamma \eta (\Delta \mathrm{St})^2 \langle \boldsymbol{k} \cdot \overline{\boldsymbol{a}} \rangle \\ & + 2 \gamma \eta \overline{\mathrm{St}} \Delta \mathrm{St} \langle \boldsymbol{k} \cdot \boldsymbol{\Delta a} \rangle - 2 \tau_\eta^2 \overline{\mathrm{St}} \Delta \mathrm{St} \langle \overline{\boldsymbol{a}} \cdot \boldsymbol{\Delta a} \rangle . \end{aligned}$$

Chang et al. 2015

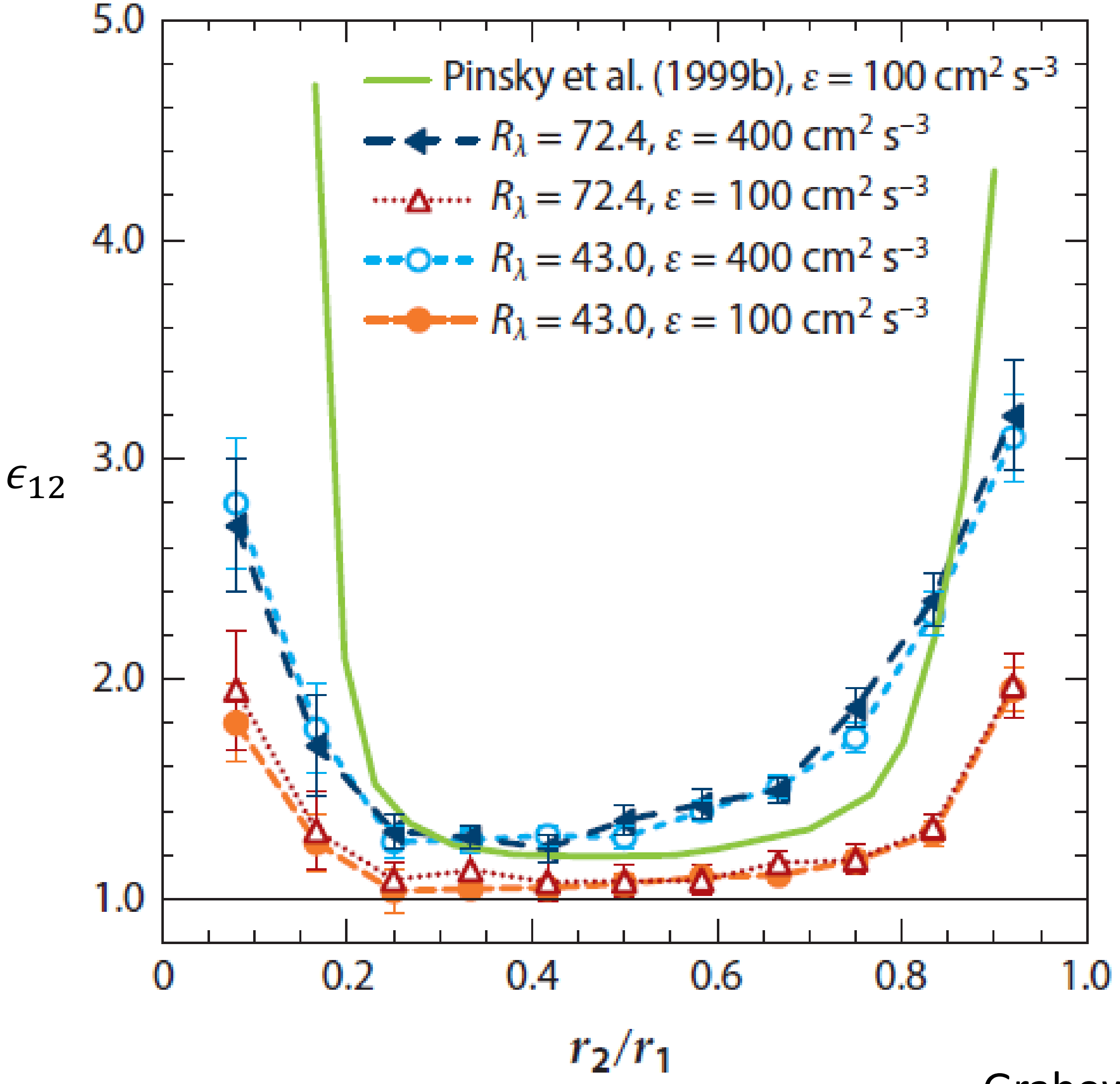
Relative velocity…





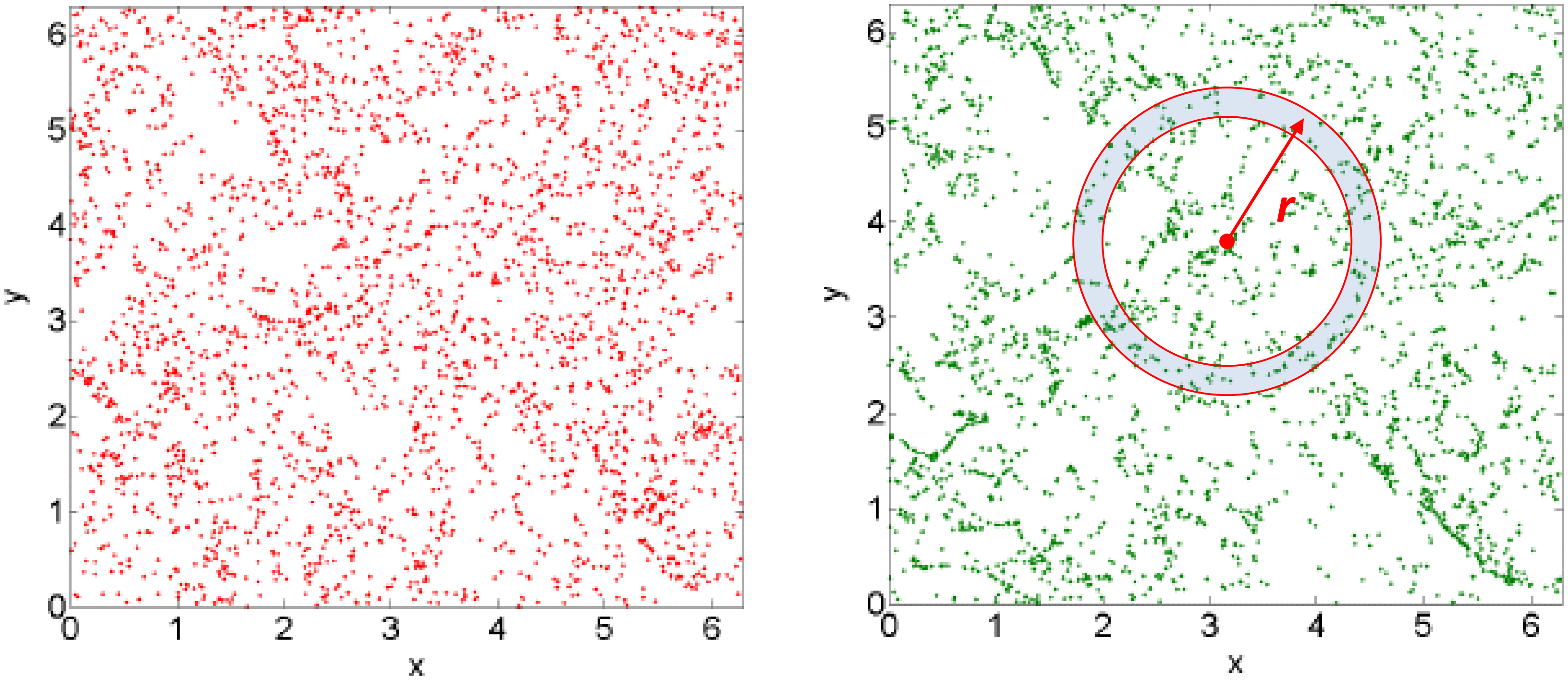
Bewley et al. 2014

Collision efficiency



Grabowski & Wang 2013

Inertial clustering & radial distribution function



$$g(r) = \frac{\psi(r)/N}{(N-1)\,\delta V_r/V} \qquad g_{ij}(r) = \frac{\psi_{ij}(r)/N_i}{N_j\,\delta V_r/V}$$

$$g(r, a \leqslant St \leqslant b) = \int_a^b \int_a^b g_{ij}(r, St_i, St_j)\,\rho(St_i)\,\rho(St_j)\,dSt_i\,dSt_j$$

Drift-diffusion theory for clustering…
(Chun et al. … similar in spirit to Falkovich et al., Zaichik & Alipchenkov)

$$0 = -\langle w\rangle_{\mathrm{p}} g_{12} + \mathcal{D}_{12}\frac{\mathrm{d}g_{12}}{\mathrm{d}r}$$

from Fokker-Planck eqn for particle pair probability *g(r)*
… *steady state*

$$\langle w\rangle_{\mathrm{p}} = -\frac{St_2}{3\tau_\eta}\left[\langle\mathcal{S}^2\rangle_{\mathrm{p}} - \langle\mathcal{R}^2\rangle_{\mathrm{p}}\right] r$$

$$\mathcal{D}_{12} = \left(\frac{B_{\mathrm{nl}}}{\tau_\eta}\right) r^2 + \mathcal{D}_{\|}$$

$$\mathcal{D}_{\|}^{\mathrm{a}} = \left(\tau_{\mathrm{p}}^{[2]} - \tau_{\mathrm{p}}^{[1]}\right)^2 a_0\, a_\eta^2\, \tau_{\mathrm{a}}$$

monodisperse:

$$g(r) = c_0\left(\frac{\eta}{r}\right)^{c_1}$$

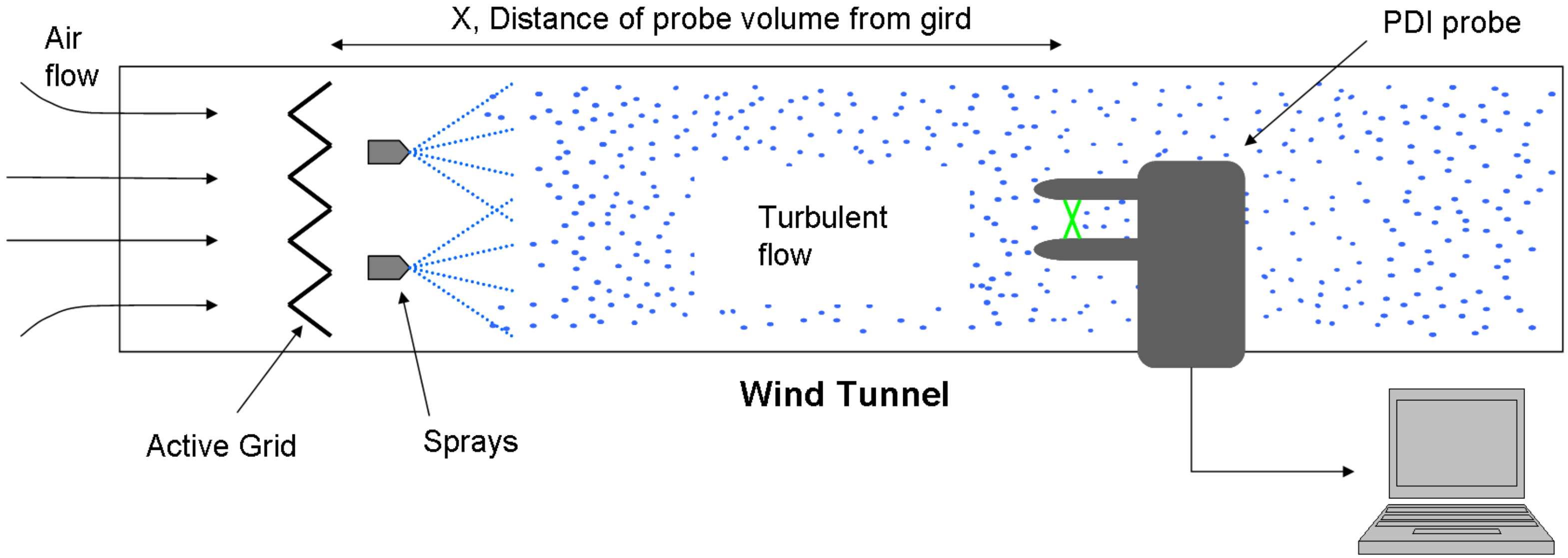
$$c_1 = \frac{\langle w\rangle_p}{v_{\mathrm{diff}}}$$

bidisperse:

$$g_{12}(r) = c_0\left(\frac{\eta^2 + r_{\mathrm{c}}^2}{r^2 + r_{\mathrm{c}}^2}\right)^{c_1/2}$$

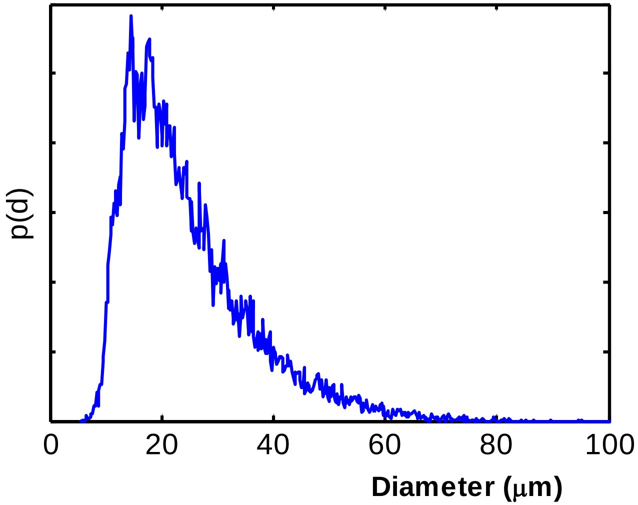
$$\left(\frac{r_{\mathrm{c}}}{\eta}\right)^2 = \frac{1}{B_{\mathrm{nl}}}\left(\frac{\tau_{\mathrm{a}}}{\tau_\eta}\right)\left[a_0\left(St_2 - St_1\right)^2\right.$$

Active-grid wind tunnel with spray

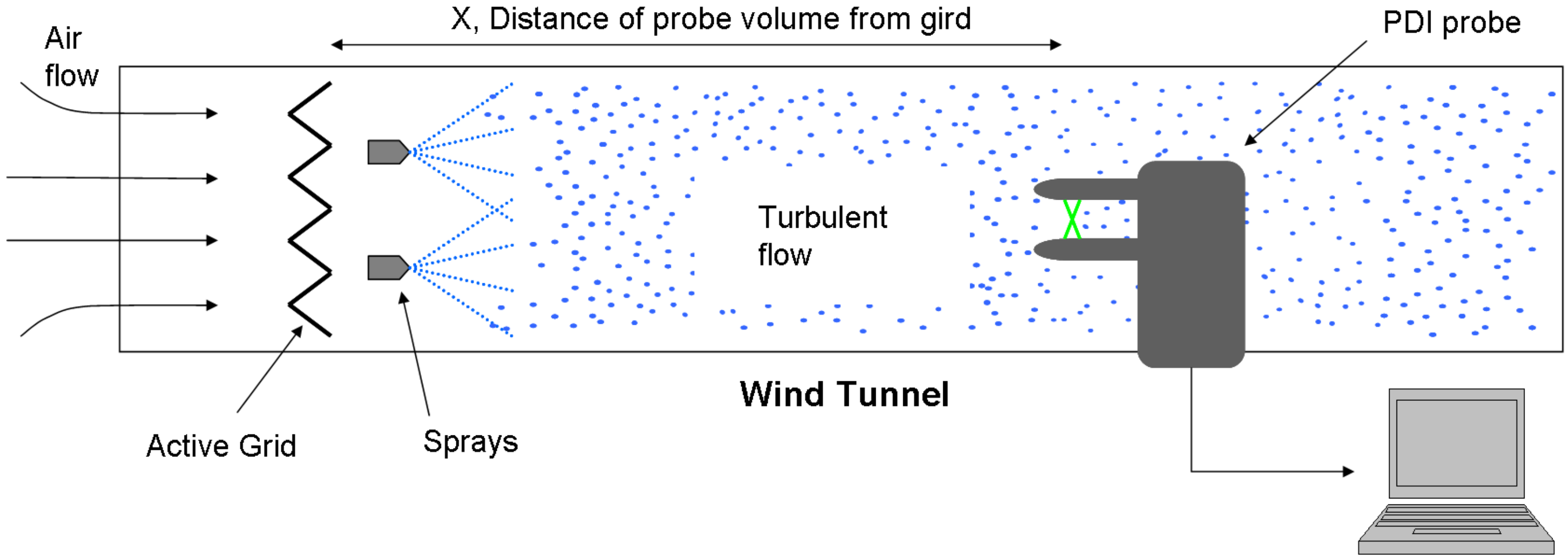


Cornell active-grid wind tunnel: Variable flow speed U and position of probe X.

- Length 15 m, Cross section 1 m^2
- Longitudinal mean speed 1-10 m/s
- rms speed ~ 10% mean
- R_λ = 300-900
- Mean diameter ~ 20 μm



Active-grid wind tunnel with spray



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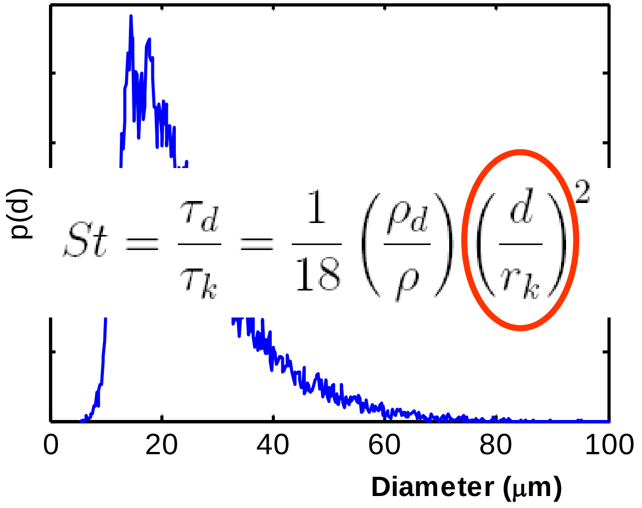
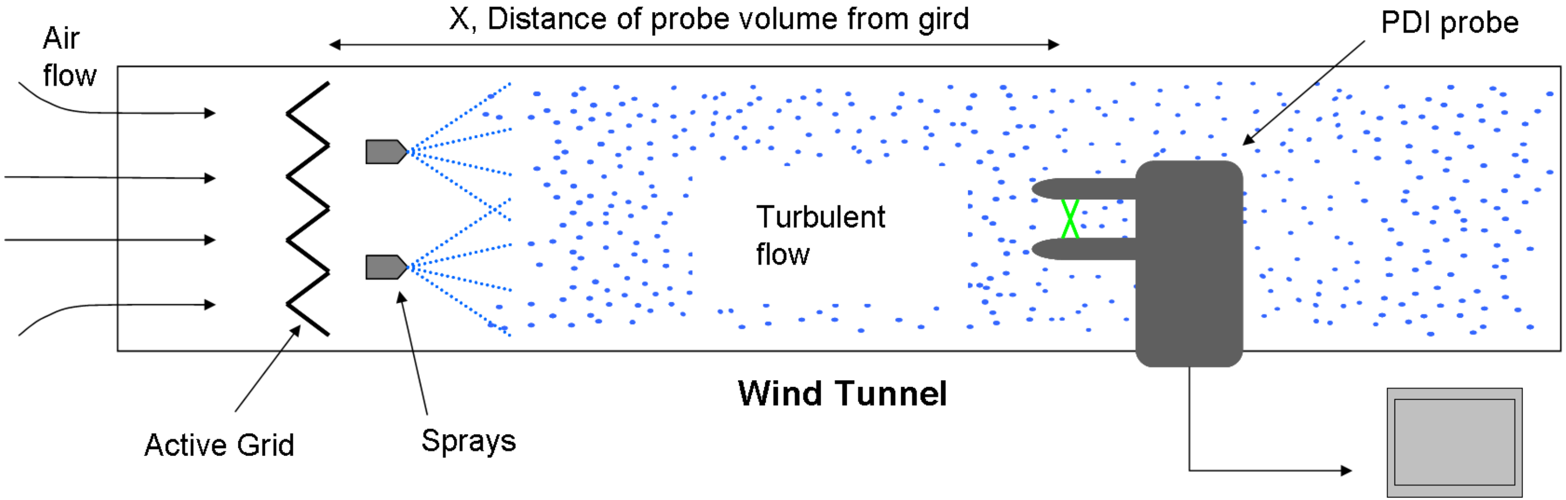
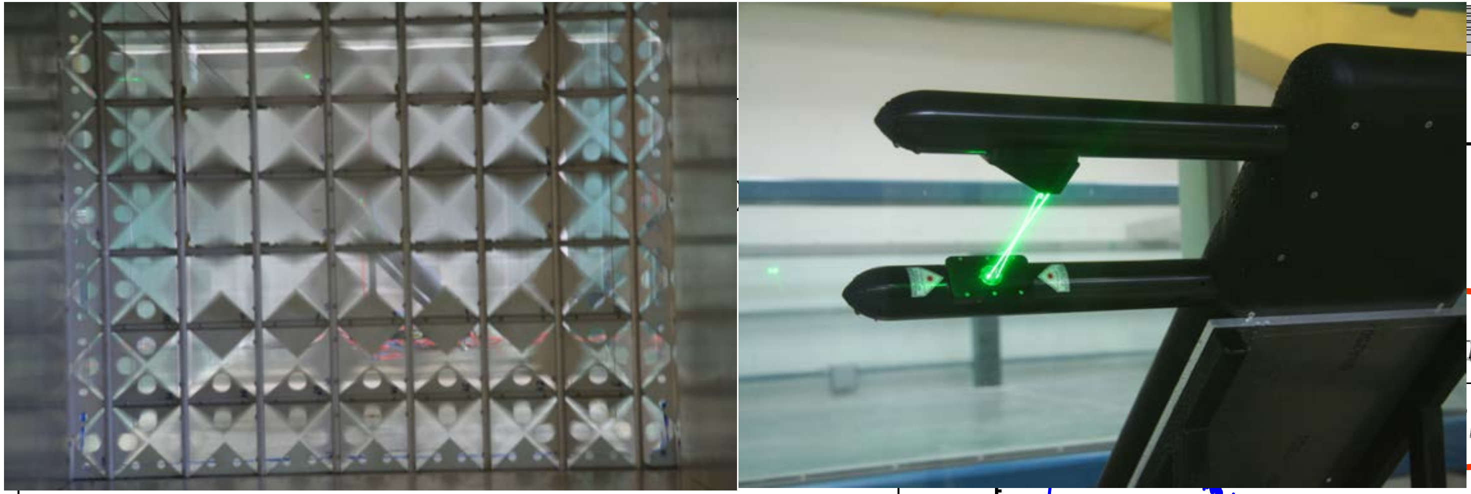
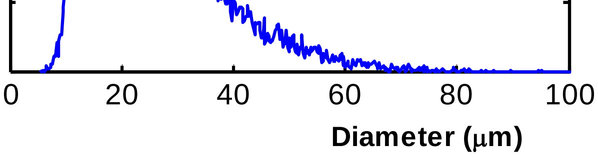


$$St = \frac{\tau_d}{\tau_k} = \frac{1}{18}\left(\frac{\rho_d}{\rho}\right)\boxed{\left(\frac{d}{r_k}\right)}^2$$

Active-grid wind tunnel with spray

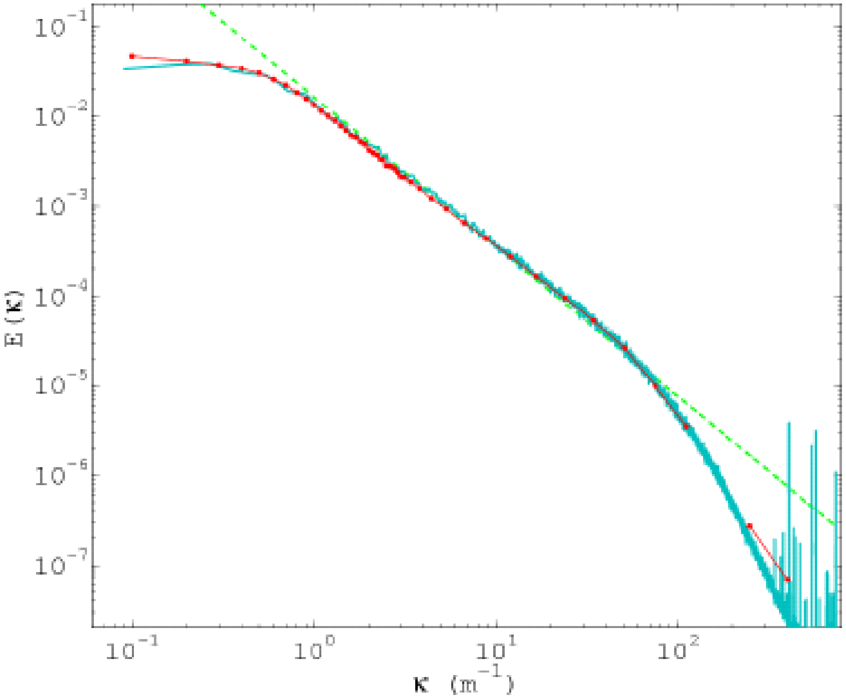


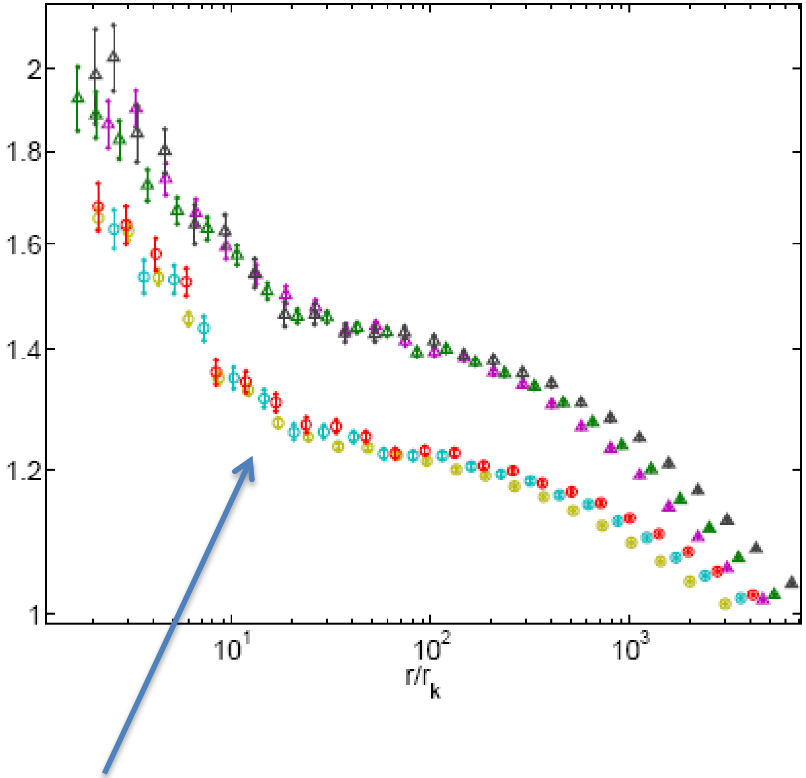




- R_λ = 300-900
- Mean diameter ~ 20 μm

Turbulence energy spectrum and RDF…

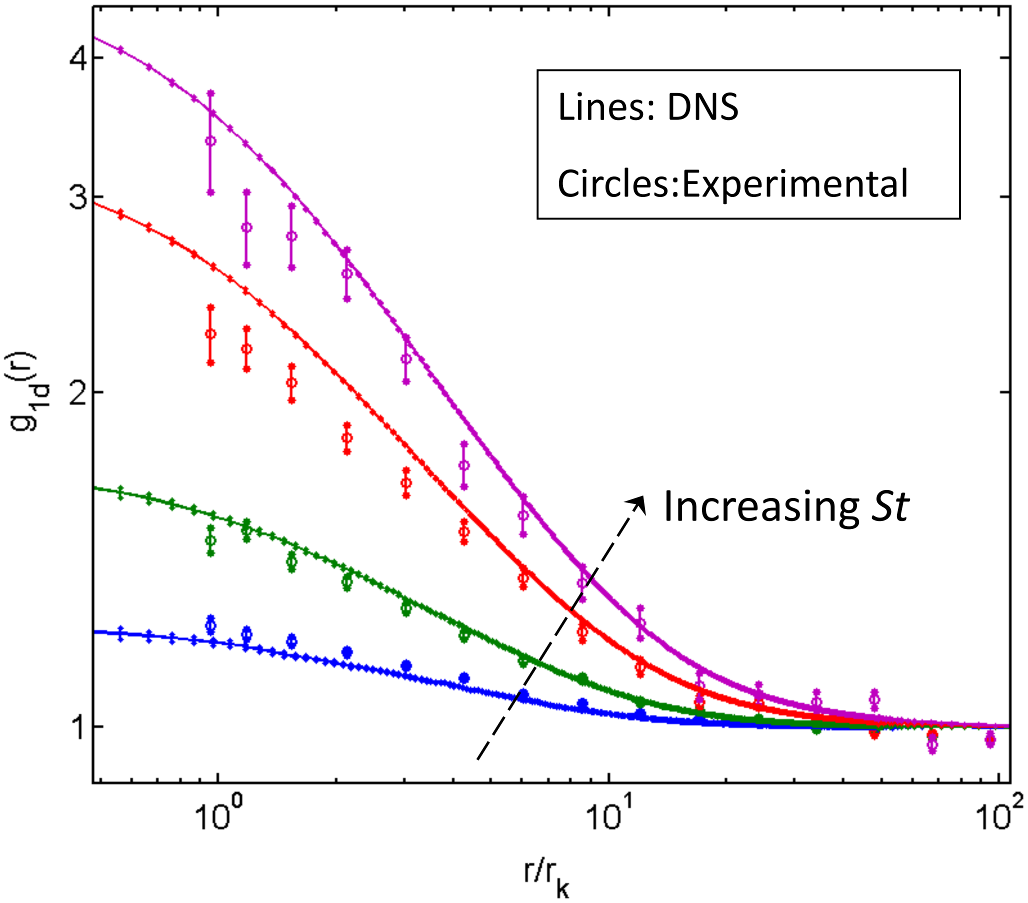




Inertial clustering scale break

Saw et al., *PRL* 2008 & *NJP* 2012

Comparison: Experiments & DNS



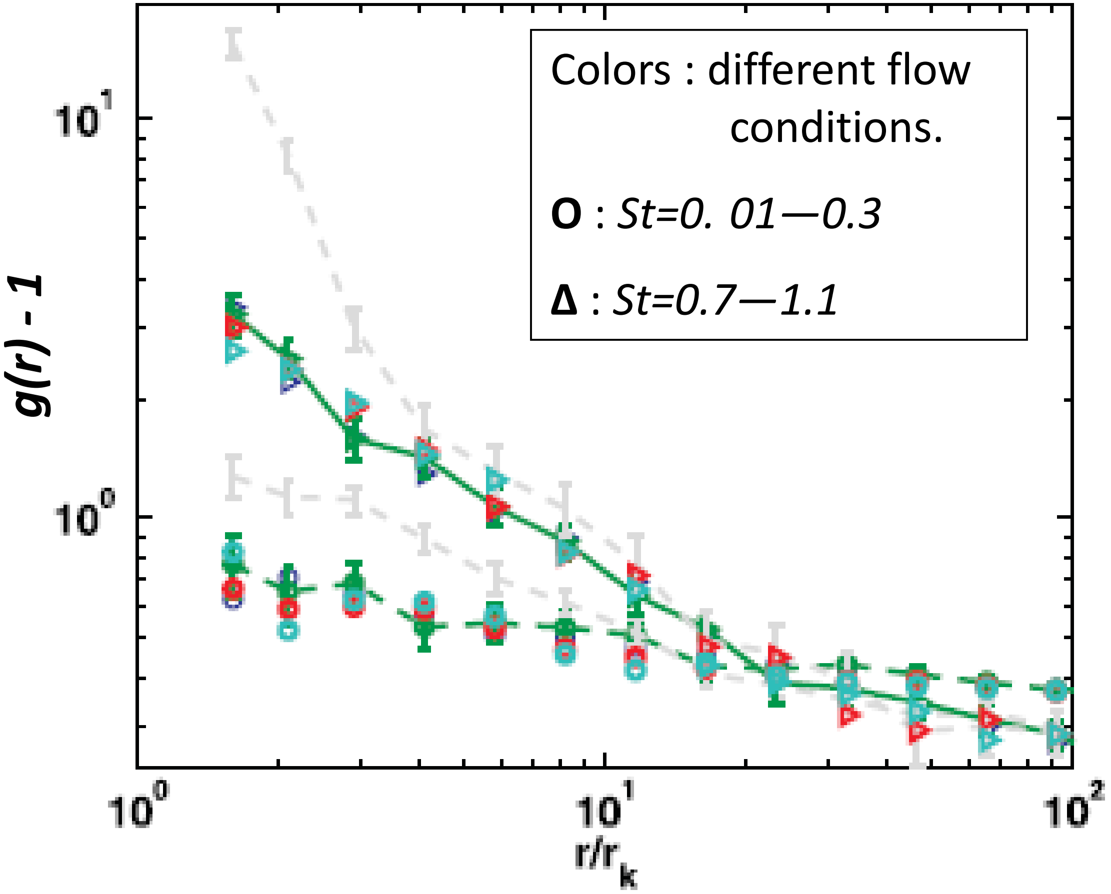
- Polydisperse
- Reasonable agreement, but small deviation at large St.

$$g(r, a \le St \le b) = \iint\limits_{a \le St \le b} g_{12}(r, St_1, St_2)\rho(St_1)\rho(St_2)\, dSt_1 dSt_2$$

Saw et al. 2012

$$St = \frac{\tau_p}{\tau_k} = \frac{1}{18}\left(\frac{\rho_p}{\rho}\right)\left(\frac{d}{r_k}\right)^2$$

Stokes number scaling…



Colors : different flow conditions.

O : *St=0. 01—0.3*

Δ : *St=0.7—1.1*

$$St = \frac{\tau_d}{\tau_k} = \frac{1}{18}\left(\frac{\rho_d}{\rho}\right)\left(\frac{d}{r_k}\right)^2$$

- Results from different flow conditions coincide when *St* are matched.
- *St* effects dominate over gravity, *Re* effects.
- For triangles, *Sv* goes from 0.2 – 1.0 .
- R_λ= 440—800 .

Saw et al., *PRL* 2008

Drift-diffusion… with charge

$$\langle w\rangle_{\mathrm{p}} = -\frac{St_2}{3\tau_\eta}[\langle \mathcal{S}^2\rangle_{\mathrm{p}} - \langle \mathcal{R}^2\rangle_{\mathrm{p}}]r + Ct_{12}\, u_\eta \left(\frac{\eta}{r}\right)^2$$

$$Ct_{12} \equiv \frac{u_{\mathrm{q}}}{u_\eta}$$

$$g(r) = c_0 \left(\frac{\eta}{r}\right)^{c_1}$$

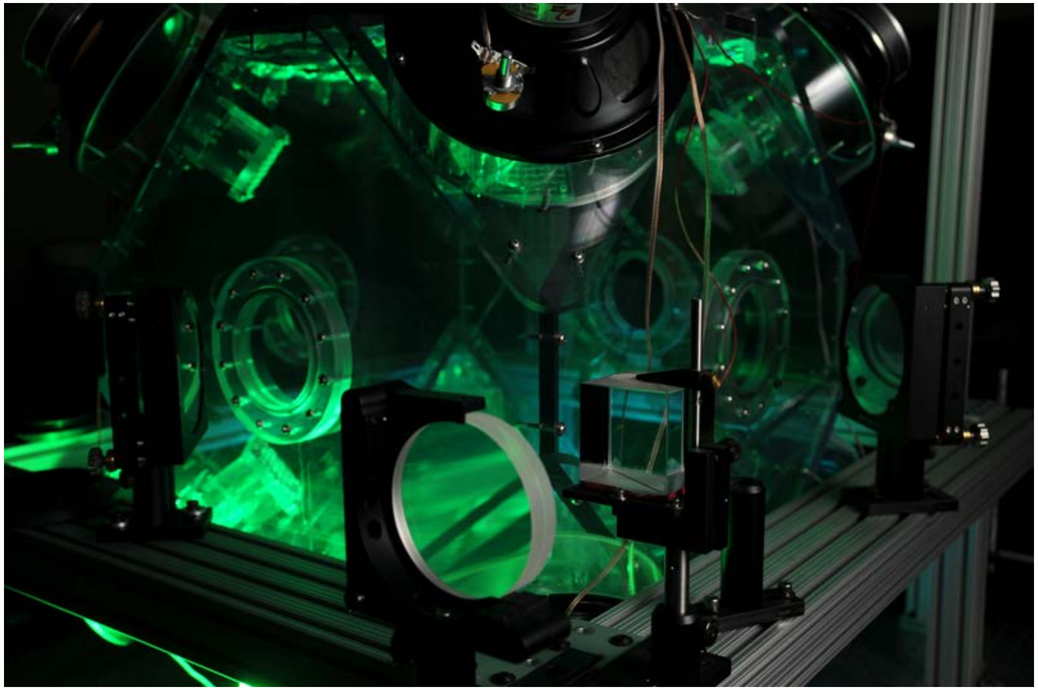
$$g(r) = c_0 \left(\frac{\eta}{r}\right)^{c_1} \exp\left[-c_{2,\mathrm{mono}}\, Ct \left(\frac{\eta}{r}\right)^3\right]$$

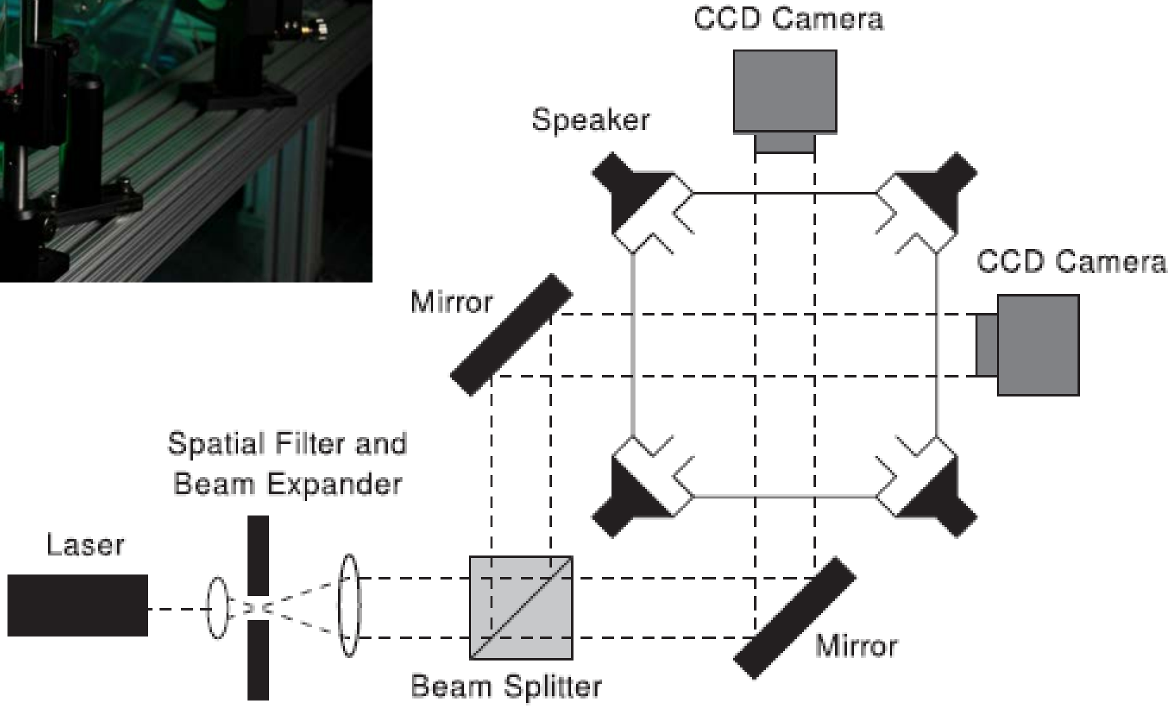
$$Ct \equiv \left(2\,\frac{kq^2}{\eta^2}\right)\left(\frac{u_\eta}{\beta}\right)^{-1}$$

$$\exp(-2v_{\mathrm{charge}}/3v_{\mathrm{turb}})$$

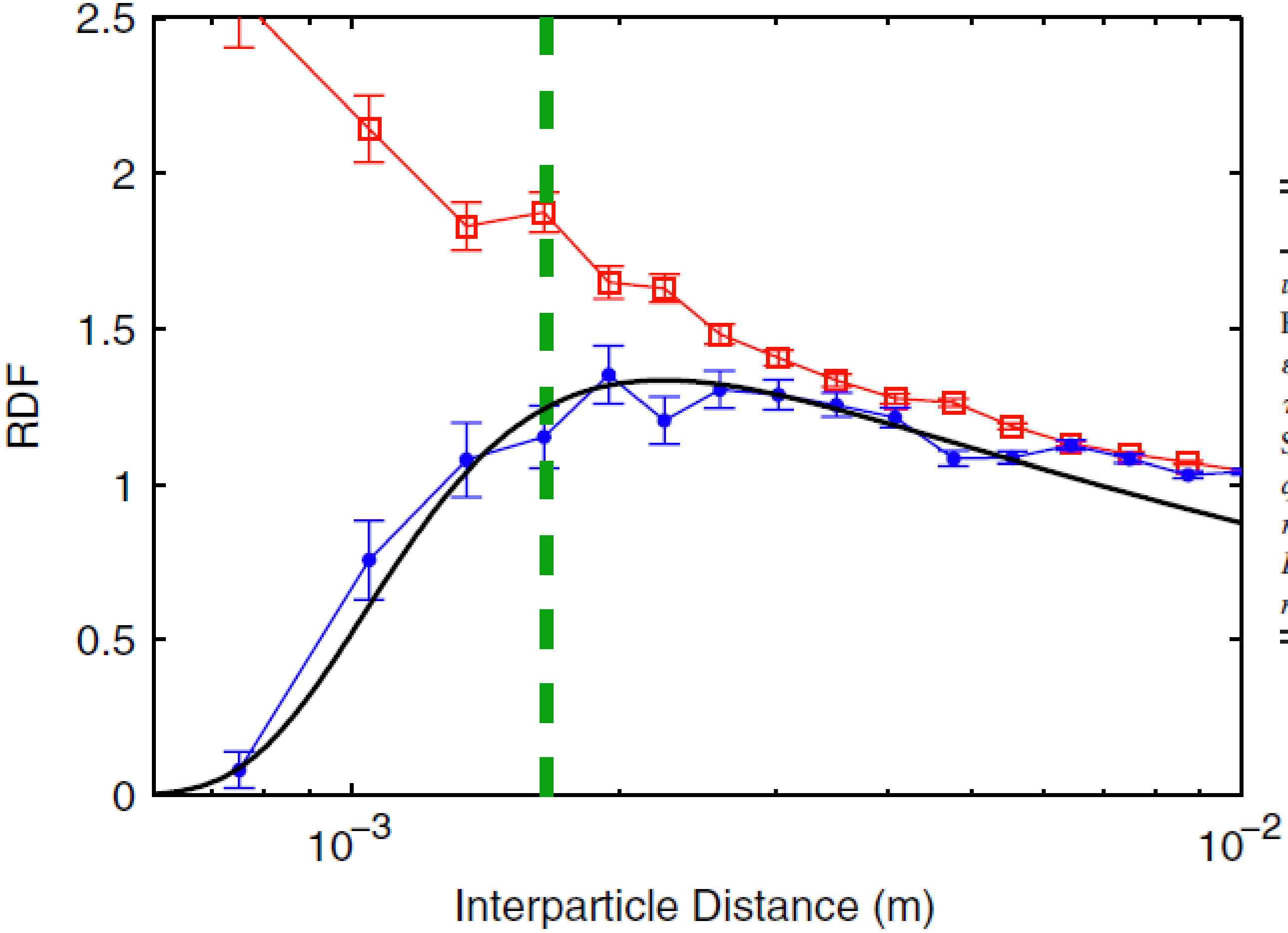
Lu et al. *New J. Phys.* 2010

Experiment: 3D particle positions in homogeneous, isotropic turbulence…





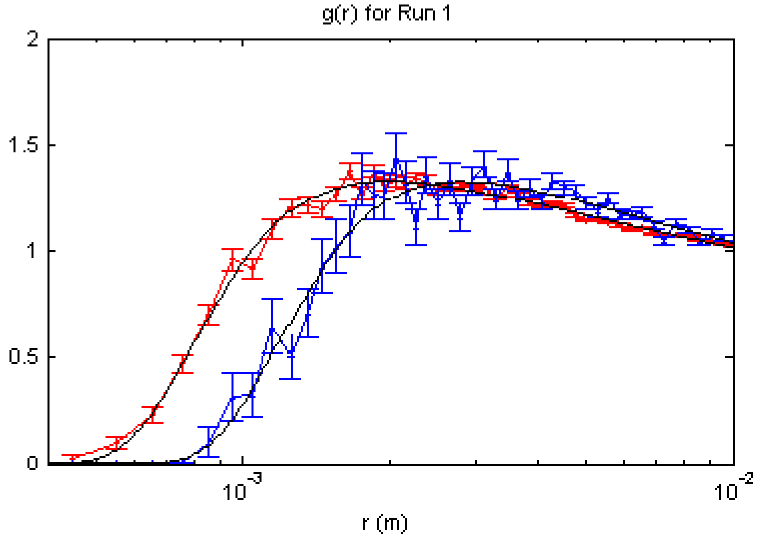
Charged inertial particles in turbulence…

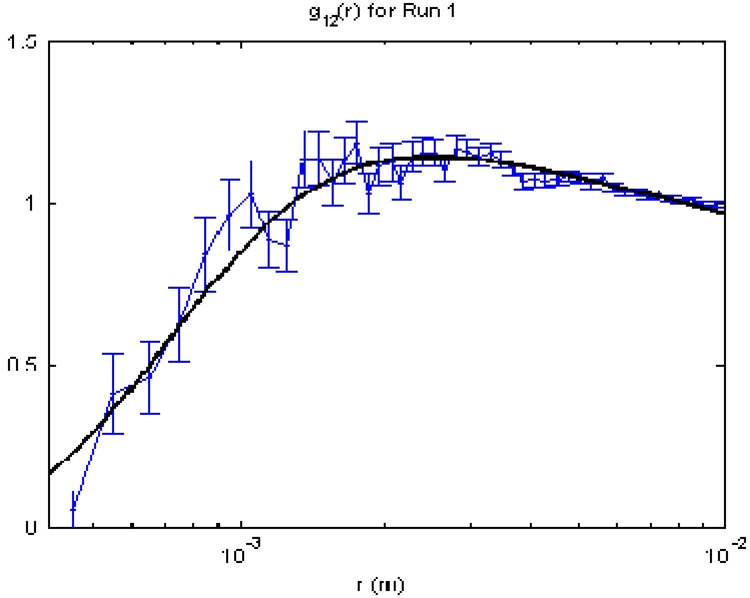


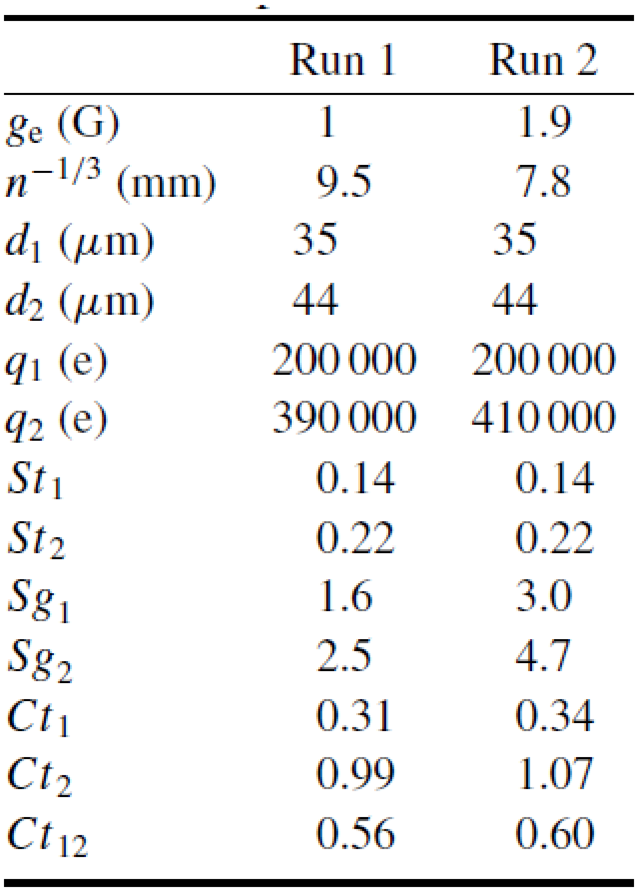
	1c	1n
u_{rms} (cm s^{-1})	15	14
Re_λ	84	82
ε (W kg^{-1})	0.06	0.06
η (μm)	510	510
St	0.3	0.3
q (e)	380 000	~80 000
r_* (mm)	1.7	~0.6
$E_{\mathrm{charge}}/E_{\mathrm{turb}}$	2.0	~0.08
$n^{-1/3}$ (mm)	10	7

Lu et al. *New J. Phys.* 2010

Effect of gravitational settling…







	Run 1	Run 2
g_e (G)	1	1.9
$n^{-1/3}$ (mm)	9.5	7.8
d_1 (μm)	35	35
d_2 (μm)	44	44
q_1 (e)	200 000	200 000
q_2 (e)	390 000	410 000
St_1	0.14	0.14
St_2	0.22	0.22
Sg_1	1.6	3.0
Sg_2	2.5	4.7
Ct_1	0.31	0.34
Ct_2	0.99	1.07
Ct_{12}	0.56	0.60

Lu et al. *New J. Phys.* 2010

Summary

- Cloud droplet collisions are influenced by turbulence in several ways: spatial distribution, relative velocity, and modified collision efficiency.

- Experiment shows strong *St* scaling for inertial clustering. Effects of *Re* & gravity are relatively weak (below experimental uncertainty) for the range covered (R_λ=430--700, Sv~ 0.1--1).

- Experiment and DNS agree well, when proper averaging is performed; this takes some effort. Drift-diffusion theory for *St*<<1 seems to work up to *St*~0.3 for the monodisperse case.

- Inclusion of gravity is important for bidisperse clustering (and collision velocity) – test accomplished through addition of electric charge.

- Turbulence enhancements can reach factor of 2 for plausible conditions.

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