Droplet-size distribution in turbulent clouds:

stochastic microphysics at unresolved scales

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Workshop on Eulerian vs. Lagrangian methods for cloud microphysics

Cracow, April 16, 2019

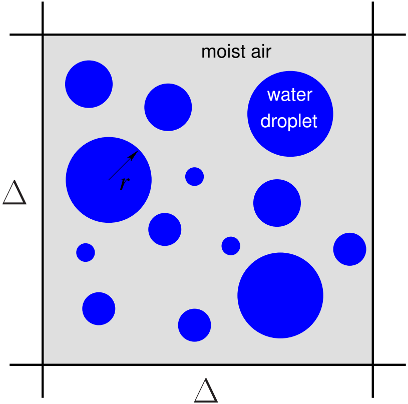
Diffusional growth

Droplet growth

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{r}\, D\, \langle S \rangle$$

$\langle S \rangle$ - mean-field supersaturation

LES grid box



$\Delta \in$ inertial range

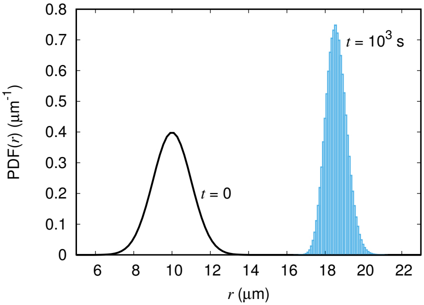
Diffusional growth

driven by mean-field supersaturation

- Droplets exposed to the same $\langle S \rangle$

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{r} D \langle S \rangle$$

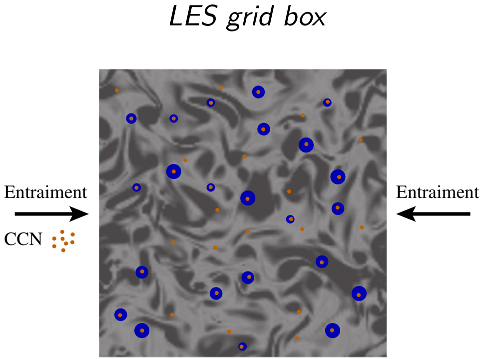
- Narrow size distribution!



Microphysical variability

at sub-grid scales (SGS)

- $S = \langle S \rangle + S'$
- Mixing
- Activation/deactivation
- Superdroplets



Köhler potential

Growth equation:

$$r\frac{\mathrm{d}r}{\mathrm{d}t} = D\left[\langle S\rangle - \frac{A}{r} + \frac{B}{r^3}\right]$$

$$x \equiv r^2$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{\partial V}{\partial x}$$

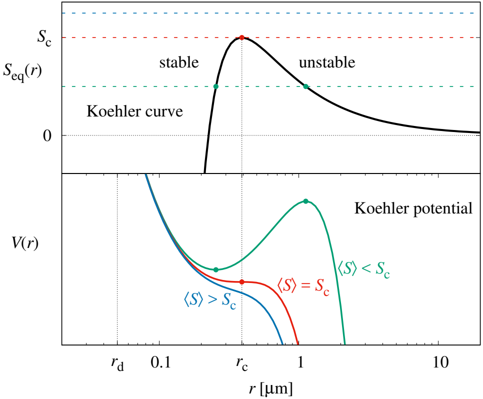
Köhler potential

Deterministic activation

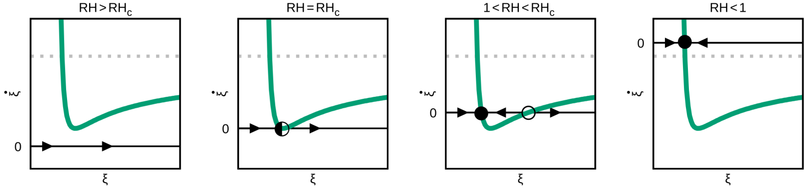
$$r\frac{\mathrm{d}r}{\mathrm{d}t} = D\left[\langle S\rangle - \frac{A}{r} + \frac{B}{r^3}\right]$$

$$x \equiv r^2$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{\partial V}{\partial x}$$



Phase portraits



$$\mathrm{RH} = S + 1, \qquad \xi = x \equiv r^2$$

Stochastic activation

Köhler potential plus fluctuations

$$r\frac{\mathrm{d}r}{\mathrm{d}t} = D\left[\langle S\rangle + S' - \frac{A}{r} + \frac{B}{r^3}\right]$$

$$x \equiv r^2$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{\partial V}{\partial x} + 2DS'$$

Abade, Grabowski and Pawlowska, JAS, **75** (2018)

Stochastic activation

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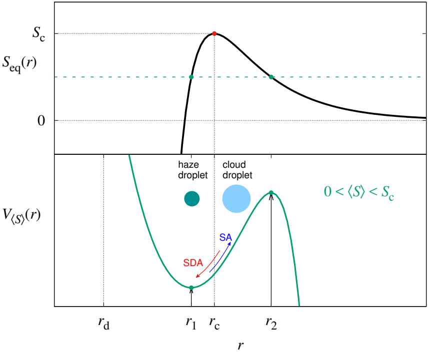
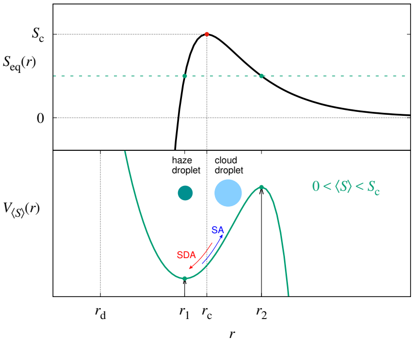


Abade, Grabowski and Pawlowska, JAS, **75** (2018)

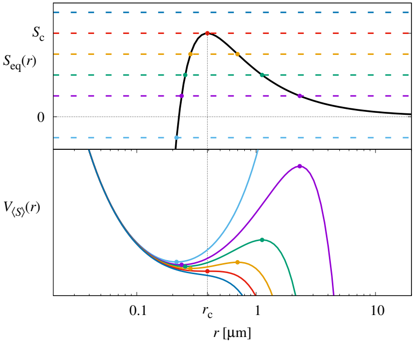
Stochastic activation

$S = \langle S \rangle + S'$

Köhler potential

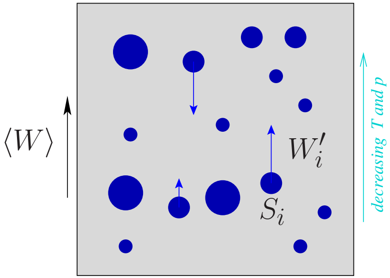


Feedback on $\langle S \rangle$



Abade, Grabowski and Pawlowska, JAS, **75** (2018)

Supersaturation and velocity fluctuations



$$\frac{\mathrm{d}S_i'}{\mathrm{d}t} = -\frac{S_i'}{\tau_\mathrm{c}} - \frac{S_i'}{\tau_\mathrm{m}} + aW_i'(t)$$

$$\tau_\mathrm{c} \sim \frac{1}{N\langle r \rangle} \quad \text{(condensation)}$$

$$\tau_\mathrm{m} \sim \text{eddy turnover time} \quad \text{(mixing)}$$

- Statistical model for $W'(t)$

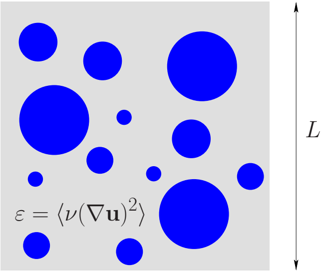
Celani *et al.*, EPL, **70** (2005); Grabowski and Abade, JAS, **74** (2017)

Vertical velocity fluctuations

Stationary homogeneous isotropic turbulence

$$\langle W'(t) \rangle = 0$$

$$\langle W'(0)W'(t) \rangle = \sigma_{W'}^2 \exp\left(-|t|/\tau_m\right)$$



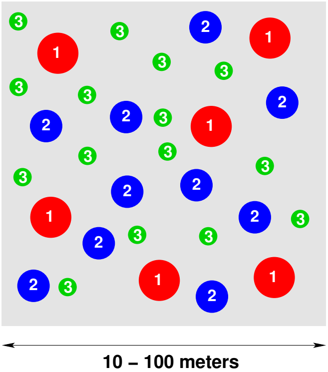
Kolmogorov scaling (inertial subrange)

$$\sigma_{W'}^2 \sim (L\varepsilon)^{2/3} \qquad \tau_m \sim \frac{L^{2/3}}{\varepsilon^{1/3}}$$

Super-droplets (SDs)

Shima *et al.* (2009), Arabas *et al.* (2015), Hoffmann *et al.* (2015)

$$N_{\text{droplets}} \sim 10^{11} - 10^{14}$$



- Multiplicities:

$$\xi_1 = 6, \quad \xi_2 = 10, \ldots$$

- SDs have the same attributes

$$(r, \ldots, S', W', \ \ldots)$$

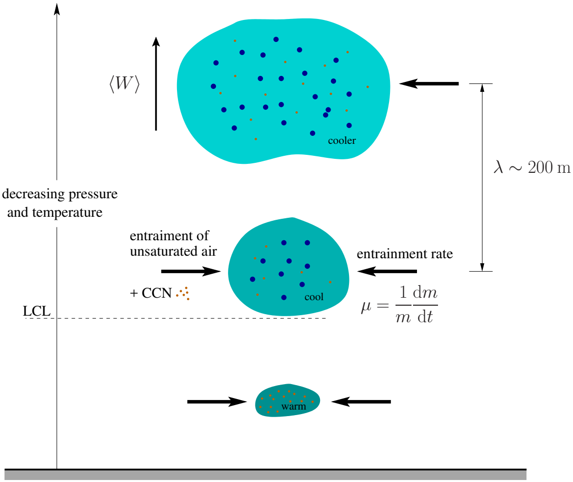
- Well-mixed

Frameworks

- Entraining cloud parcel
- Synthetic turbulent-like ABL flow
- Eulerian + SGS $\equiv$ stochastic Lagrangian

Entraining cloud parcel

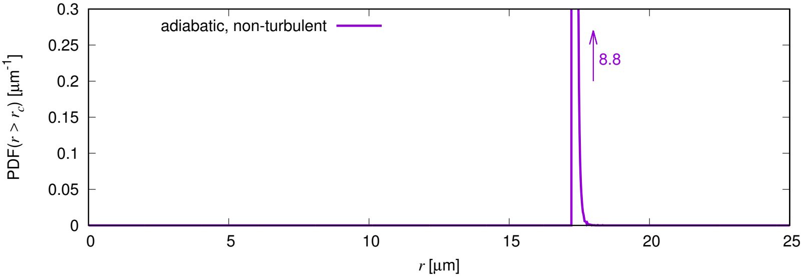
stochastic entrainment events



Krueger *et al.*, JAS, **54** (1997); Romps and Kuang, JAS, **67** (2010)

Droplet-size distribution

after a 1-km parcel rise



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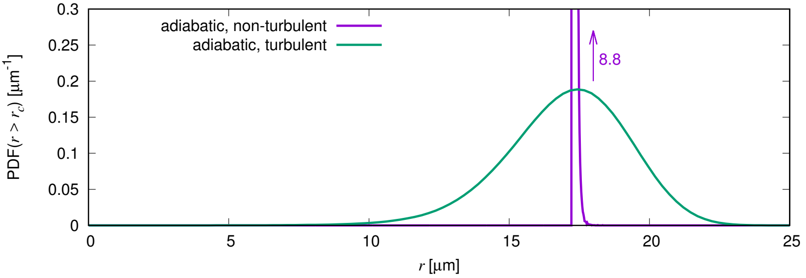

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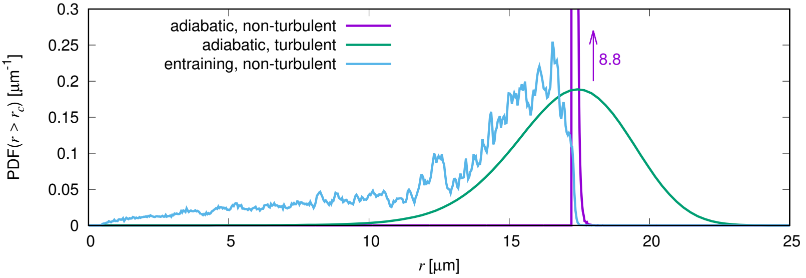

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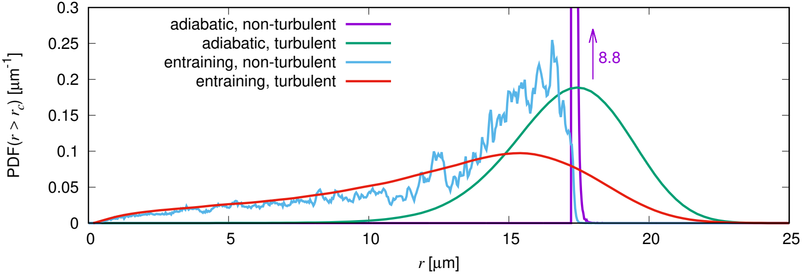

Droplet-size distribution

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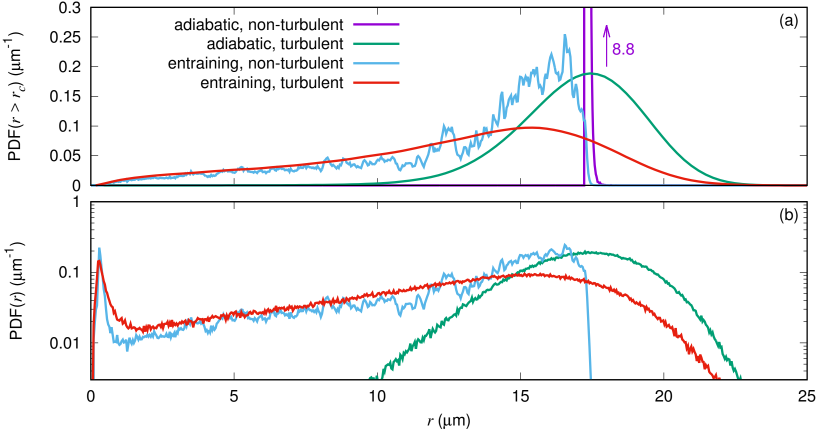
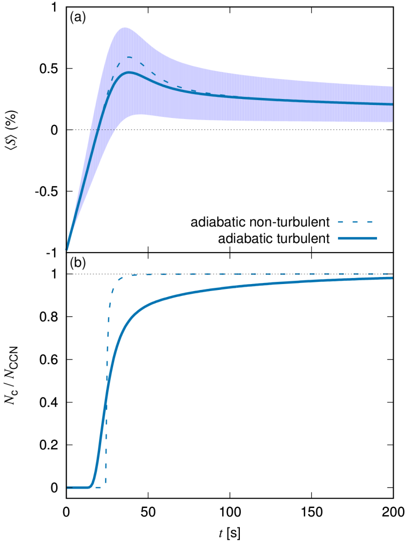
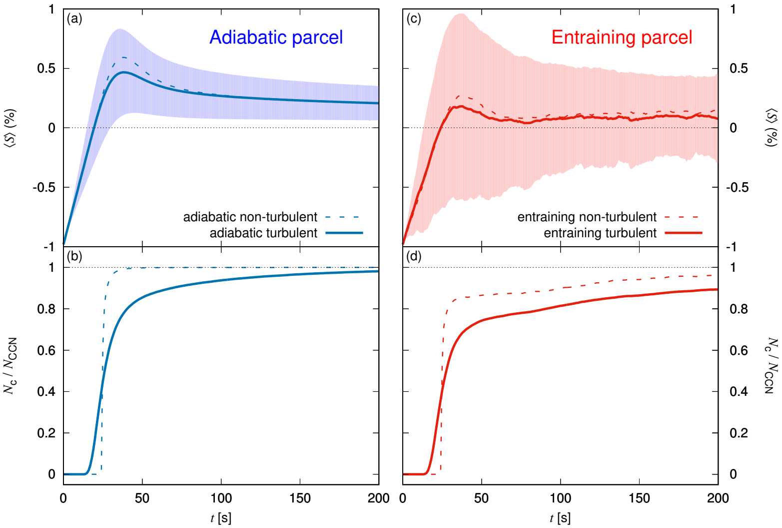


Stochastic activation and feedback on $\langle S \rangle$

Adiabatic parcel

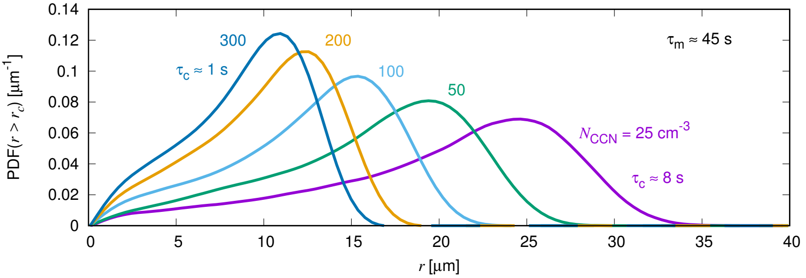


Stochastic activation and feedback on $\langle S \rangle$



Aerosol indirect effect

induced by turbulence



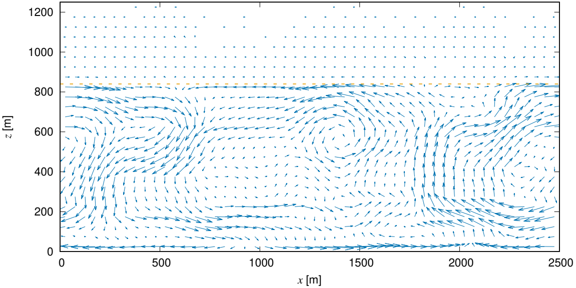
- fast × slow microphysics

$$\frac{dS'}{dt} = -\frac{S'}{\tau_S} + aW'(t), \qquad \tau_S \sim \min\{\tau_{\text{condens}}, \tau_{\text{mixing}}\}$$

Chandrakar *et al.*, PNAS, **113** (2016); Siebert and Shaw, JAS, **74** (2017)

Turbulent-like ABL flow

$$\mathbf{u}(\mathbf{r},t) = \sum_{|\mathbf{k}_n|<K} \text{random modes}$$

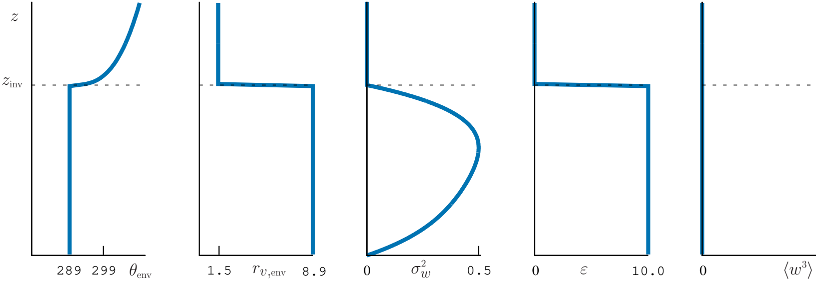


$$\langle w^2 \rangle = \sigma_w^2(z) \qquad \langle w(x',z)w(x'+x,z)\rangle = \hat{C}_w(x)\,\sigma_w^2(z)$$

Pinsky *et al.*, JAS, **65** (2008), ..., Magaritz-Ronen *et al.*, ACP, **16** (2016)

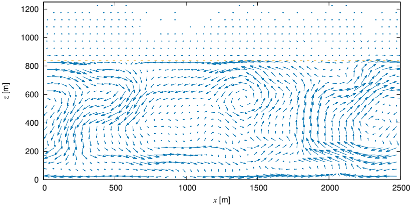
Turbulent-like ABL flow

Vertical structure



Turbulent-like ABL flow

- Prescribed flow $\mathbf{u}(\mathbf{r}, t)$



- Balance equations for entropy and water vapor

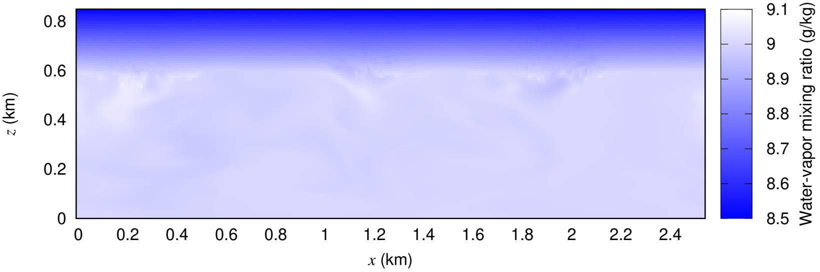
- Superdroplets

$$\boldsymbol{X}_i(t + \delta t) = \boldsymbol{X}_i(t) + \delta \boldsymbol{X}_i$$

$$\delta \boldsymbol{X}_i = \delta \boldsymbol{X}_i^{(\text{mean})} + \delta \boldsymbol{X}_i^{(\text{sgs})}$$

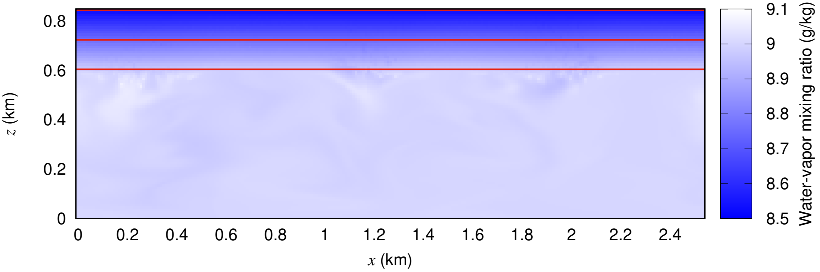
Turbulent-like ABL flow

Droplet-size PDF



Turbulent-like ABL flow

Droplet-size PDF



Turbulent-like ABL flow

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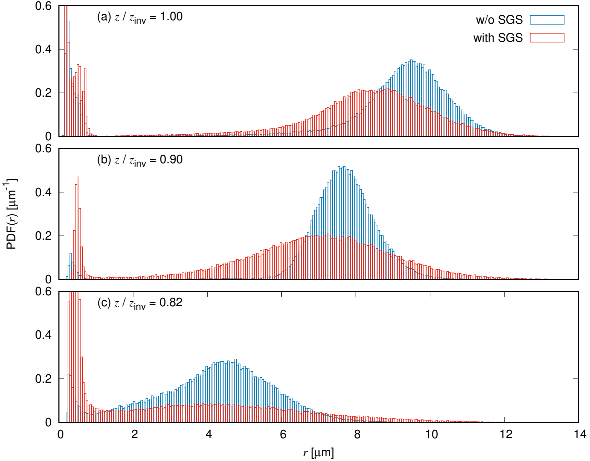
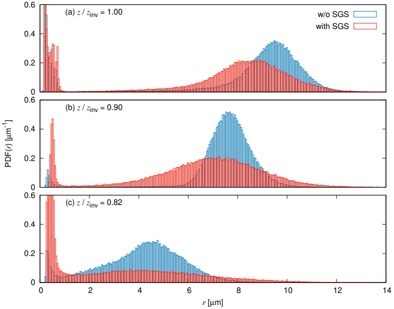


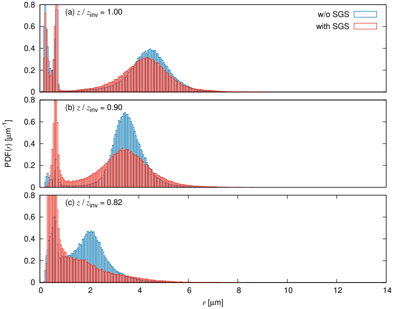
Turbulent-like ABL flow

Droplet-size PDF

PRISTINE conditions

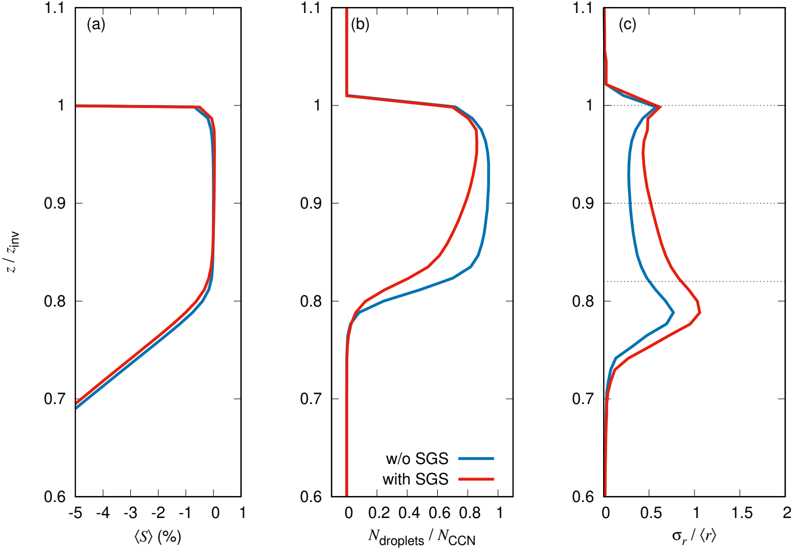


POLLUTED conditions



Microphysical profiles

horizontally averaged



Eulerian + SGS model $\equiv$ stochastic Lagrangian

Transported PDF methods (Stephen B. Pope and co-workers)

Eulerian description

Thermodynamic scalar $\theta = \langle\theta\rangle + \theta'$

$$\frac{\partial\theta}{\partial t} + \mathbf{u}\cdot\nabla\theta = S_\theta$$

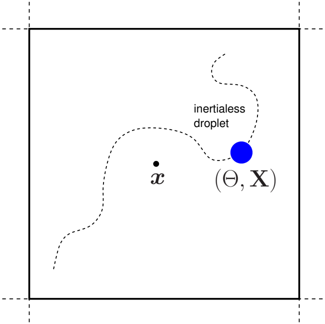
$$\frac{\partial\langle\theta\rangle}{\partial t} + \langle\mathbf{u}\rangle\cdot\nabla\langle\theta\rangle = -\nabla\cdot\mathbf{J} + \langle S_\theta\rangle$$

SGS turbulent flux:

$$\mathbf{J} = \langle\mathbf{u}'\theta\rangle \approx -K\,\nabla\langle\theta\rangle$$

Lagrangian description

Stochastic variables $(\Theta, \mathbf{X})$



Lagrangian description

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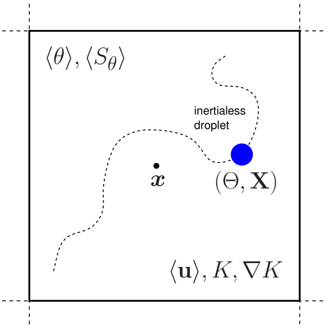
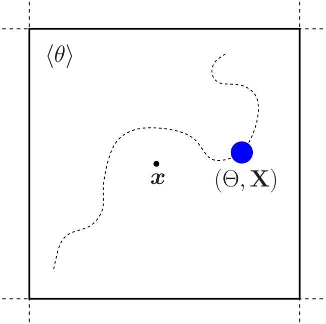


- Langevin equations:

$$\mathrm{d}\Theta = \underbrace{-\frac{\Theta - \langle\theta\rangle}{\tau_m}\,\mathrm{d}t}_{\text{SGS mixing}} + \langle S_\theta\rangle\,\mathrm{d}t,$$

$$\mathrm{d}\mathbf{X} = \left[\langle\mathbf{u}\rangle + \nabla K\right]\mathrm{d}t + \sqrt{2K}\,\mathrm{d}\mathbf{W}$$

Probability description



- Probability that $\theta < \Theta < \theta + \mathrm{d}\theta$

$$f(\theta; \boldsymbol{x}, t)\, \mathrm{d}\theta$$

- f - probability density function
- Average

$$\langle\theta\rangle = \int \theta\, f(\theta; \boldsymbol{x}, t)\, \mathrm{d}\theta$$

Fokker-Planck equation for $f(\theta; \mathbf{x}, t)$:

$$\frac{\partial f}{\partial t} = - \frac{\partial}{\partial \theta}\left[\left(-\frac{\theta - \langle\theta\rangle}{\tau_m} + \langle S_\theta \rangle\right) f\right]$$

$$- \frac{\partial}{\partial \mathbf{x}} \cdot \left[\left(\langle \mathbf{u} \rangle + \nabla K\right) f\right] + \frac{\partial}{\partial \mathbf{x}} \cdot \frac{\partial}{\partial \mathbf{x}} \left(K f\right) \tag{1}$$

Performing

$$\int \theta \, [\text{Eq. } (1)] \, \mathrm{d}\theta \, \ldots$$

… one recovers the Eulerian equation for the average:

$$\frac{\partial \langle \theta \rangle}{\partial t} + \langle \mathbf{u} \rangle \cdot \nabla \langle \theta \rangle = -\nabla \cdot \mathbf{J} + \langle S_\theta \rangle$$

$$\mathbf{J} = -K \nabla \langle \theta \rangle$$

Scalar variance:

$$\frac{\mathrm{d}\langle\theta'^{\,2}\rangle}{\mathrm{d}t} = \text{“turbulent fluxes”} + \text{“production”}$$

$$- \quad 2\,\langle\epsilon_\theta\rangle$$

Scalar dissipation:

$$\langle\epsilon_\theta\rangle = \langle\alpha\nabla\theta' \cdot \nabla\theta'\rangle \approx \frac{\langle\theta'^{\,2}\rangle}{\tau_m}$$

α - molecular diffusivity

Summary

- Simple models to mimic SGS variability
- Broadening of the droplet-size distribution
- Thermodynamic feedback: extends the distance of activation
- Statistical equivalence: Eulerian $\times$ Lagrangian

Acknowledgements

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