UNIVERSITY OF WARSAW

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Cloud droplets in small vortices

Workshop on Eulerian vs. Lagrangian methods for cloud microphysics

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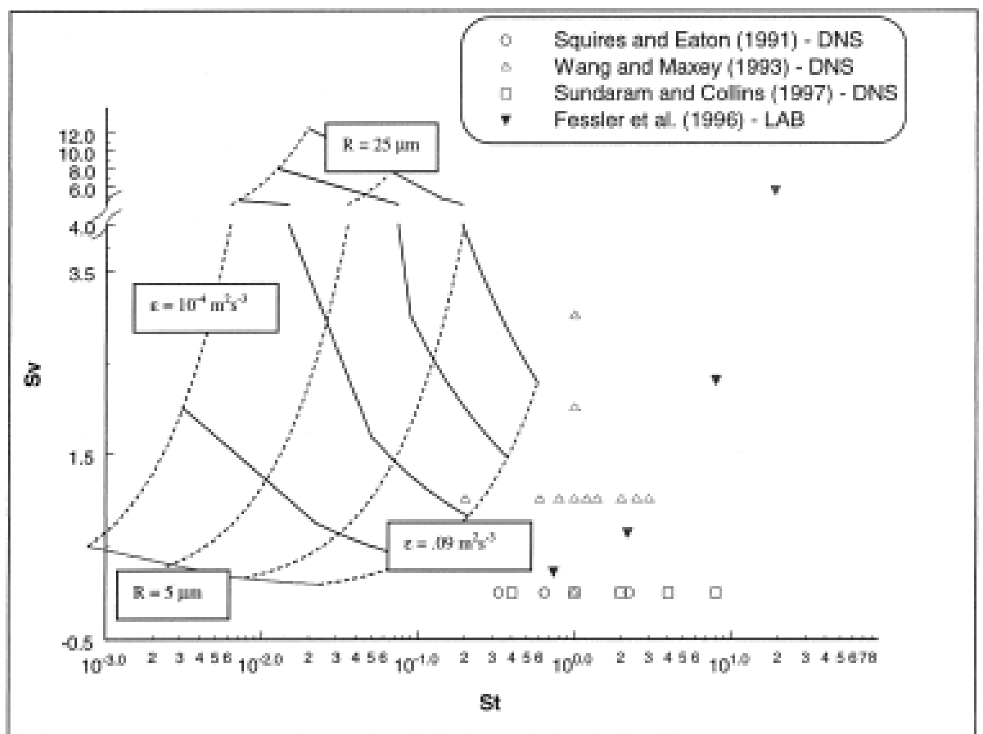
Preferential concentration in clouds

» Interaction between atmospheric turbulence and cloud droplets may lead to preferential concentration of droplets.

» Preferential concentration occur when the spatial distribution of particles does not display features of Poisson distribution.

» Recent studies do not describe well preferential concentration in clouds.



Vaillancourt, Yau,
Bulletin of Am. Met. Soc., 2000

What causes preferential concentration?

https://www.youtube.com/watch?v=T2R6XAJTJYY&feature=kp

What causes preferential concentration?

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- High vorticity regions of tubulent fluid are organized in structures called vortex tubes. They appear in high Re turbulence - such as atmospheric tubulence - occuping small fraction of fluid volume.
- From visualization experiments using air bubbles and DNS (smaller than atmospheric Re_λ):
  - radius of such tubes is of the order of Kolmogorov lengthscale,
  - length is of the order of integral scale,
  - lifetime is several times longer than turnover time of the largest eddies

  (look e.g. Fung, Journal of Geophysical Research,2000).
- **Research idea: presence of the worms can signifiantly influence cloud droplets spatial distribution.**
- First step: simple model of 3d motion of sedimenting particles in line vortex with stretching aligned at the angle to gravity.
- Similar analysis has been already conducted in burgers vortex.

Droplet motion in oblique line vortex

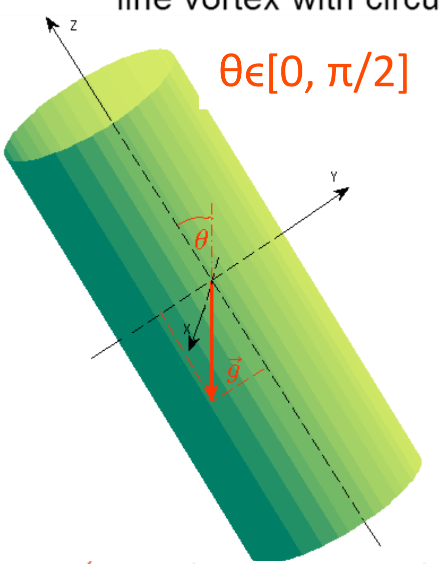
Vortex tube model

Vortex tube is modeled by constant velocity field of line vortex with stretching:

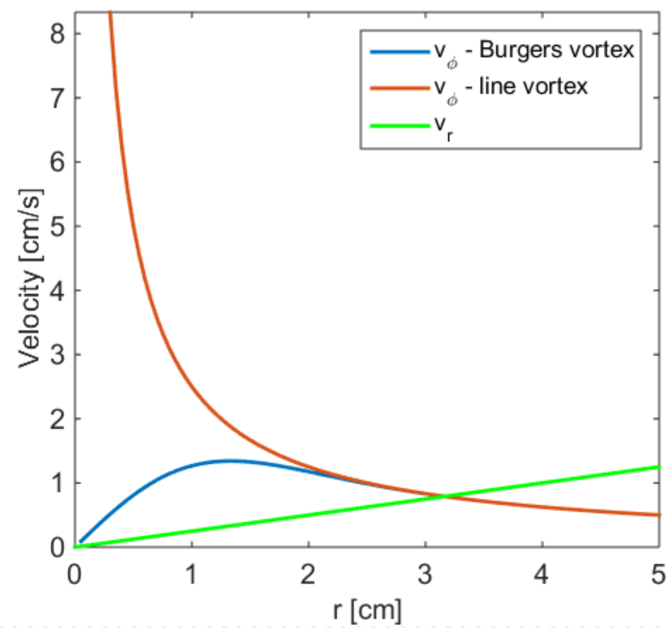
stretching with force γ

$$\vec{v_a} = -\frac{\gamma}{2} r \hat{e}_r + \frac{L_w}{r} \hat{e}_\phi + \gamma z \hat{e}_z$$

line vortex with circulation $2\pi L_w$



$$\vec{g} = -g(sin\theta \hat{e_y} + cos\theta \hat{e_z})$$



Simulation cases	"weak vortex" $\gamma = 0.5\frac{1}{s}, \quad L_w = 0.00025\frac{m^2}{s}$
	"strong vortex" $\gamma = 30\frac{1}{s}, \quad L_w = 0.025\frac{m^2}{s}$

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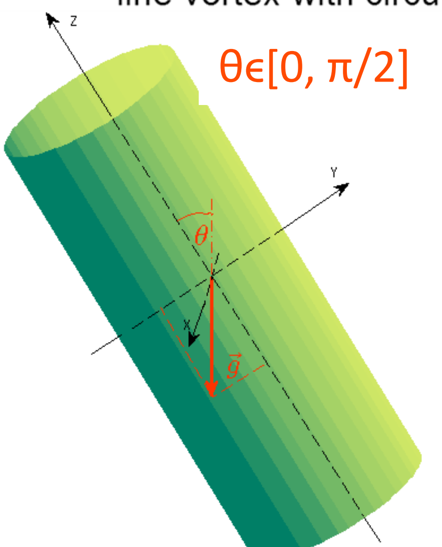
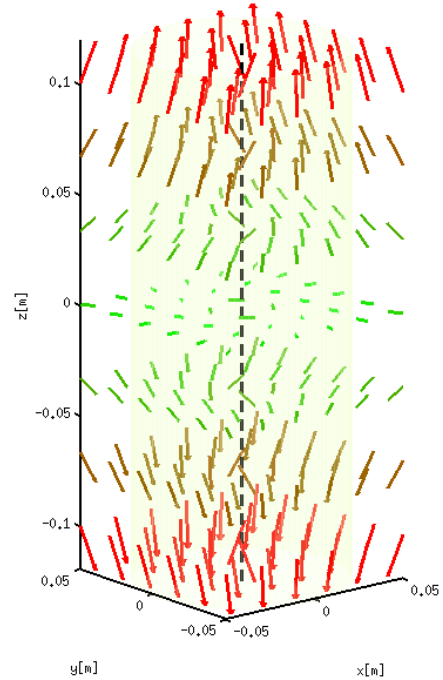


θϵ[0, π/2]

$$\vec{g} = -g(sin\theta \hat{e_y} + cos\theta \hat{e_z})$$



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Droplet dynamics

stretching with force γ

$$\vec{v_a} = -\frac{\gamma}{2} r\hat{e}_r + \frac{L_w}{r}\hat{e}_\phi + \gamma z\hat{e}_z$$

line vortex with circulation $2\pi L_w$

- » Droplet motion is influenced by viscous and gravitational forces only, no interaction between droplets was included.
- » Motion in a plane perpendicular to vortex axis separates from the motion in direction parallel to it.
- » Effectively there is a dependence on three parameters: St, Sv, θ.

$$m\ddot{\vec{r}} = \frac{m}{\tau}(\vec{v}_a - \dot{\vec{r}}) + m\vec{g}$$

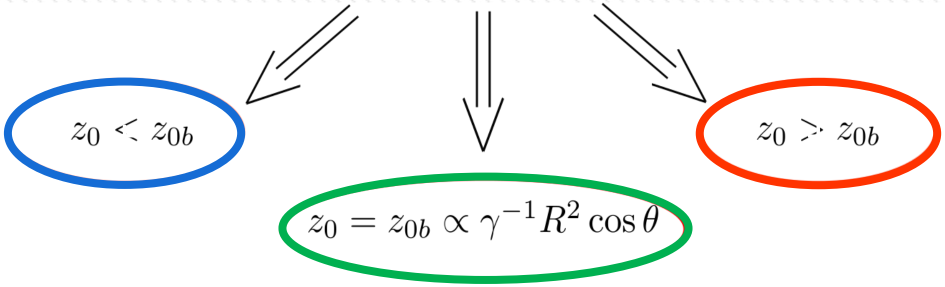
$$\ddot{\vec{r}} = \frac{1}{St}(\overrightarrow{v_a} - \dot{\vec{r}}) - \frac{Sv}{St} e_g$$

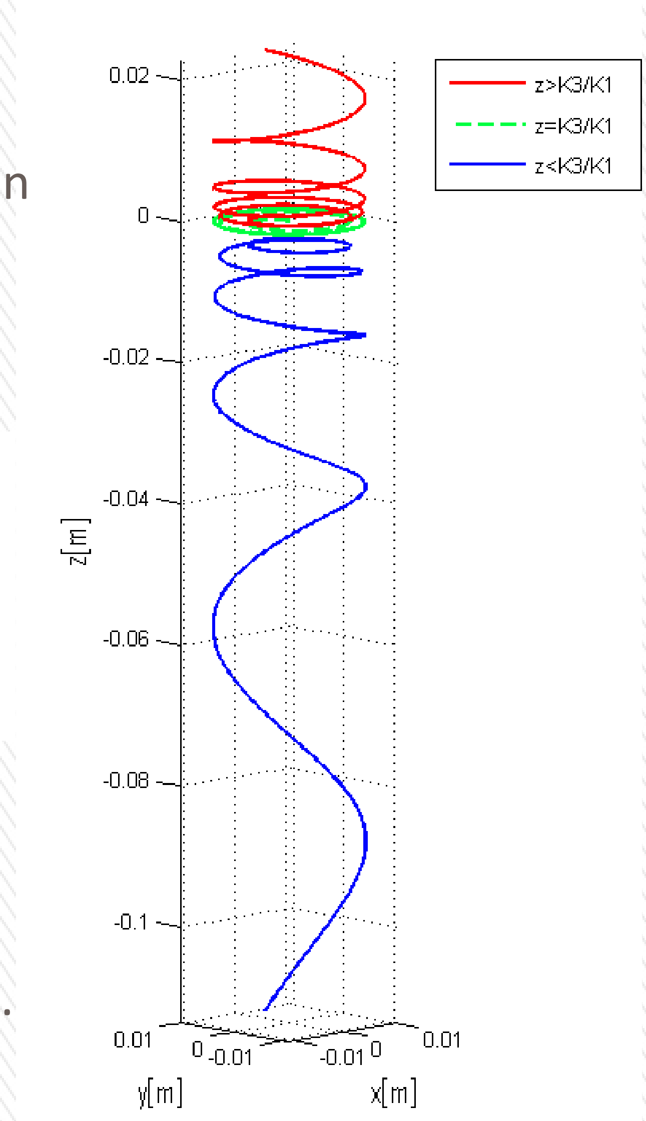
$$\delta = \sqrt{\frac{2\nu}{\gamma}}$$ - core size of Burgers vortex

$$\tau_f = \frac{\delta^2}{\Gamma}$$ - turnover time of Burgers vortex

Motion in Z-direction –along vortex axis

Under the influence of stretching and gravity forces motion of droplet in Z-direction with initial condiitions $z(0) = z_0, \ \dot{z}(0) = 0$ depends only on initial position z_0 in respect to **steady, unstable** position z_{0b}.





The most promising is the case when droplets start close to their steady position.

Motion in XY-plane – vertical vortex

» Motion in a plane perpendicular to vortex axis shows strong dependence on the angle of vortex - gravity alignment θ.

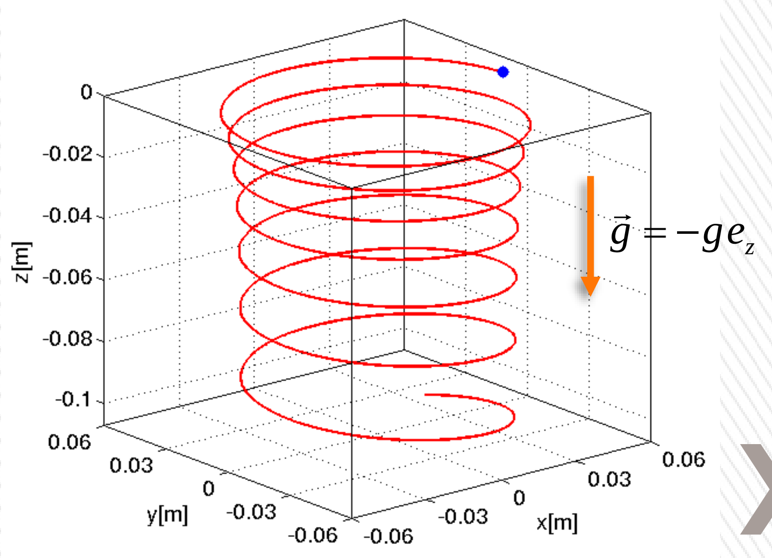
» For θ=0 we have „vertical" vortex, in which every droplet gets on its stationary orbit (stable, periodic)

Orbit radius

$$r_{orb} = \sqrt[4]{2L_w^2\tau\gamma^{-1}} \propto \sqrt{R}$$

Orbit angular velocity

$$\omega_{orb} = \sqrt[2]{\frac{\gamma}{2\tau}} \propto R$$



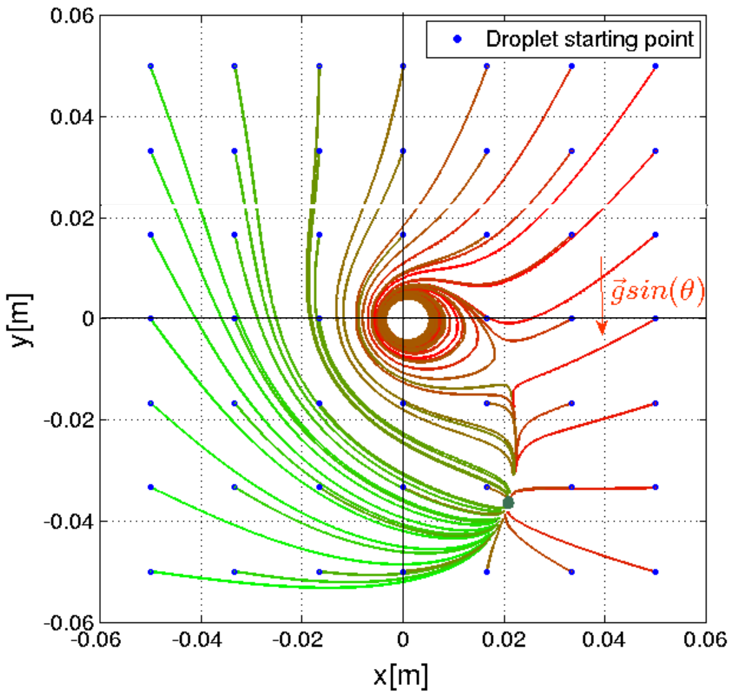
Motion in XY-plane – oblique vortex

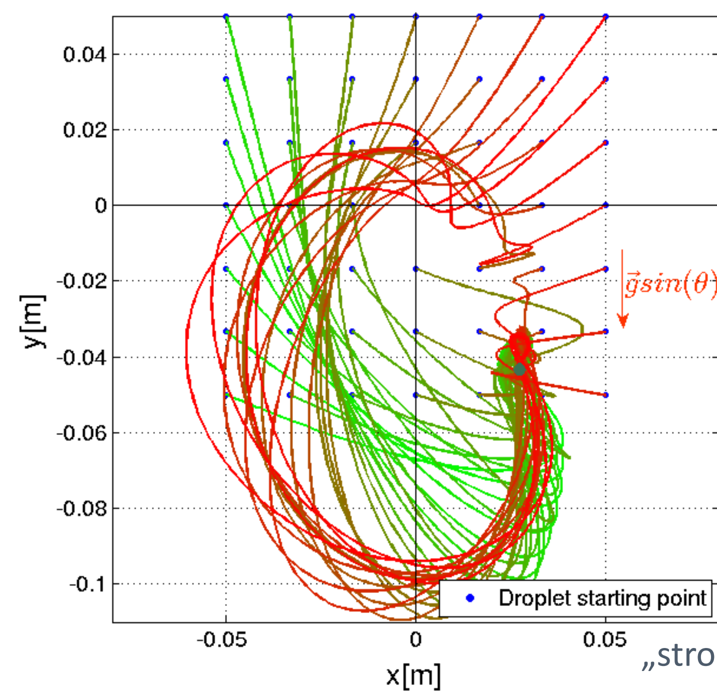
When θ≠0 gravity influence destroys axial symmetry present in vertical vortex.

no stationary orbits

stationary points

limit cycles

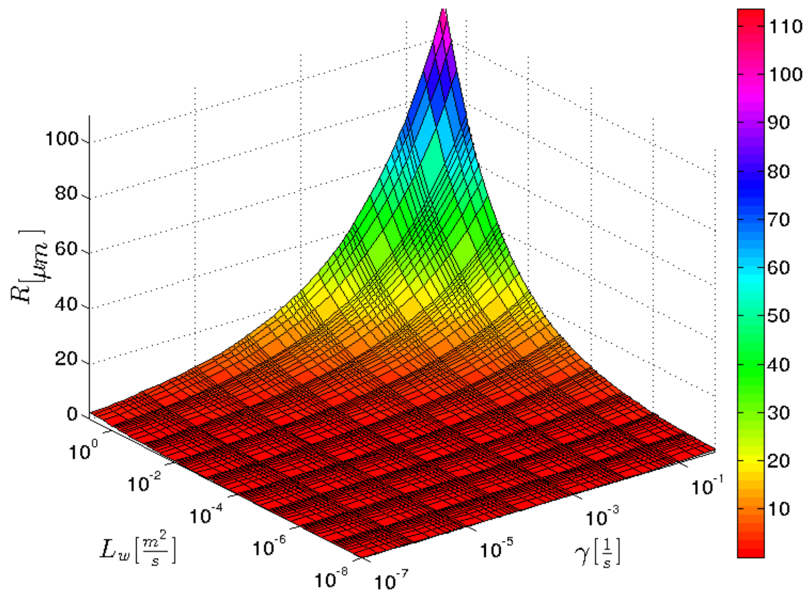




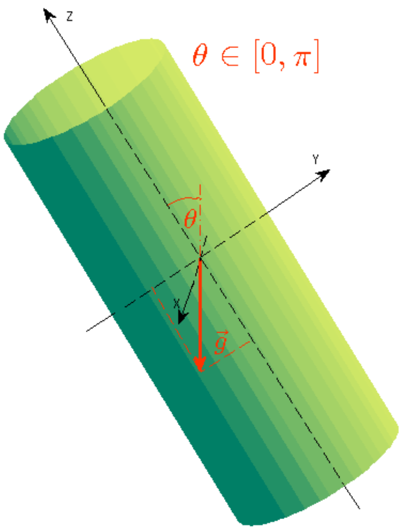
R=86 μm
„strong” vortex

Stationary points

- Stationary points are steady positions where forces of viscosity and gravity balance.
- Existence of stationary points is limited to certain ranges of parameters.



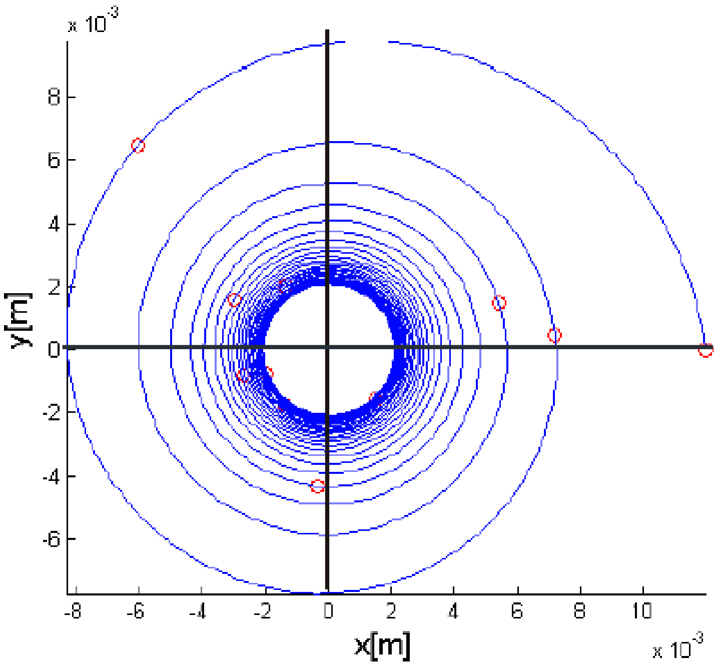
$$x_{st} \geq 0, \quad y_{st} \leq 0$$



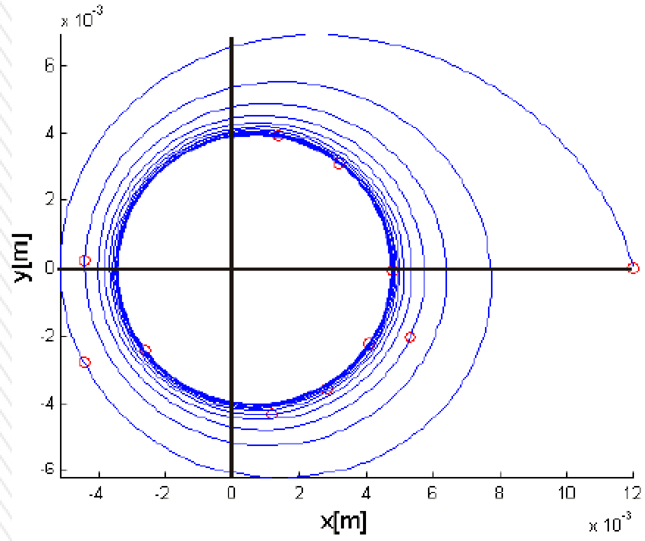
Existence condition -boundary radius R_{gr} surface

$$f(L_w, \gamma, R) \leq 1 \longrightarrow \sin(\theta) \geq f(L_w, \gamma, R)$$

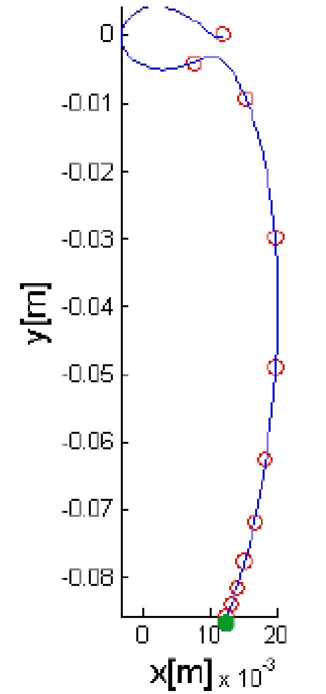
$$\vec{g} = -g(sin\theta\hat{e_y} + cos\theta\hat{e_z})$$



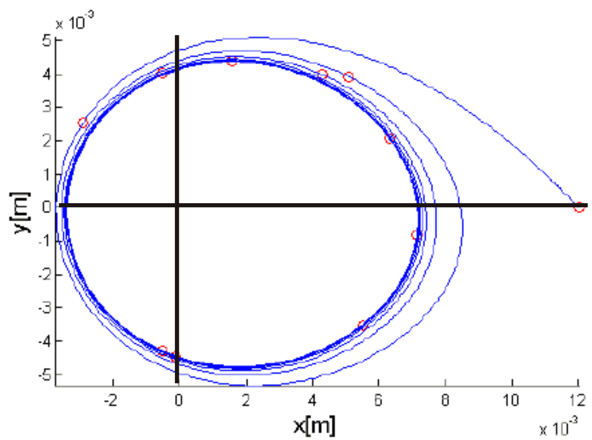
(a) R=2.5 μm



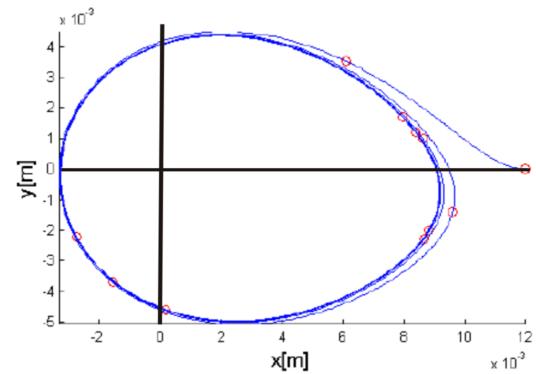
(c) R=9 μm



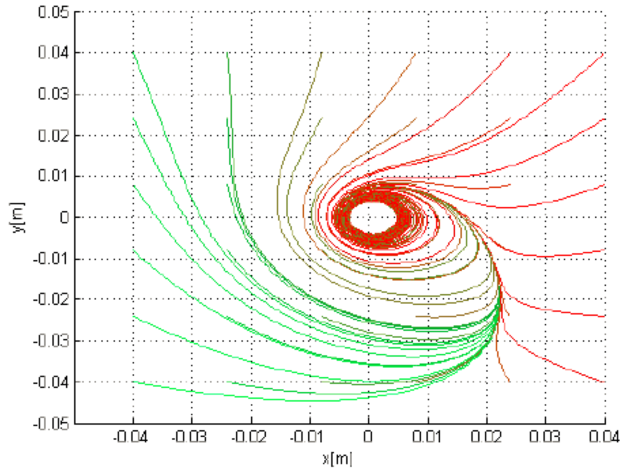
(e) R=13.5 μm



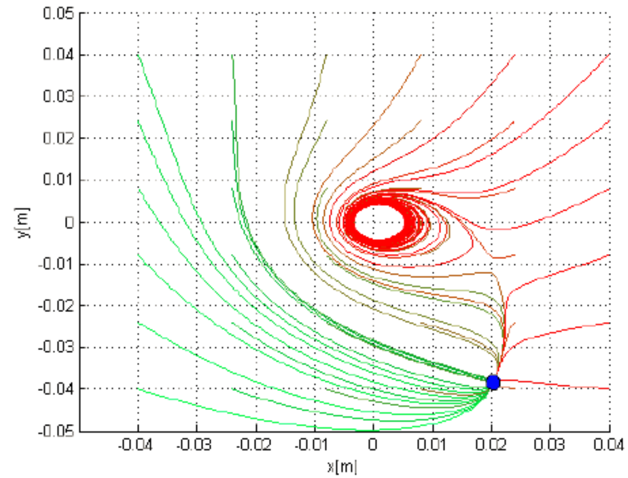
(a) R=12 μm



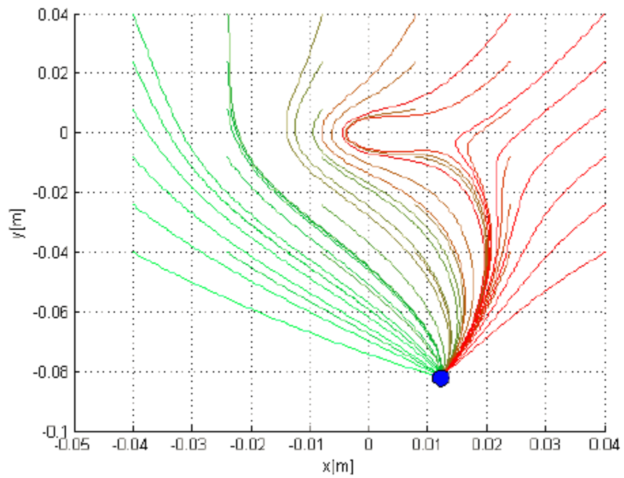
(c) R=13 μm



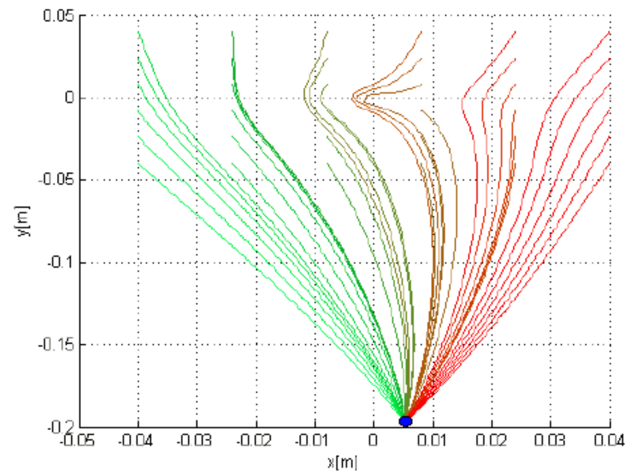
(a) R=9 μm



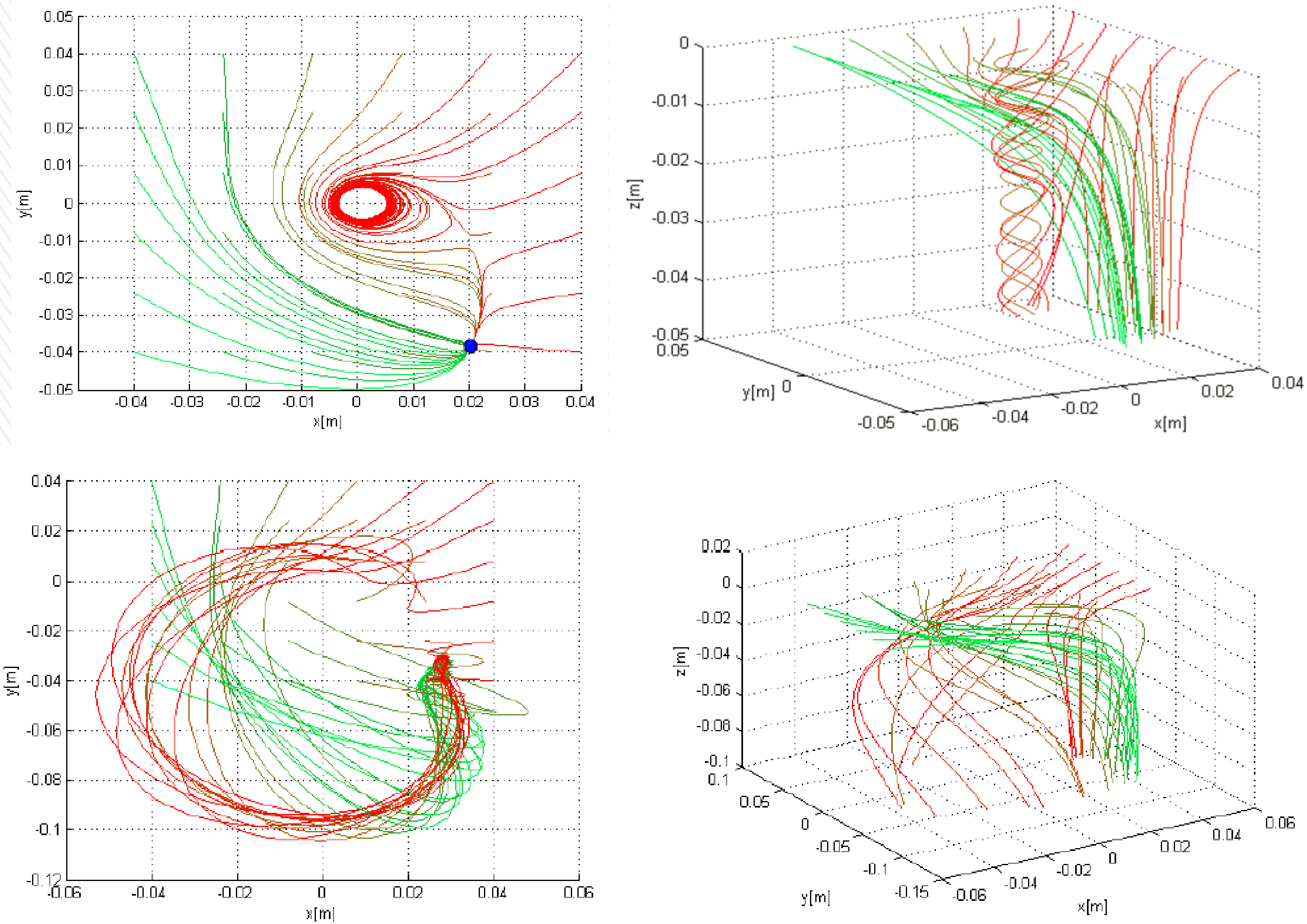
(b) R=10 μm



(c) R=13 μm



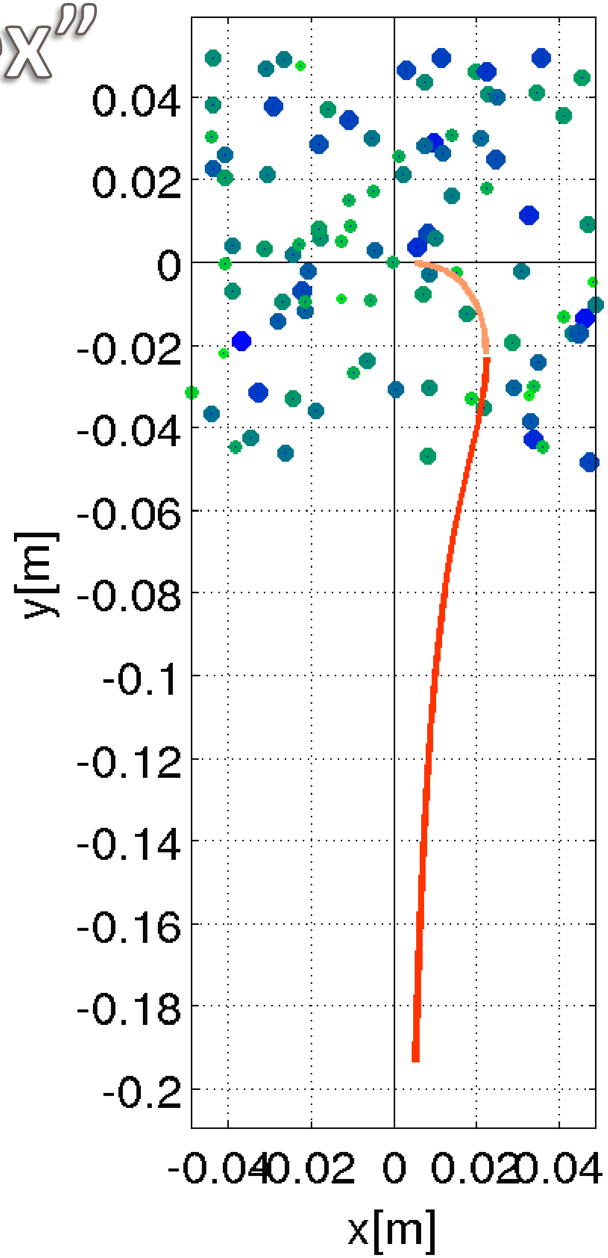
(d) R=20 μm



(a) z_0=K2/K1 (b) z_0=0

Rysunek 3.10: Zbiory trajektorii 36 kropel rozmieszczonych równomiernie w płaszyźnie o stałym z, na kwadracie o boku l=8cm wokół osi wiru, o promieniach $R = 85\ \mu m$ w wirze ukośnym standardowym, przy $\theta = 0.45\pi$.

Different size droplets: trajectories in „weak vortex”

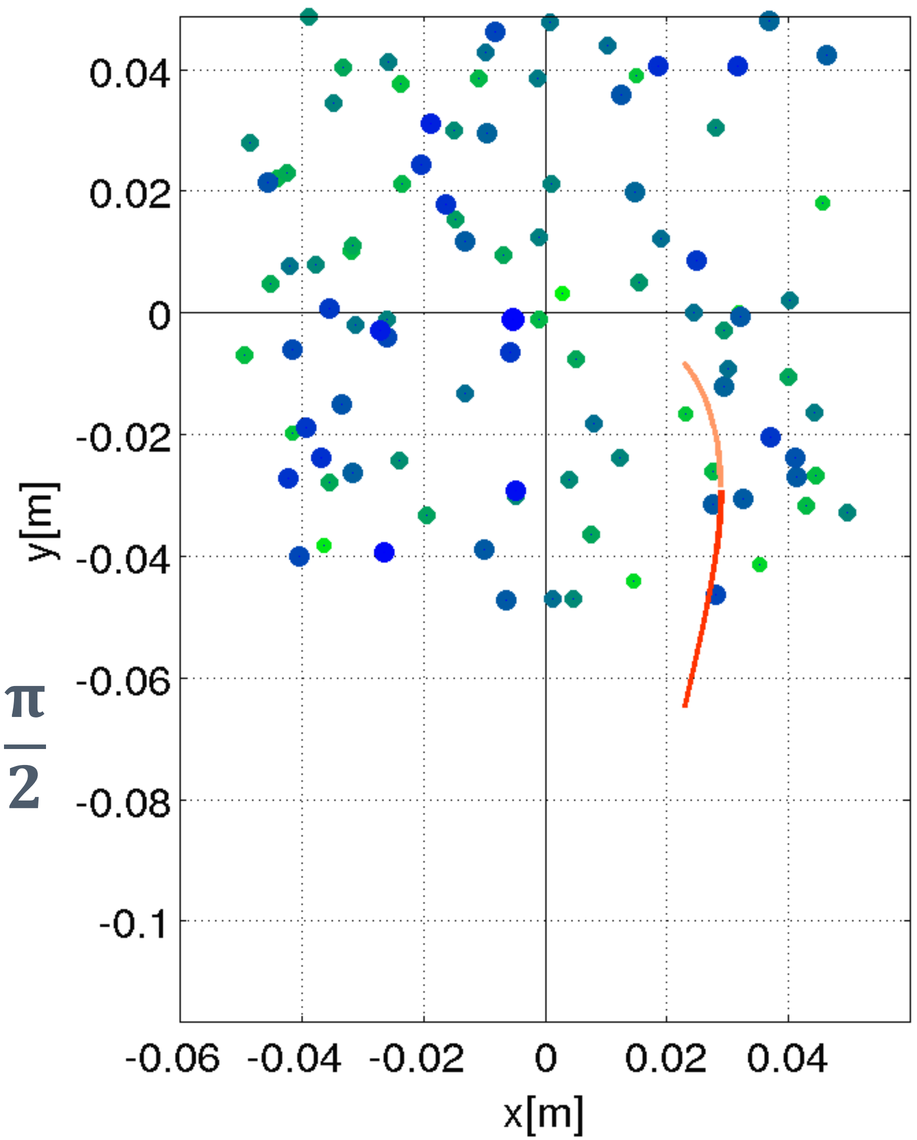


$$R \in [1\mu m, 20\mu m]$$

Boundary droplet radius values	"weak vortex" $R_{gr} = 9,56\mu m$
	"strong vortex" $R_{gr} = 84,07\mu m$

$$\theta = 0.9 \cdot \frac{\pi}{2}$$

Different size droplets: trajectories in „strong vortex”



$$\theta = 0.9 \cdot \frac{\pi}{2}$$

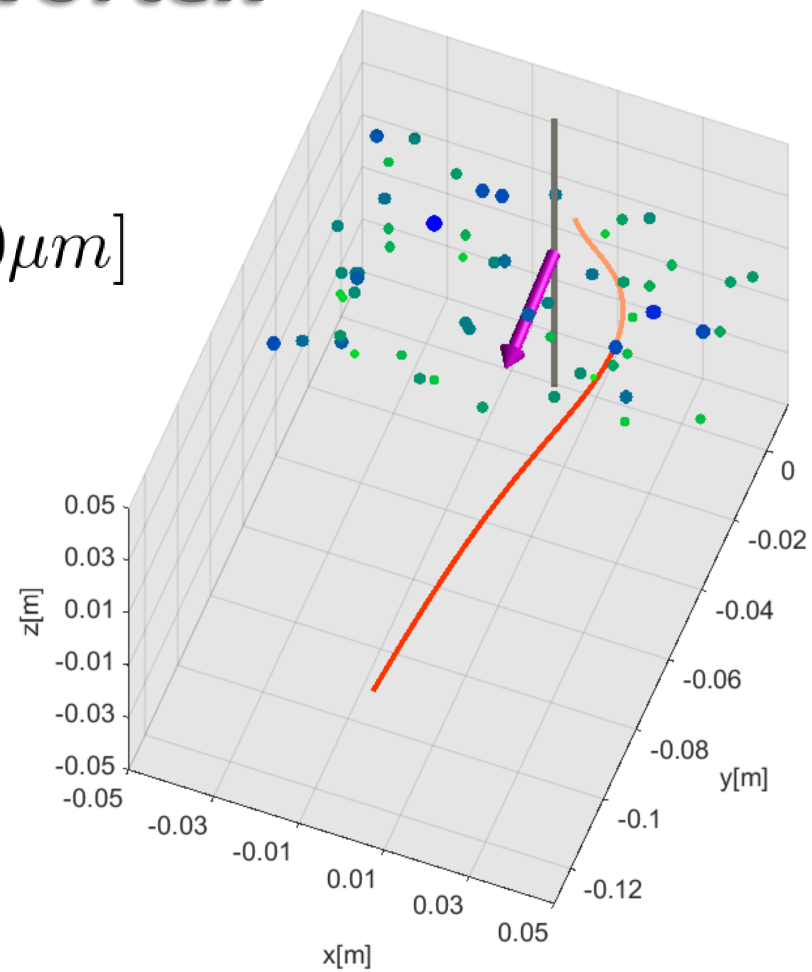
$$R \in [75\mu m, 95\mu m]$$

<table>
<tr><td rowspan="2">Boundary droplet radius values</td><td>"weak vortex"
$R_{gr} = 9,56\mu m$</td></tr>
<tr><td>"strong vortex"
$R_{gr} = 84,07\mu m$</td></tr>
</table>

Different size droplets: 3D trajectories in „weak vortex"

$$R \in [1\mu m, 20\mu m]$$

t=0s

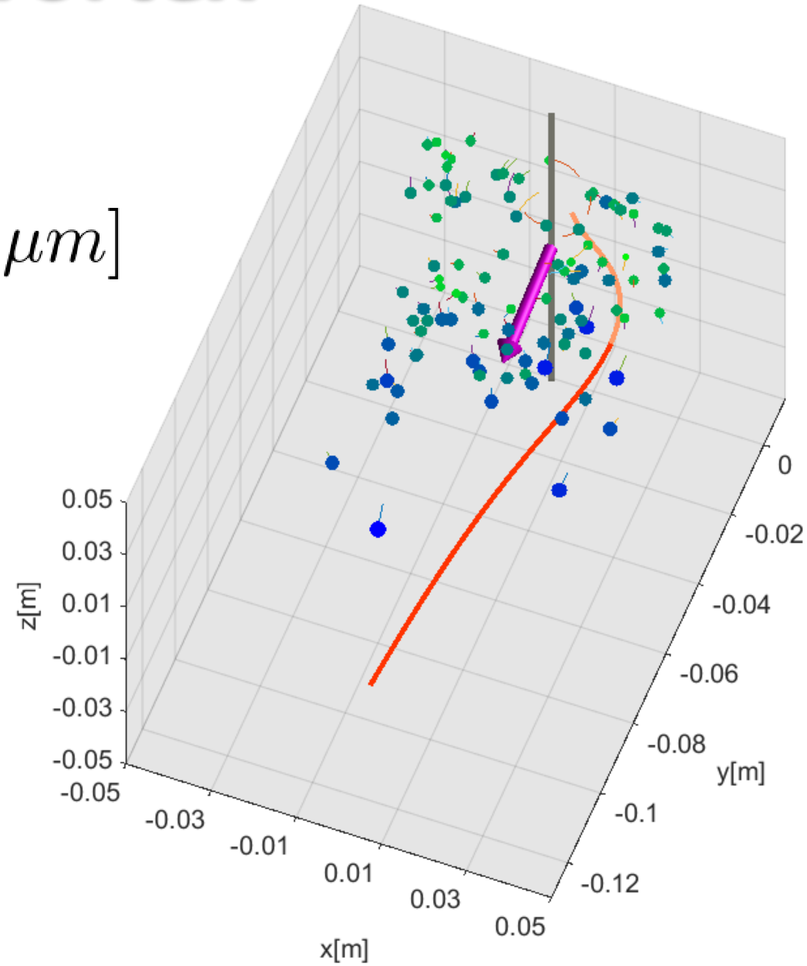


Droplet sorting occur also in three dimensions.

Different size droplets: 3D trajectories in „weak vortex”

$$R \in [1\mu m, 20\mu m]$$

t=2s

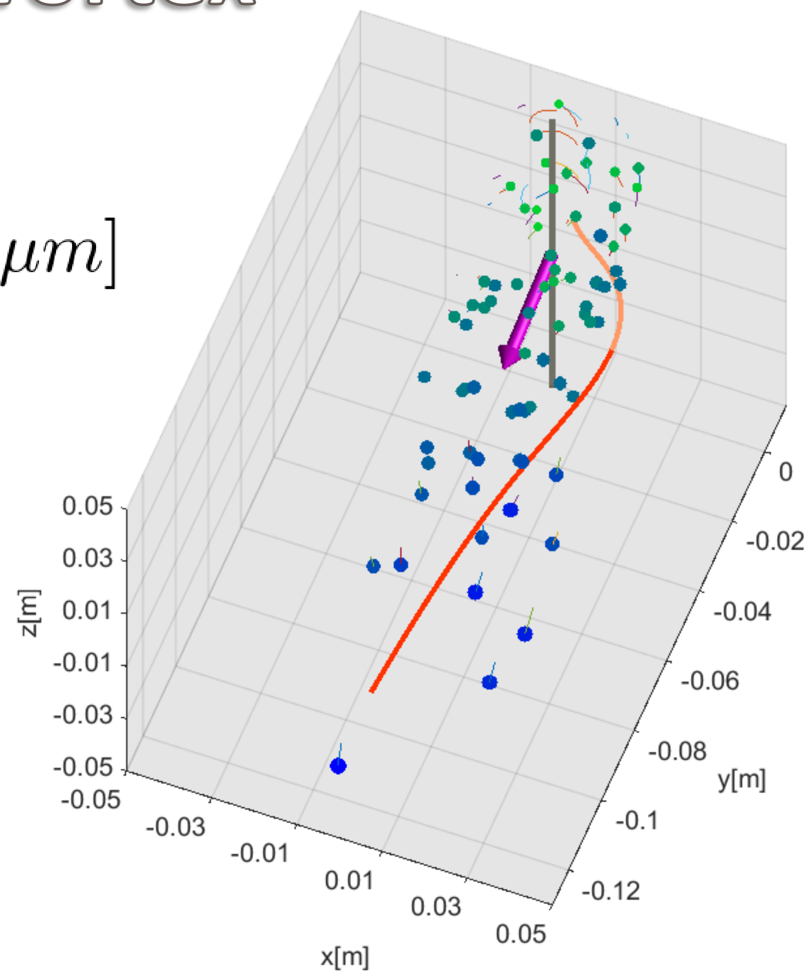


Droplet sorting occur also in three dimensions.

Different size droplets: 3D trajectories in „weak vortex”

$$R \in [1\mu m, 20\mu m]$$

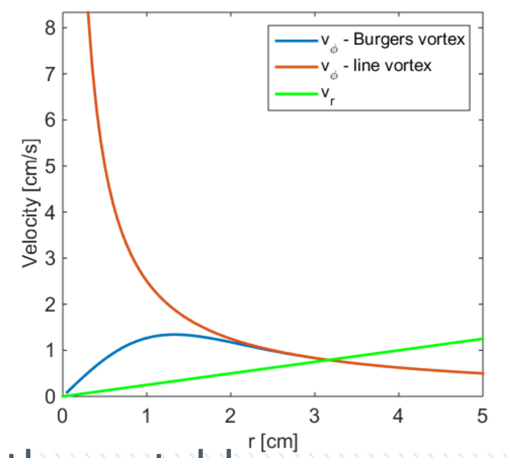
t=4s



Droplet sorting occur also in three dimensions.

Droplet motion in oblique Burgers vortex

Stationary solutions in Burgers vortex



- No gravity: stationary point on the axis, over St_{cr} stationary (periodic stable) orbit.
- With gravity: one or multiple steady points, some of them stable.

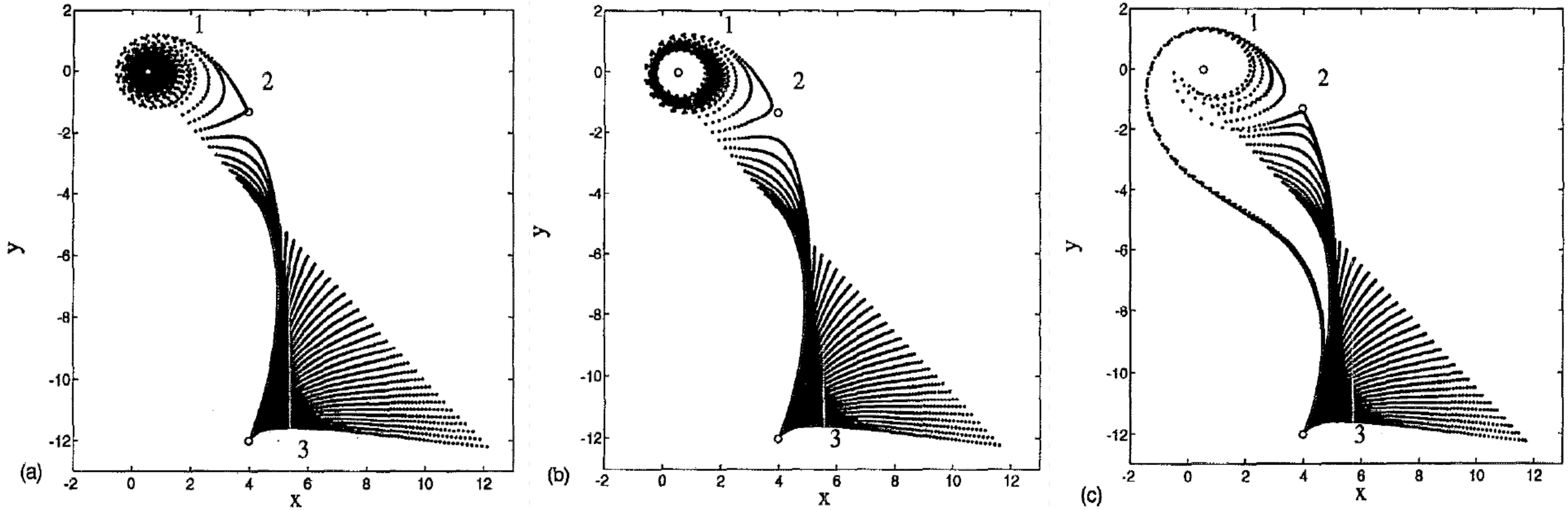


FIG. 4. Particle trajectories for the case $St/Fr^2=0.04$ and $A=0.003$. (a) St=0.5, Fr=3.5355. (b) St=1.0, Fr=5.0. (c) St=10.0, Fr=15.8114.

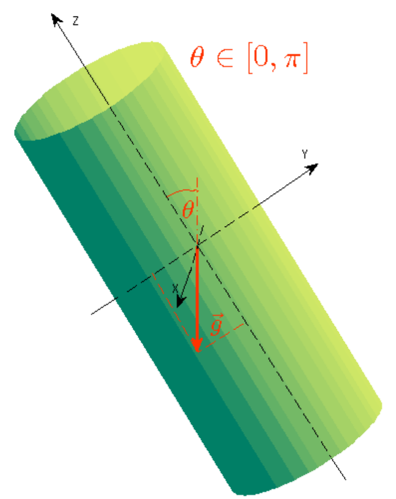
Marcu, Mieburg and Newton, 1995

Preferential concentration in vortex tube

How to measure preferential concentration in a model of a vortex tube?

» Droplets inserted on the sidewalls of the cylinder of chosen sizes in a vortex velocity field.

» Positions of droplets are randomized to obtain Poisson ditribution with certain concentration outside of the vortex.

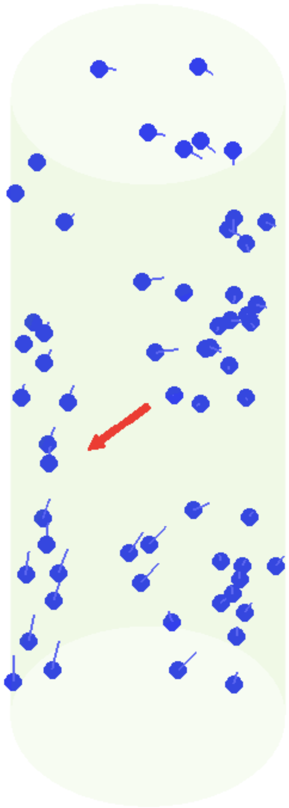


» Timestep of inserting is adjusted to the radial inflow, radial initial velocity is the same as fluid flow.

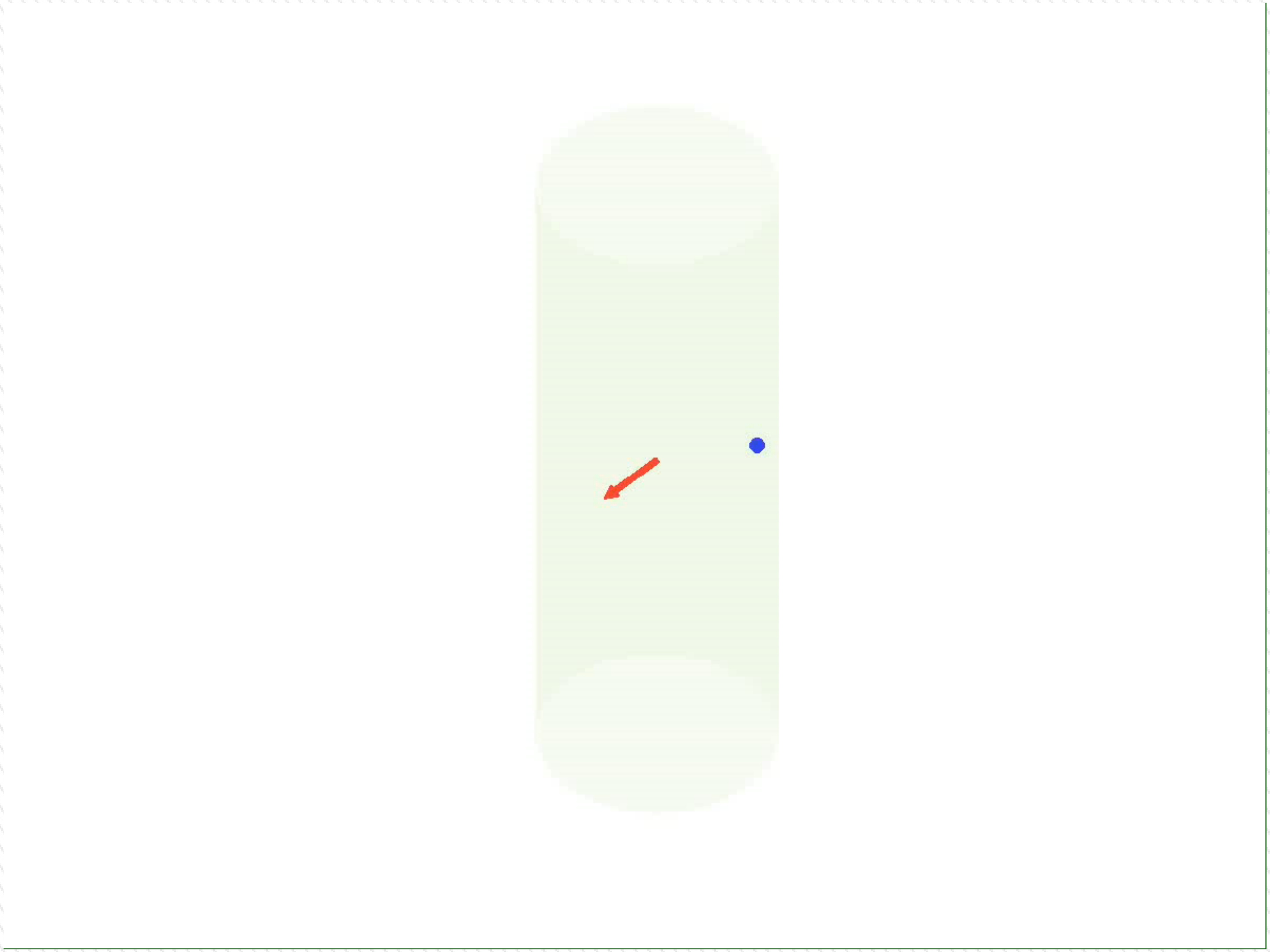
! The idea: compare statistical properties of droplets spatial distribution in the vortex with Poisson distribution.

Multi-droplet simulations

Parameter	Value
Cylinder radius	D=2cm
Cylinder length	Z=12cm
Droplet radius	R=10um
Stretching strenght	$L_w=2.4\cdot10^{-4}m^2/s$
Circulation	γ=0.5/s
Angle of alignement	θ=41º
Droplet number concentration	$n=50/cm^3$
Stokes number	**$St=2.7\cdot10^{-2}$**
Sedimentation parameter	**$Sv=4.9\cdot10^{-2}$**
Time of simulation	$T_k=10s$

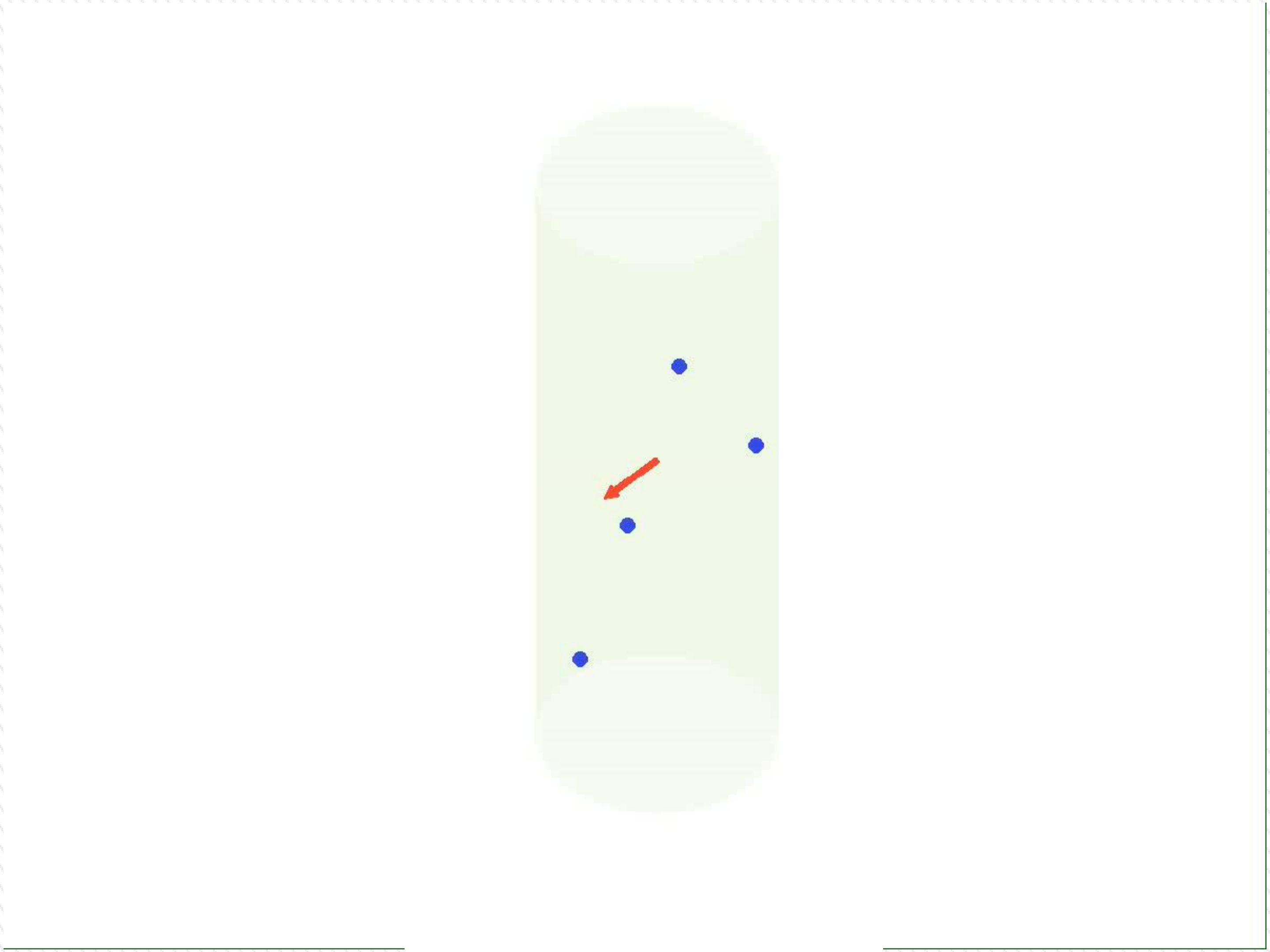


Multi-droplet simulations



Line vortex

Multi-droplet simulations



Burgers vortex

Preferential concentration analysis

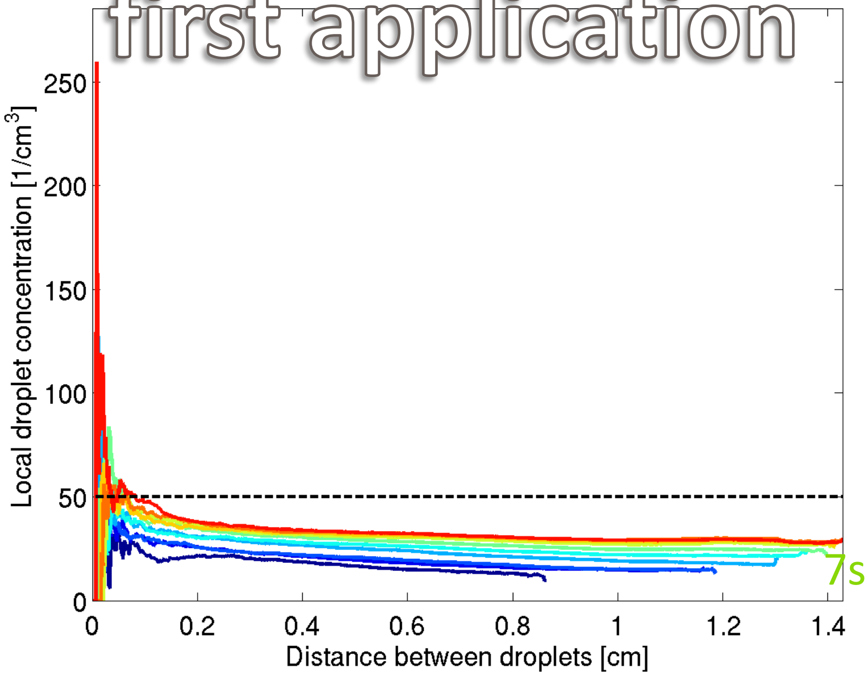
$$C(r) = \frac{1}{\frac{4}{3}\pi r^3} \frac{\sum_{1}^{N} k_{jr}}{N}$$

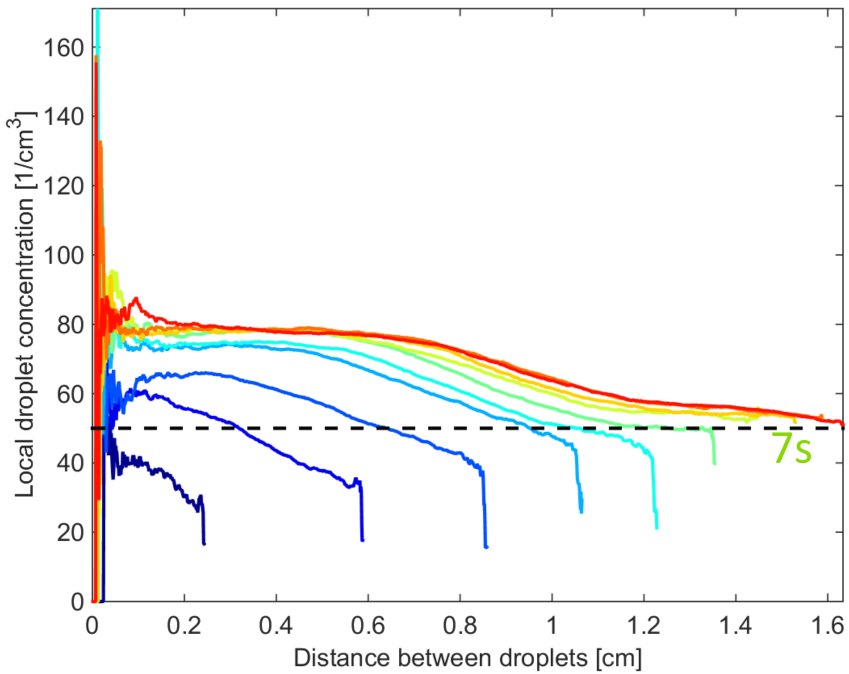
j-droplet number of neighbours in the sphere of radius r

number of droplets with the distance to the edge smaller than r

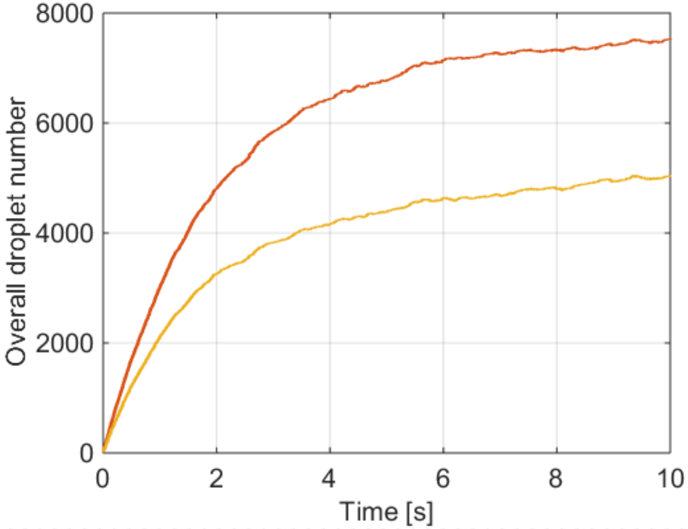
For Poisson droplet spatial distribution, this function should be constant and equal droplet concentration chosen outside the vortex.

Preferential concentration analysis – first application

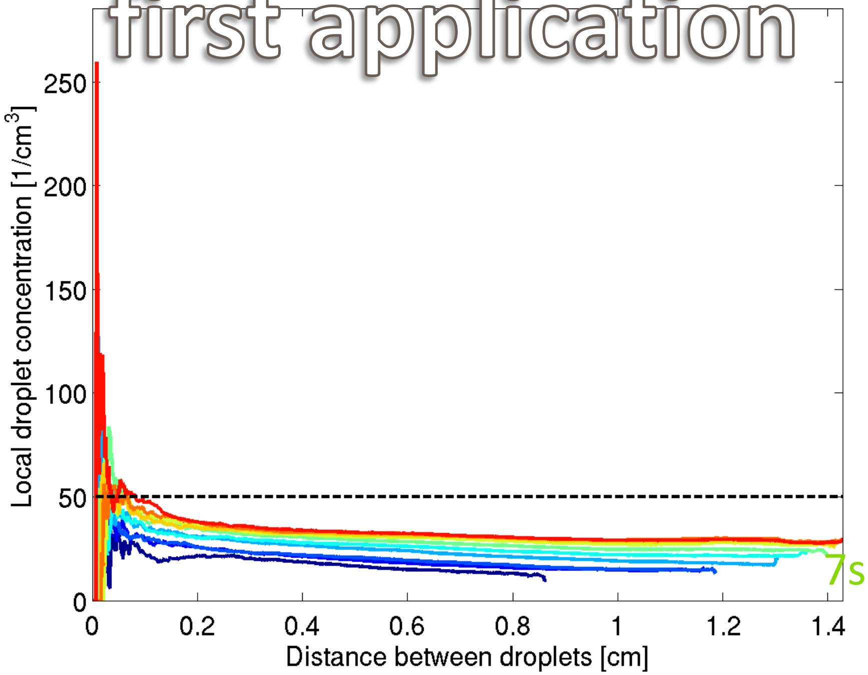


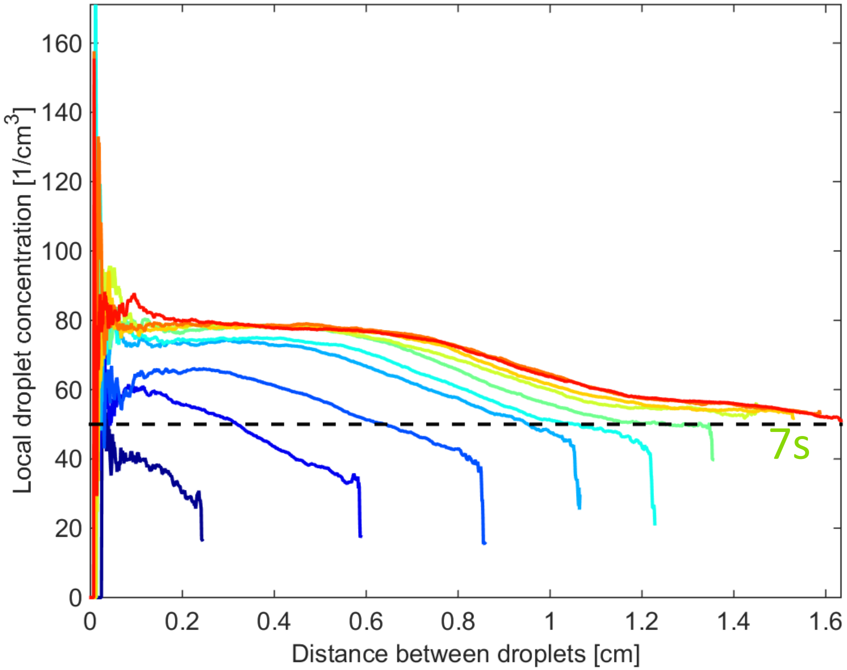


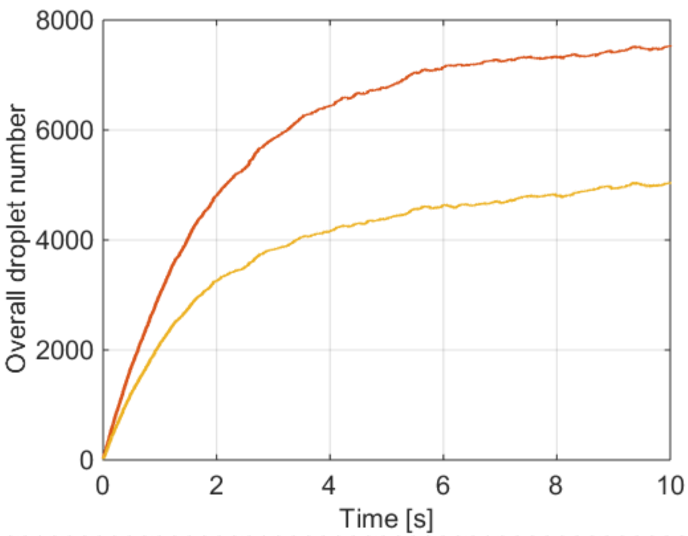
Line vortex



Preferential concentration analysis – first application







Burgers vortex

Summary

- Features such as stationary orbits, stationary points and limit cycles were identified qualitatively in a line vortex as three-dimensional structures that may lead to enhancement of preferential concentration of inertial particles as well as sorting them in space by size.
- Conditions for existence of stationary orbits and points for a line vortex may be derived analytically. For a given set of parameters there exist always one or more of the identified solutions.
- Preferential concentration is clearly visible in local concentration function for our simulations.
- Influence of processes described on cloud droplets spatial distribution may strongly depend not only on the vortex stretching strenght and circulation, but also on its size and time of life. The urgent need for experimental data has emarged.

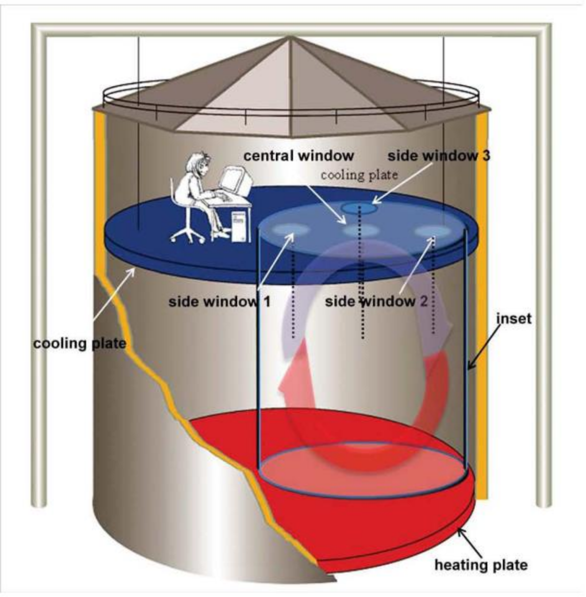

Thank you for your attention.

PCCDrop experiment

PCCDrop – general overview

„The Barrel of Ilmenau (BOI) presents a large-scale Rayleigh-Bènard experimental setup to study highly turbulent convection in its pure form. Roughly isotropic turbulence is obtain with zero mean velocity in the center, different from all the wind-tunnel-like facilities.”
http://www.euhit.org/infrastructures/barrel-of-ilmenau

By looking with the camera through transparent walls of the small barrel inserted in the big one we took pictures of nonevaporating 5 micrometer droplets injected into close to isotropic turbulence of dissipation rates around atmospheric values.



https://www.tu-ilmenau.de/en/barrel-of-ilmenau/external-users/

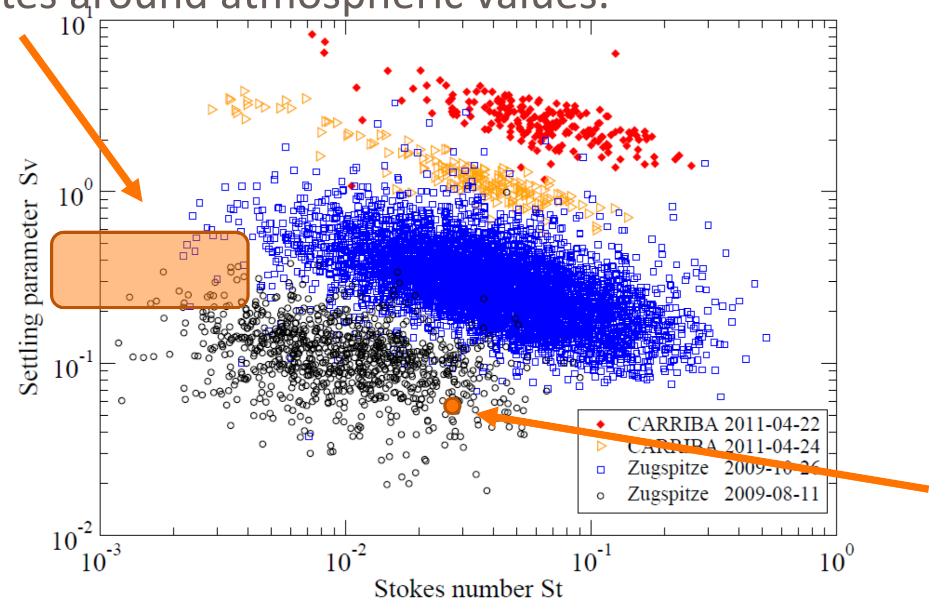


Figure 10. Dimensionless Stokes and settling parameter space. Each ⟨St|Sv⟩-point is based on a one-second average of cloud data. The CARRIBA data represents typical conditions for clean (red) and slightly more polluted (yellow) cases and provide a reference for typical trade wind cumuli (see Siebert et al., 2013, for more details). Data from small cumulus and stratocumulus clouds under continental conditions are not shown, but are nearly coincident with the Zugspitze values (Siebert et al., 2010a).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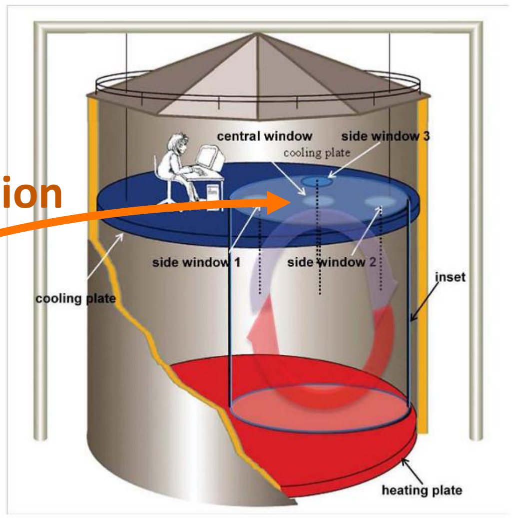
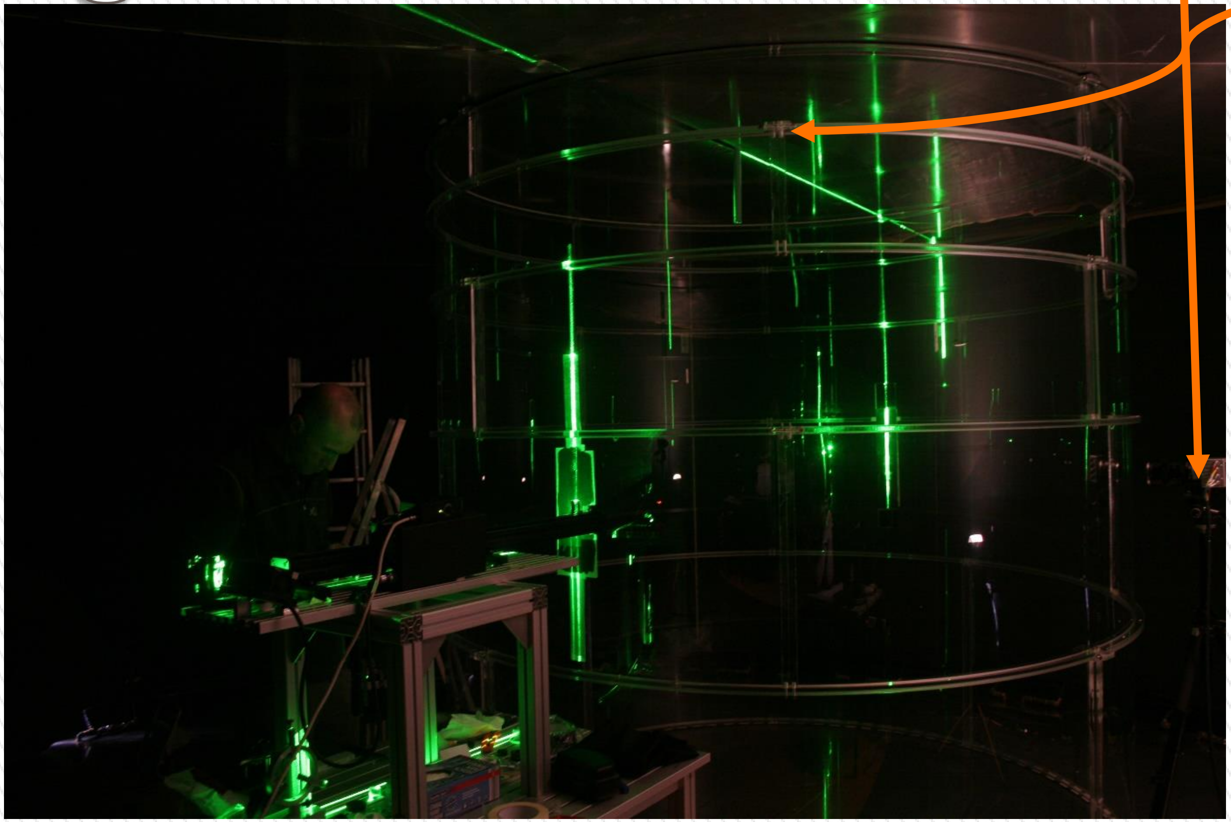
PCCDrop experiment – general overview

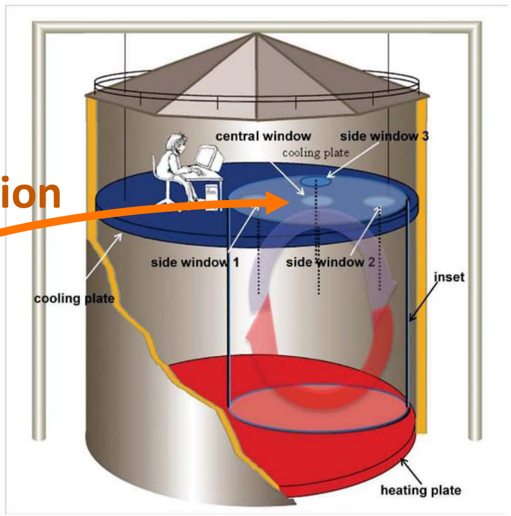
Droplets injection



PCCDrop experiment – general overview

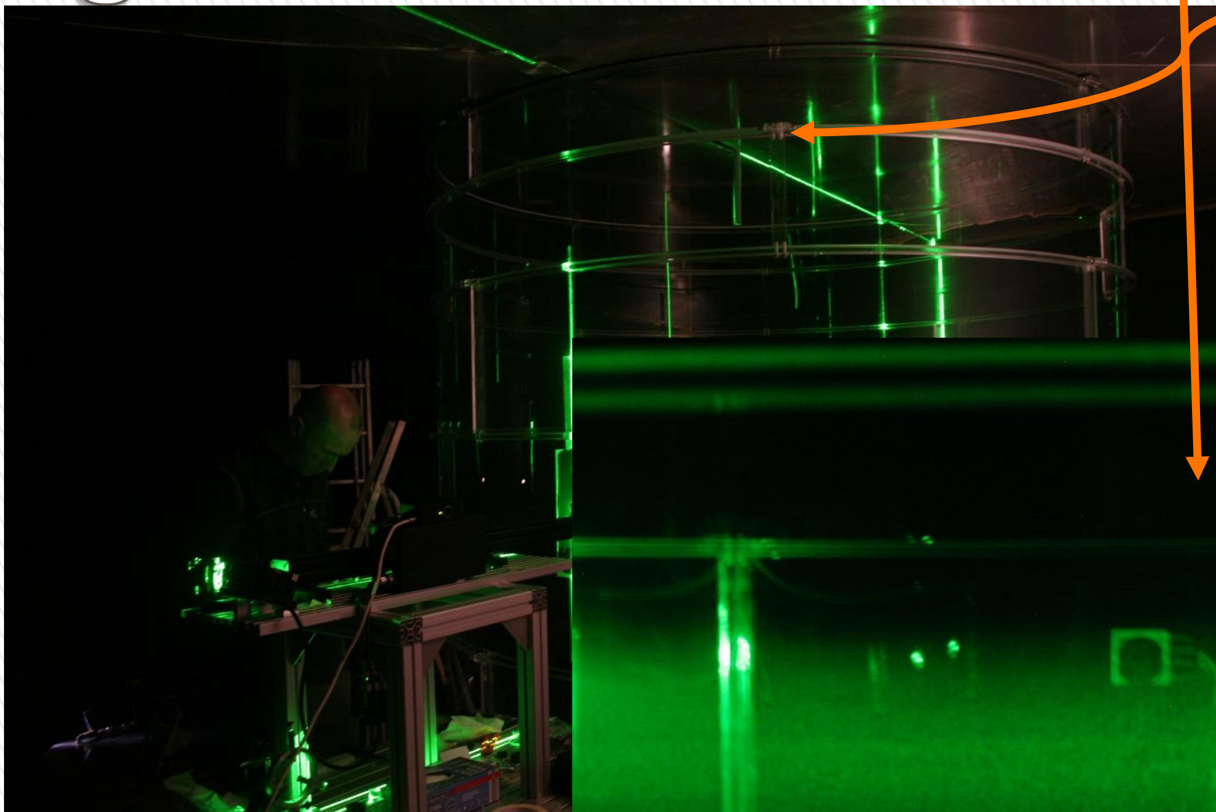
Droplets injection

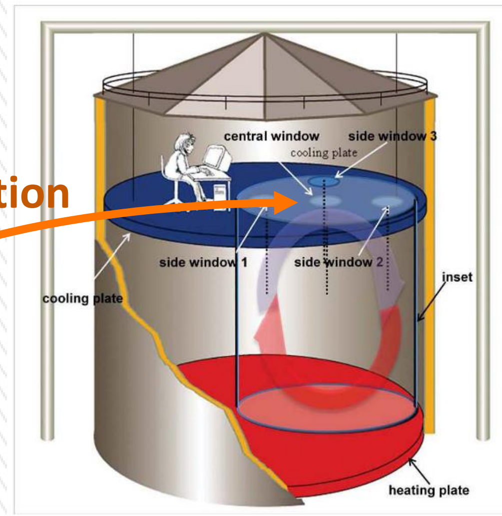




PCCDrop experiment – general overview

Droplets injection



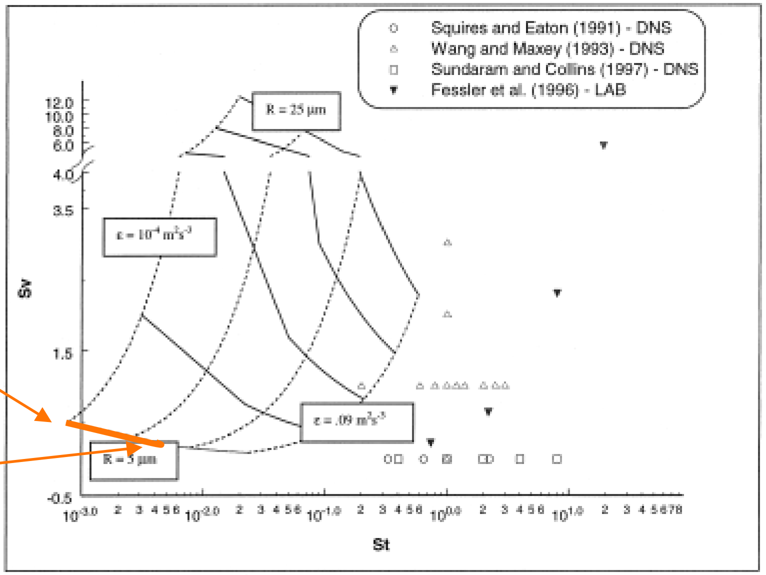


PCCDrop experiment – general overview

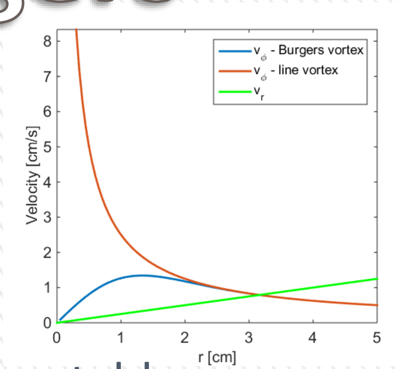
- Droplet radius: R=5 µm,
- Droplets liquid: Di-Ethyl-Hexyl-Sebacat (DEHS), density similar to water density (912 kg/m³), almost no evaporation (boiling temperature at around 240 ºC).
- Picture size: 25cm *25 cm (85mm lens), 43cm*43cm (50mm lens)
- Lasersheet width: 3mm
- Every picture shows independent spatial distribution of droplets.

diameter:	7,15 m
plate distance:	0,20 ... 6,30 m
temperature of the heating plate:	10 ... 80°C
temperature of the cooling plate:	10 ... 80°C
sidewall:	no heat flux
working fluid:	air
Rayleigh number:	10^5 ... 10^{12} 10^7 ... 10^{11}
aspect ratio:	1,13 ... 40
Prandtl number:	0,7
Nusselt number:	... 650
Reynolds number:	... 3x10^5

Δ T [K]	ε [m^2/s^3]
3,8	0,00013
6,8	0,00028
17,5	0,00100
29,4	0,00200
37	0,00271



Stationary points in Burgers vortex



- No gravity: stationary point on the axis, over St_{cr} stationary (periodic stable) orbit.
- With gravity: one or multiple steady points, some of them stable.

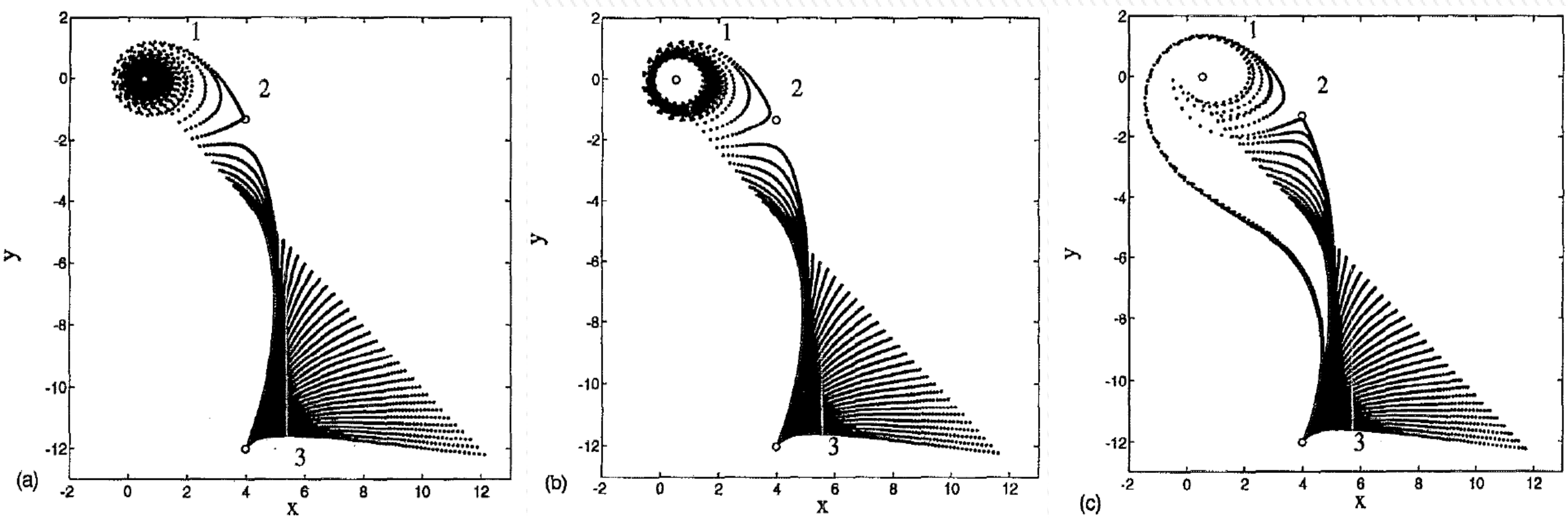


FIG. 4. Particle trajectories for the case $St/Fr^2=0.04$ and $A=0.003$. (a) St=0.5, Fr=3.5355. (b) St=1.0, Fr=5.0. (c) St=10.0, Fr=15.8114.