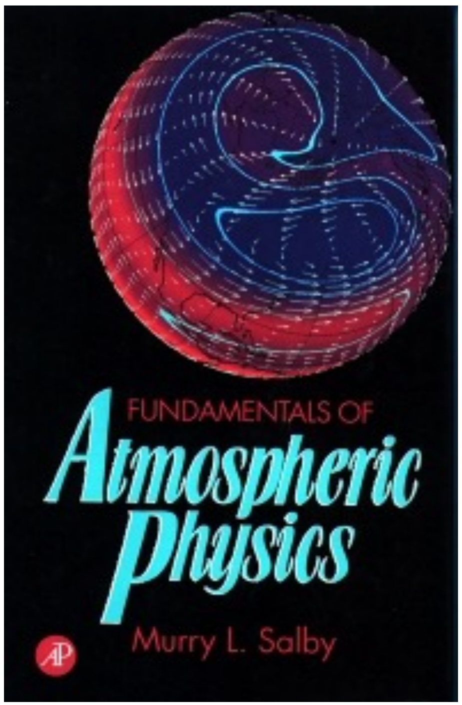
STABILITY IN THE ATMOSPHERE

LECTURE OUTLINE

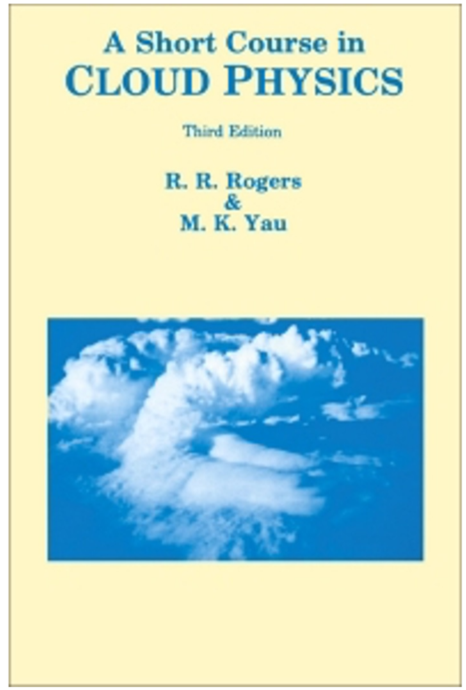
1. Conditional stability
2. Entrainment
3. Modification of stability

Salby, Chapter 6



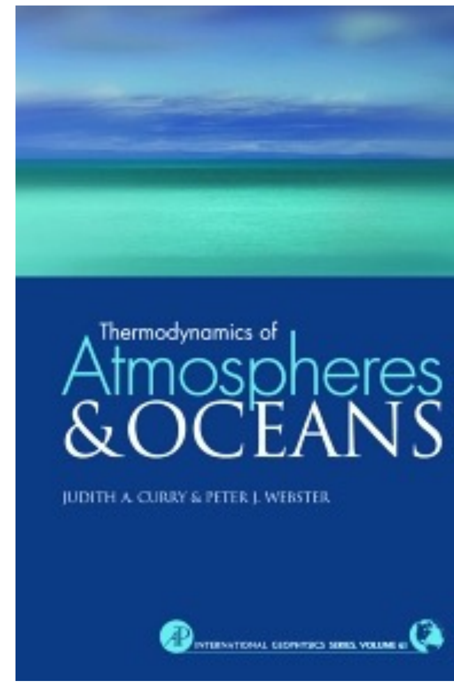
Fundamentals of Atmospheric Physics,
M.L. Salby; Salby

R&Y, Chapter 4



A Short Course in Cloud Physics,
R.R. Rogers and M.K. Yau; R&Y

C&W, Chapter 7

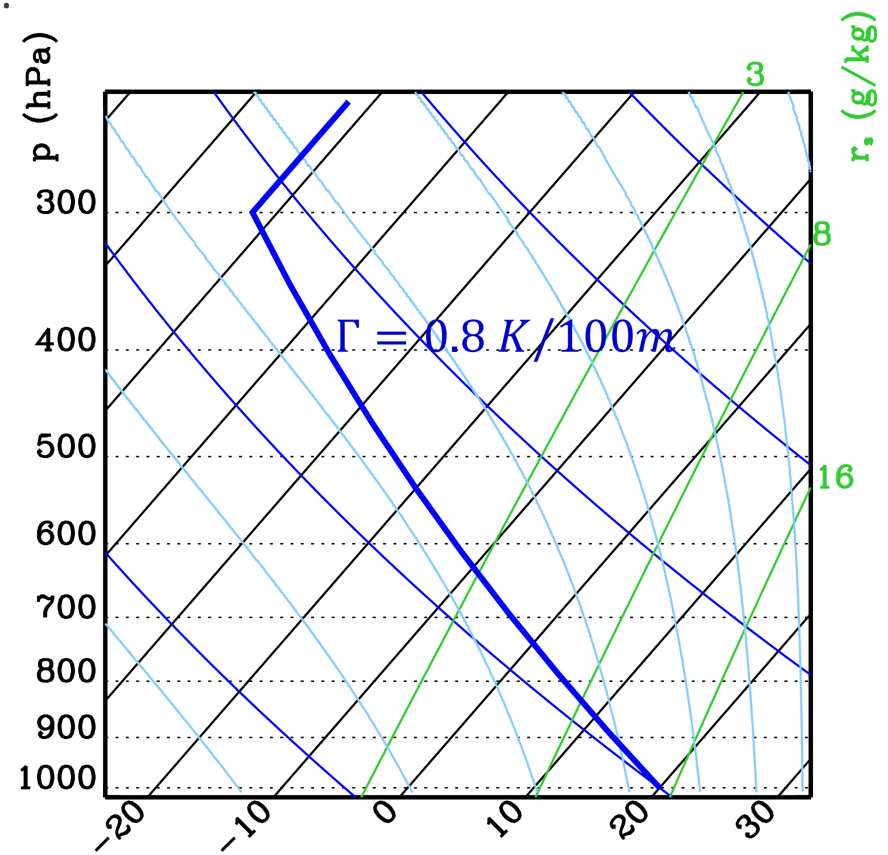


Thermodynamics of Atmospheres
and Oceanes,
J.A. Curry and P.J. Webster; C&W

Conditional instability

Consider an unsaturated parcel that is displaced upward inside the conditionally unstable layer.

It represents a typical stratification of the tropical atmosphere: a constant and conditionally unstable lapse rate ($\Gamma_s < \Gamma < \Gamma_d$) in the troposphere and an lapse rate of zero in the lower stratosphere.



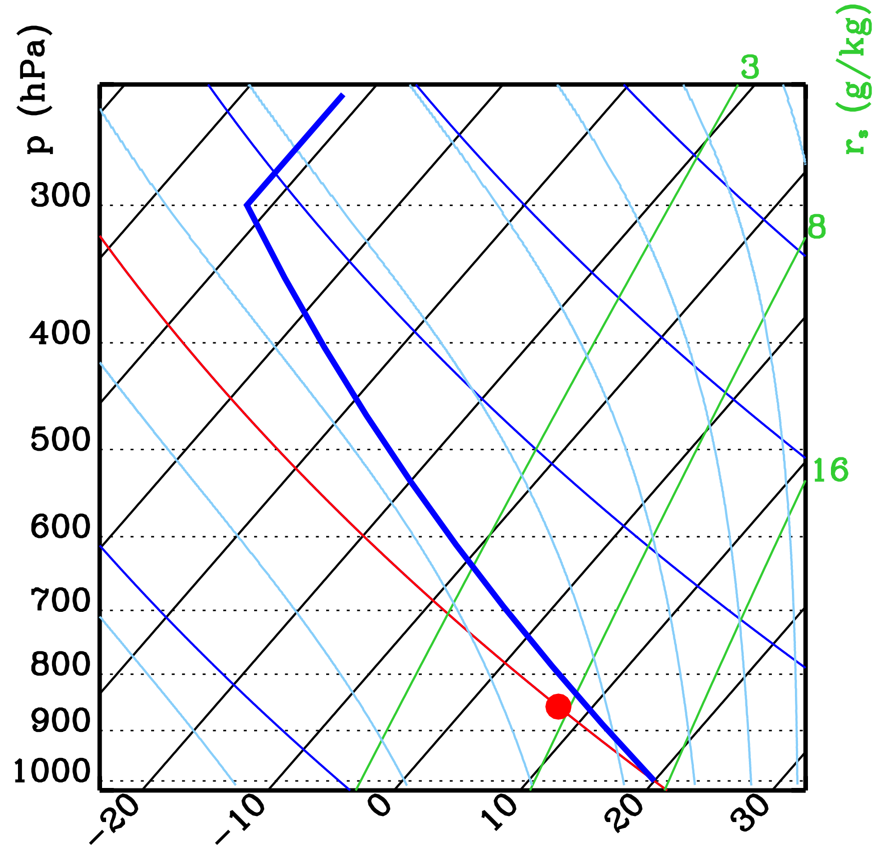
$$p_0 = 1000\ hPa$$

$$T_0 = 20°C$$

$$q_s = 7.48\ g/kg$$

Below the LCL the displaced parcel cools at the dry adiabatic lapse rate (Γ_d) and therefore cools more rapidly than its surroundings ($\Gamma<\Gamma_d$).

The parcel experiences a positive restoring force, one which increases with upward displacement.



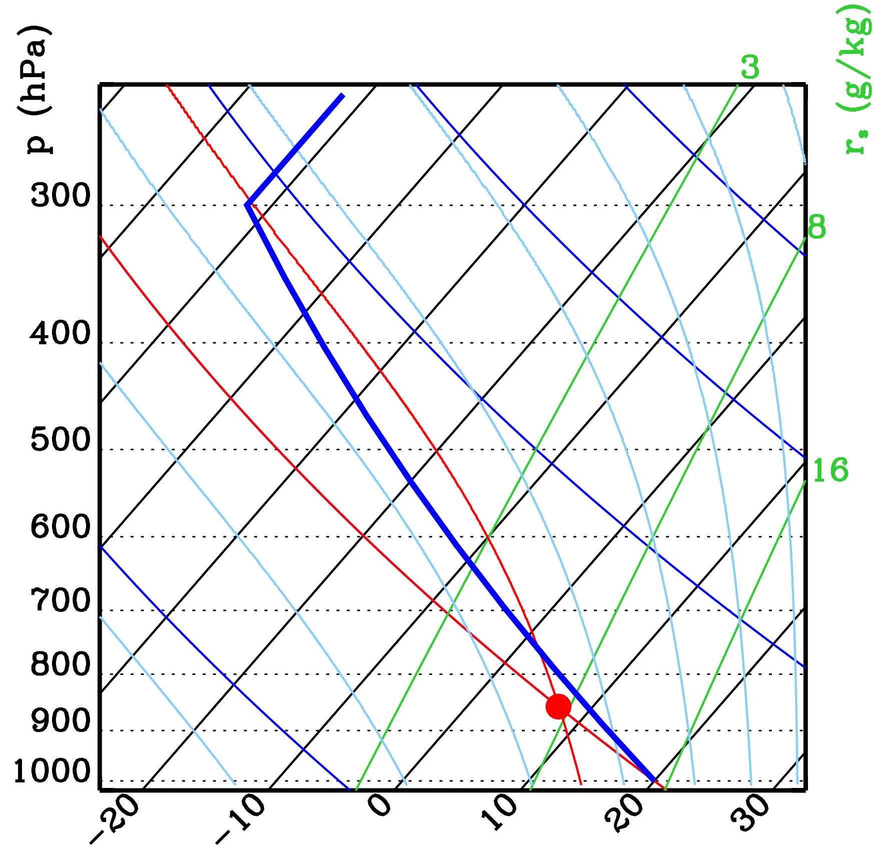
$$T_{LCL} = 7.27°\text{C}$$
$$p_{LCL} = 856\ hPa$$

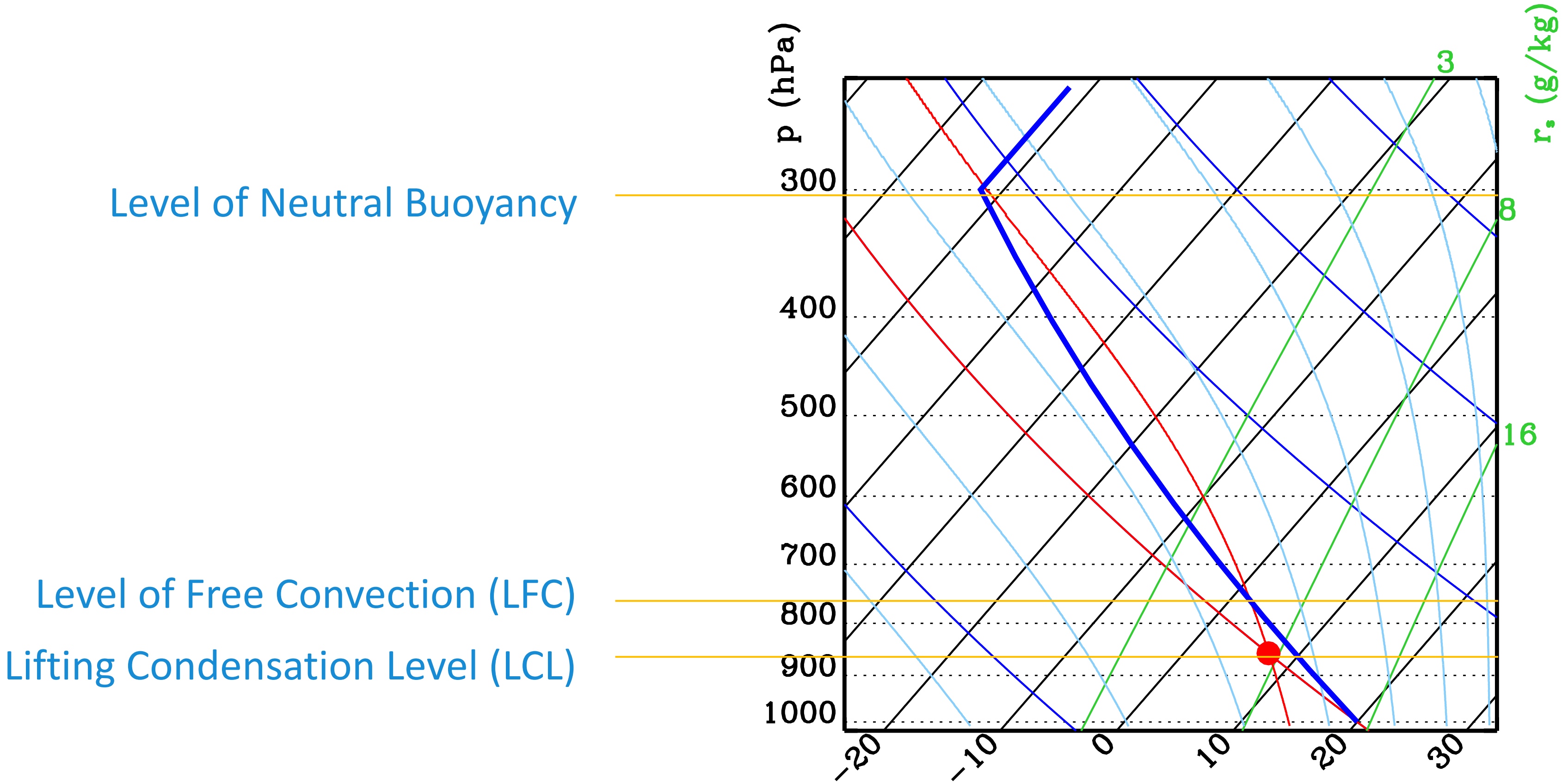
Below the LCL the displaced parcel cools at the dry adiabatic lapse rate (Γ_d) and therefore cools more rapidly than its surroundings ($\Gamma<\Gamma_d$).

The parcel experiences a positive restoring force, one which increases with upward displacement.

Above the LCL the parcel cools slower at the saturated adiabatic lapse rate (Γ_s), due to the release of latent heat. Since the layer is conditionally unstable, then the parcel cools slower than its surroundings, $\Gamma_s<\Gamma$.

The temperature difference between the parcel and its environment and hence the positive restoring force of buoyancy diminish with height.





A level where the temperature profile crosses the profile of environmental temperature is the level of free convection (LFC). The parcel becomes warmer than its surroundings, so it experiences a negative restoring force and thus can ascend of its own accord.

In the lower stratosphere the temperature does not vary with altitude. The temperature of the parcel again crosses the profile of environmental temperature at the crossing level p_c called a level of neutral buoyancy. Above that level, the parcel is again cooler than its surroundings, so buoyancy opposes further ascent.

Under adiabatic conditions, the buoyancy force is conservative (the work performed along a cyclic path vanishes).

The potential energy P of the displaced parcel is: $dP = \delta w_b = -f_b dz$

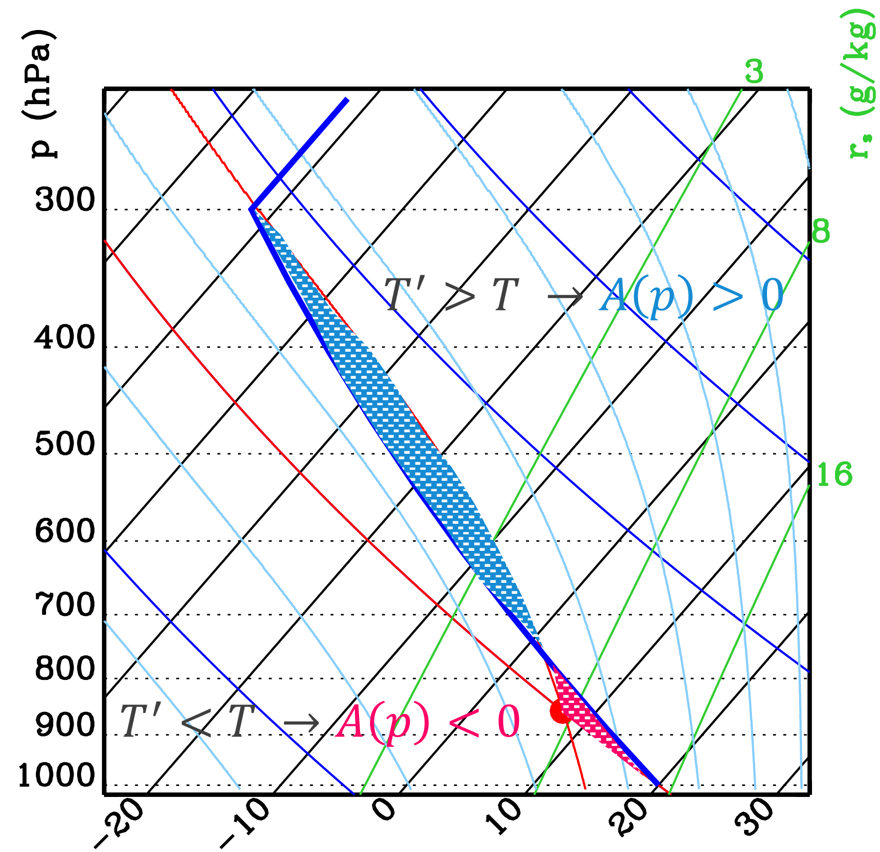
δw_b is the incremental work performed against buoyancy.

Let's define a reference value of zero potential energy at the undisturbed height z_0.

$$P = \int_{z_0}^{z} \left(\frac{\rho - \rho'}{\rho'}\right) g dz = \int_{p_0}^{p} (v' - v) dp$$

$$dp = -\rho g dz \qquad pv = RT \qquad pv' = RT'$$

$$P(p) = -R \int_{p_0}^{p} (T' - T)(-d\ln p) = -R \cdot A(p)$$



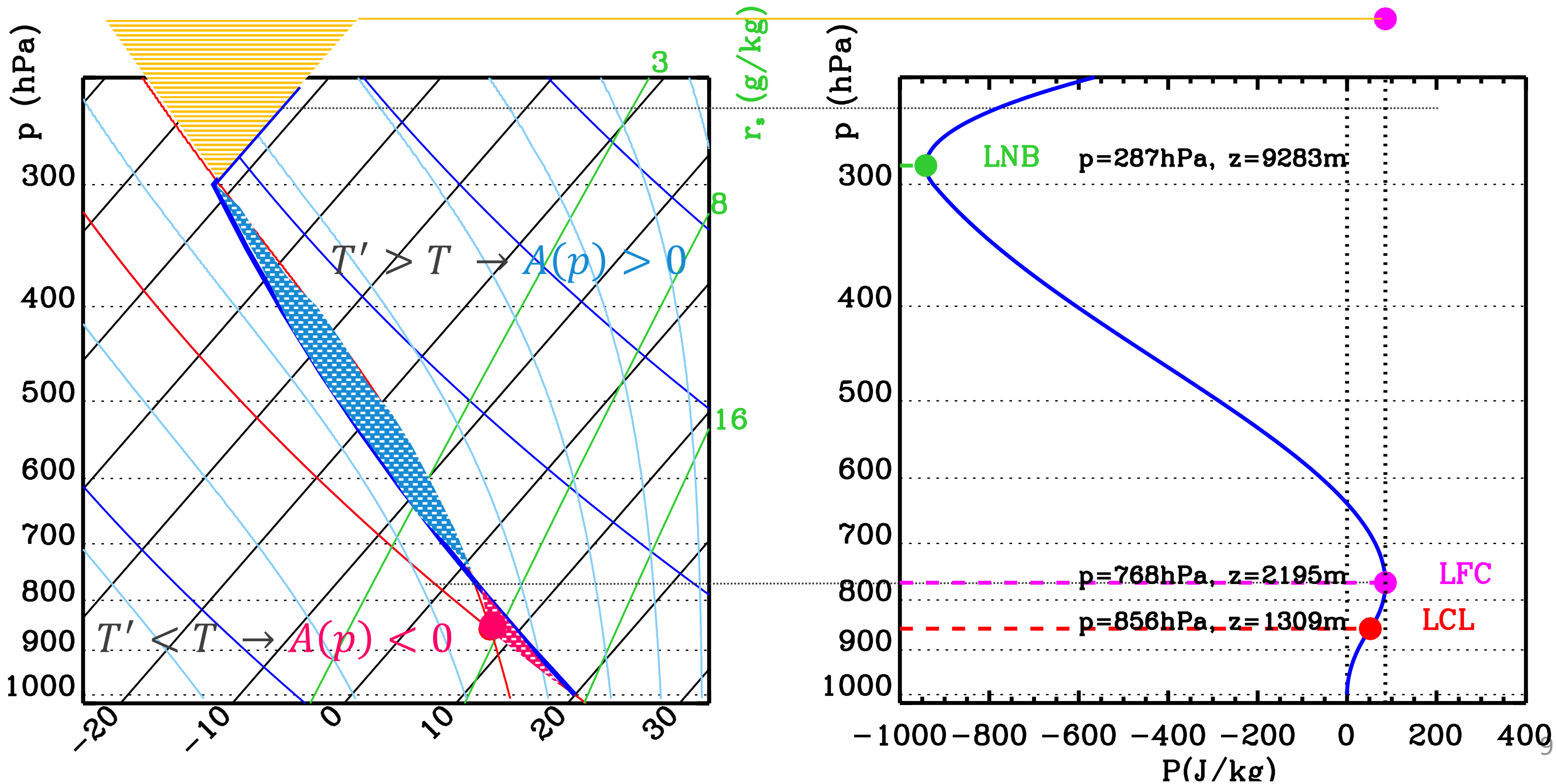
A(p) is the cumulative area between the temperature profile of the parcel T' and that of the environment T to the level p. A(p) can be positive or negative.

Below the LCL, T‘ <T, so potential energy increases upward.

Work must be performed against the positive restoring force of buoyancy to liberate the parcel from this potential well.

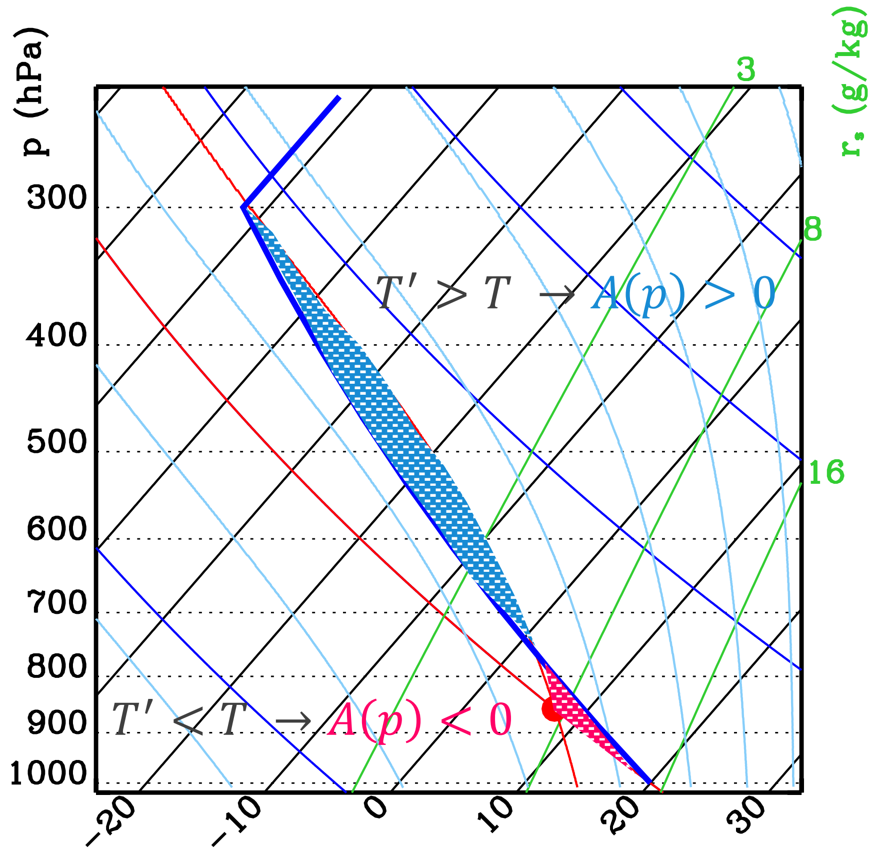
Above the LCL (where P is a maximum) the work that has been performed below is available for conversion to kinetic energy K. Above the LCL, T’>T, so P decreases upward. Under conservative conditions ΔK=-ΔP.

Decreasing P above the LCL represents a conversion of potential energy to kinetic energy, which drives deep convection through buoyancy work.



Convective Available Potential Energy (CAPE)

The total potential energy available for conversion to kinetic energy is termed the convective available potential energy (CAPE). It is represented by the blue area in the figure.



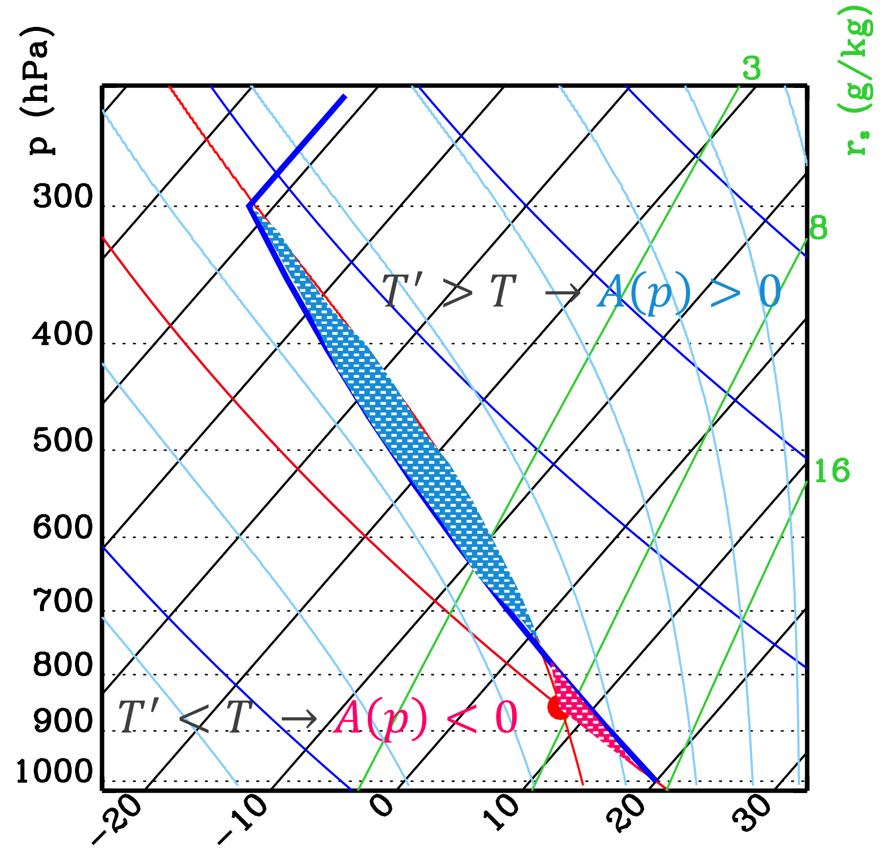
CAPE=1027 J/kg

Since the parcel's temperature crosses the environmental temperature profile a second time CAPE is necessarily finite, as is the kinetic energy that can be acquired by the parcel.

An upper bound on the parcel's kinetic energy is:

$$\frac{w'^2}{2} = P(p_{LFC}) - P(p_c)$$

$$= R\int_{p_{LFC}}^{p_c} (T' - T)(-d\ln p) = \text{CAPE}$$



In practice mixing with the surroundings makes the behavior inside convection inherently nonconservative, so the upward velocity presented above is seldom observed.

Above the crossing level (LNB – level of neutral buoyancy) T' and T are again reversed, so P increases upward above p_c. The parcel becomes negatively buoyant and is bound in another potential well.

Despite opposition by buoyancy, the parcel overshoots its new equilibrium level p_c due to the kinetic energy it acquired above the LCL. Because T diverges rapidly from T' the penetration into the stable layer aloft is shallow compared to the depth traversed through the conditionally unstable layer below. Like the maximum updraft the estimate of the penetration level is only an upper bound.

Convective domes overshooting: a photo and simulation results

Figure 3. (Left) Jumping cirrus photographed by Martin Setvák on 24 May 1996 late afternoon from an airplane above Alabama and Georgia (Courtesy of Martin Setvák). (Right) RHi 30% contour surface of the simulated storm at t = 1440 s. The vertical dimension is enhanced to match the perspective view of the photograph.

Wang, P.K., 2004: A cloud model interpretation of jumping cirrus above storm top. Geophys. Res. Let. Vol. 31, L18106, doi: 10.1029/2004GL020787.

Overshooting tops

Acknowledgments
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Entrainment -1

Cooler and dryier air environmental air that is entrained into and mixed with a moist thermal depletes ascending parcels of positive buoyancy and kinetic energy.

Only in the cores of broad convective towers are adiabatic values ever approached.

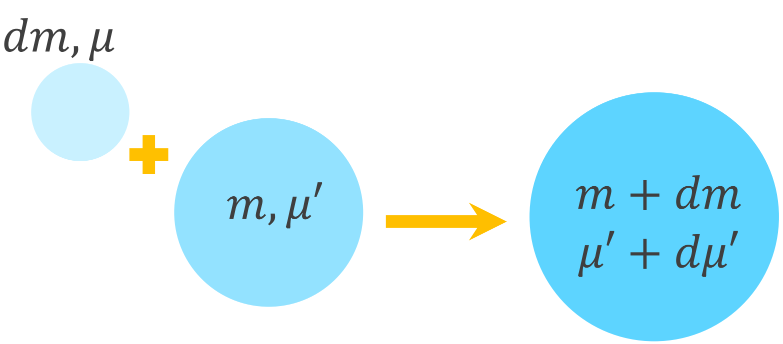
Mixing also modifies the surroundings of a cumulus tower, which modifies gradients of temperature and moisture that control buoyancy.

Let’s consider a process where entrained environmental air is uniformly mixed with ascending air of a convective cell. Through mixing, additional mass is incorporated into a parcel.

Entrainment -2

If μ is a conserved property (e.g. $\mu = q_v$ beneath the LCL), $m\mu$ represents how much of property μ is associated with the parcel under consideration.

After mass dm of environmental air has been mixed into it, the parcel will have mass $m + dm$ and property $\mu' + d\mu'$.



Primes distinguish properties of the environment from those of the parcel.

Conservation of μ requires:

$$(m + dm)(\mu' + d\mu') = m\mu' + \mu dm$$

We will neglect the higher order terms

$$\cancel{m}\mu' + md\mu' + \mu' dm + dmd\mu' = \cancel{m}\mu' + \mu dm$$

$$d\mu' = \frac{dm}{m}(\mu - \mu')$$

If the process occurs during the time dt :

$$\frac{d\mu'}{dt} = \frac{d\ln m}{dt}(\mu - \mu')$$

Entrainment -3

If μ is not conserved but, rather, is produced per unit mass at the rate S_μ its evolution inside the parcel is goverened by equation:

$$\frac{d\mu'}{dt} = \frac{d\ln m}{dt}(\mu - \mu') + S_\mu$$

The time rate of change is related to its vertical velocity $\frac{d}{dt} = w\frac{d}{dz}$

$$w\frac{d\mu'}{dz} = w\frac{d\ln m}{dz}(\mu - \mu') + S_\mu$$

$$\frac{1}{H_e} = \frac{d\ln m}{dz} \quad \longrightarrow \quad w\frac{d\mu'}{dz} = \frac{w}{H_e}(\mu - \mu') + S_\mu$$

H_e defines the mass entrainment height.
H_e reflects the rate that the entraining thermal expands due to incorporation of environmental air.
It represents the height for upwelling mass to increase by a factor of *e*.

Exemple: $\mu = \ln \theta_e$

The source term $S_{\theta e} = 0$ because θ_e is conserved.

$$\frac{d \ln \theta_e'}{dz} = \frac{1}{H_e}(\ln \theta_e - \ln \theta_e')$$

$$\frac{d \ln \theta_e'}{dz} = \frac{1}{H_e}\left[\ln \frac{\theta}{\theta'} + \frac{L_{lv}}{c_{pd}}\left(\frac{q_s}{T} - \frac{q_s'}{T'}\right)\right]$$

$$w\frac{d\mu'}{dz} = \frac{w}{H_e}(\mu - \mu') + S_\mu$$

$$\theta_e = \theta \exp\left(\frac{L_{lv} q_s}{c_{pd} T}\right)$$

$$\ln \theta_e = \ln \theta + \frac{L_{lv} q_s}{c_{pd} T}$$

Let's assume that the air in the parcel and in the environment is unsaturated.

We have to replace q_s by the actual mixing ratios and temperatures should correspond to the temperatures at the LCL.

Note that differences of temperature are small compared to differences of moisture.

EXEMPLE: $\mu = \ln \theta_e$

$$\frac{d \ln \theta'_e}{dz} \cong \frac{1}{H_e}\left[\ln \frac{\theta}{\theta'} + \frac{L_{lv}}{c_{pd} T}(q_v - q'_v)\right] = \frac{1}{H_e}\left[\ln \frac{T}{T'} + \frac{L_{lv}}{c_{pd} T}(q_v - q'_v)\right]$$

The air in the ascending thermal is usually :

- warmer than in the environment; temperatures at the LCL fulfil also: $T' > T$. The first term on the right-hand side : $\ln(T/T') < 0$
- more humid than in the environment: $q'_v > q_v$. The second term on the right-hand side $(q_v - q'_v) < 0$.

Therefore in the ascending thermal that mixes with the environment:

$$\frac{d \ln \theta'_e}{dz} < 0$$

θ'_e decreases with the ascending parcel's height. This is to be contrasted with the parcel's behavior under conservative conditions (e.g. in the absence of entrainment), in which case its mass is fixed and $\theta'_e = const$.

The two sinks of θ'_e on the right-hand side reflect transfers of sensible and latent heat to the environment.

Example: $\mu = w$ (specific momentum)

Suppose μ equals to the specific momentum w (vertical velocity): $\mu' = w$

In the environment w=0. $\mu = 0$

$$w\frac{d\mu'}{dz} = \frac{w}{H_e}(\mu - \mu') + S_\mu$$

$$w\frac{dw}{dz} = -\frac{w^2}{H_e} + S_w$$

$$\frac{1}{2}\frac{dw^2}{dz} = -\frac{w^2}{H_e} + S_w \qquad K' = \frac{w^2}{2}$$

K' is the specific kinetic energy of the parcel (energy per unit mass).

$$\frac{dK'}{dz} = -\frac{2}{H_e}K' + \frac{S_K}{w}$$

Production of momentum and kinetic energy is related as $S_k = wS_w$.

Example: $\mu = w$ (specific momentum) -2

Even in the absence of entrainment ($H_e \rightarrow \infty$), K is not conserved because the parcel's kinetic energy changes through work (δw_b) performed on it by the force of buoyancy.

$$dP = \delta w_b = -f_b dz \qquad \Delta K = -\Delta P \qquad \frac{dK'}{dz} = f_b$$

The production term can be evaluated for the case $H_e \rightarrow \infty$.
This term does not change if entrainment is present.

$$S_K = w f_b = wg\left(\frac{T' - T}{T}\right)$$

$$\frac{dK'}{dz} = -\frac{2}{H_e}K' + \frac{S_K}{w} \quad \longrightarrow \quad \frac{dK'}{dz} = g\left(\frac{T' - T}{T}\right) - \frac{2}{H_e}K'$$

The sink of K' on the right-hand side represents turbulent drag that is exerted on the ascending parcel due to incorporation of momentum from the environment.

Entrainment reduces the percel's acceleration from what it would be under the action of buoyancy alone and limits its penetration into a stable layer to about an entrainment height H_e.

LAPSE RATE MODIFICATION DUE TO ENTRAINMENT -1

Consider a mass m of saturated cloudy air which rises from a level z, entraining a mass dm of the environmental air over the distance dz.

The cloudy air has a temperature T', and the environmental air, T.

We will apply the First Law of thermodynamics to the mixture m+dm and assume that no heat transfer mechanisms occur other than condensation, evaporation, and mixing.

$$dh = \delta q + v dp$$

$$c_p dT' - RT' \frac{dp}{p} = \delta q$$

The latent heat required to evaporate just enough water from the cloudy air to saturate the mixed parcel.

$$m\left(c_{pd} dT' - R_d T' \frac{dp}{p}\right) = -mL_{lv} dq_s - c_{pd}(T' - T)dm - L_{lv}(q_s' - q_v)dm$$

The latent heat released by the cloudy parcel in ascent.

The heat required to warm the entrained air.

LAPSE RATE MODIFICATION DUE TO ENTRAINMENT - 2

Following a simillar procedure to that used in derivation of the saturated adiabatic lapse rate we obtain an expression for the lapse rate of a saturated parcel subject to entrainment.

$$\Gamma_m = \Gamma_s + \frac{\frac{1}{m}\frac{dm}{dt}\left[(T' - T) + \frac{L_{lv}}{c_{pd}}(q_s' - q_v)\right]}{1 + \frac{\varepsilon L_{lv}^2 q_s}{c_{pd} R_d T^2}}$$

Γ_m reduces to Γ_s if $dm/dz = 0$, i.e. if no entrainment takes place.

For $dm/dz > 0$ and $T' > T$, then $\Gamma_m > \Gamma_s$.

In effect, the mixing of cloudy air with dry environmental air reduces the density difference between the parcel and its environment, hence reducing the buoyancy force. Since the lapse rate in an entraining cloud is greater than the saturated adiabatic lapse rate, an entraining cloud achieves a smaller vertical velocity relative to that predicted by parcel theory.

The process of entrainment of environmental air into vertically developing clouds is not adequately understood at present $\rightarrow$ it is difficult to determine dm/dz.

Processes producing changes in stability

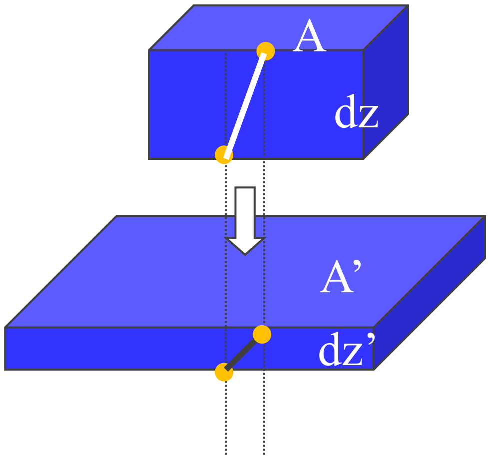
The static stability of a layer in the atmosphere is modified by:

- vertical motion in a layer; and
- differential heating or cooling of the layer.

Modification of stability due to vertical motions in a layer -1

Consider the large-scale ascent of a dry atmospheric layer, during which the mass of the layer remains constant (i.e. there is no horizontal or vertical convergence of air).

From the definition of potential temperature: $\theta = T\left(\frac{p_0}{p}\right)^{\kappa}$



$$\frac{d\ln\theta}{dz} = \frac{1}{T}\frac{dT}{dz} - \frac{R_d}{c_{pd}}\frac{1}{p}\frac{dp}{dz}$$

$$\frac{dp}{dz} = -\rho g$$

$$p = R_d T \rho$$

$$\frac{1}{\theta}\frac{d\theta}{dz} = \frac{1}{T}\frac{dT}{dz} + \frac{1}{T}\frac{g}{c_{pd}}$$

$$\frac{1}{\theta}\frac{d\theta}{dz} = -\frac{\Gamma_d - \Gamma_{env}}{T}$$

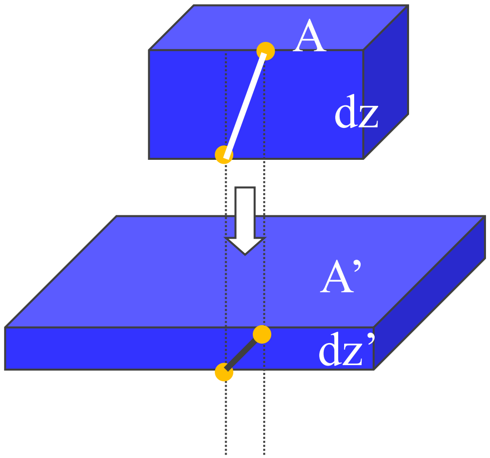
Γ_{env} is the lapse rate of the environment.

We ignore the virtual temperature effects for dry air.

Modification of stability due to vertical motions in a layer -2

In the former equation we replace dz by dp using the hydrostatic equation.

$$\frac{1}{\theta}\frac{d\theta}{dp} = -\frac{R_d}{g}\frac{\Gamma_d - \Gamma_{env}}{p}$$



During ascent or descent of the layer, the derivative $d\theta/dp$ remains constant, since θ is conserved in dry adiabatic motion and the mass of the layer remains constant. Hence we can write:

$$\Gamma_d - \Gamma_{env} = C_1 p$$

C_1 is constant

During the ascent of a layer, the lapse rate of the layer approaches the dry adiabatic lapse rate.

An initially stable layer is made less stable.

An initially unstable layer is made more stable

The reverse occurs during the descent of a layer when the pressure increases, whereby the environmental lapse rate moves further away from the Γ_d.

Potential instability -1

The changes of the layer's thermodynamic properties alter its stability.

Stratification of moisture plays a key role in this process because different levels need not to achieve saturation simultaneously.

Consider an unsaturated layer in which :

- potential temperature increases with height (stable stratified layer)
- equivalent potential temperature decreases with height.

$$\frac{d\theta}{dz} > 0\,, \qquad \frac{d\theta_e}{dz} < 0 \quad \longrightarrow \quad d(\theta_e - \theta) < 0$$

The difference θ_e - θ for an individual parcel reflects the total latent heat available for release.

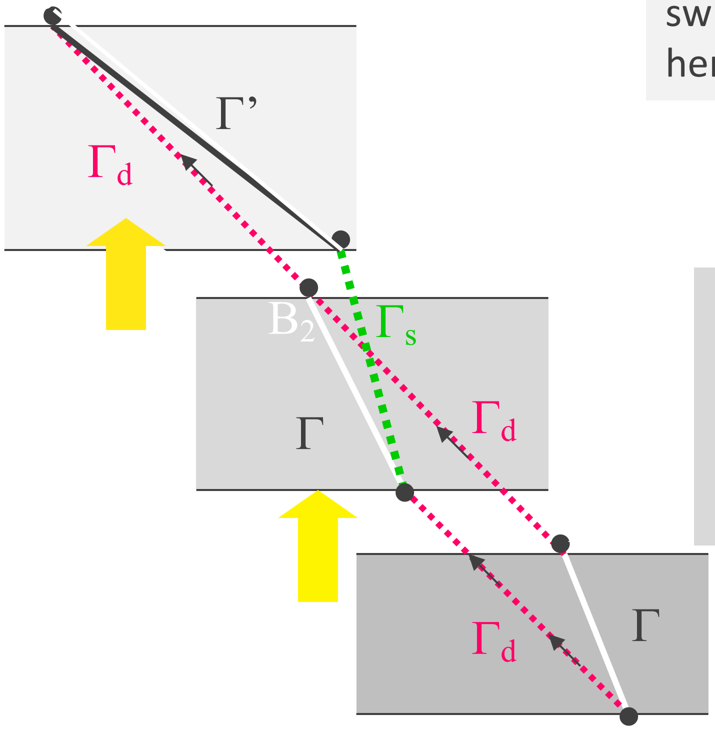
$$\theta_e - \theta = \theta \frac{L_{lv} q_v}{c_{pd} T}, \qquad \theta_e - \theta = \frac{L_{lv} q_v}{c_{pd}} \left(\frac{p_0}{p}\right)^{\kappa}$$

$$\theta_e = \theta \exp\left(\frac{L_{lv} q_s}{c_{pd} T}\right) \cong \theta \left(1 + \frac{L_{lv} q_s}{c_{pd} T}\right)$$

The specific water vapor mass decreases with height.

$$\frac{d}{dz}(\theta_e - \theta) < 0 \quad \longrightarrow \quad \frac{dq_v}{dz} < 0$$

Potential instability -2



Differential cooling between lower and upper levels swings the temperature profile counterclockwise and hence destabilizes the layer.

Lower levels achieve saturation sooner than upper levels because they have greater mixing ratios. Lower levels cool slower at the saturated adiabatic lapse rate Γ_s, whereas upper levels continue to cool at the dry adiabatic lapse rate Γ_d.

The layer is initially unsaturated. Air parcels along a vertical section all cool at the dry adiabatic lapse rate Γ_d. The layer’s profile of temperature is preserved through the displacement, as are the layer’s lapse rate (Γ) and stability.

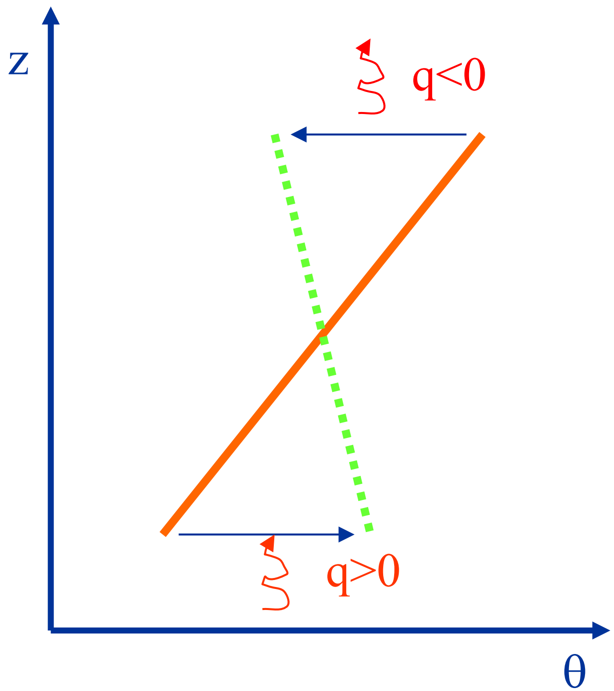
Potential instability -3

A layer characterized by $\dfrac{d\theta}{dz} < 0$, $\dfrac{d\theta_e}{dz} > 0$ is potentially stable.

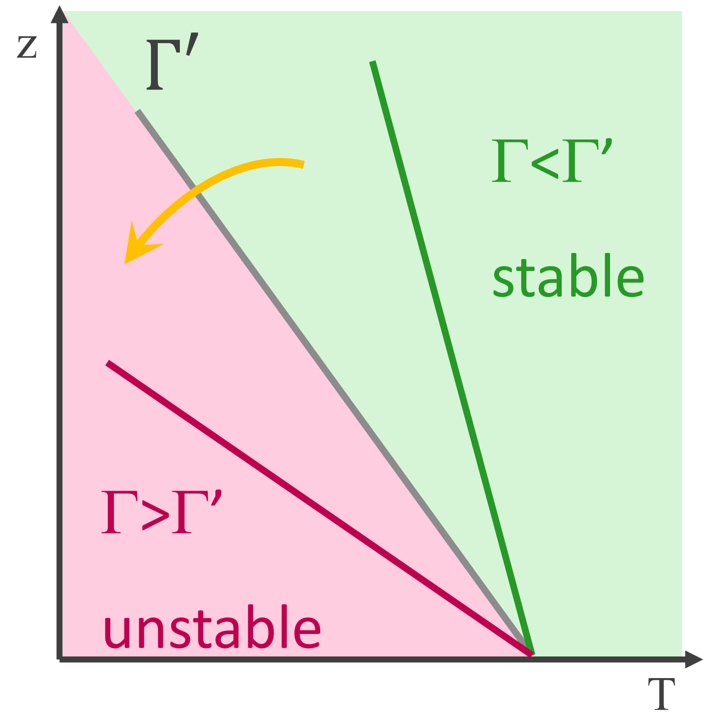
A layer characterized by $\dfrac{d\theta}{dz} > 0$, $\dfrac{d\theta_e}{dz} < 0$ is potentially unstable.

Destabilizing influences

Emission of longwave radiation at upper levels cools the troposphere from above.



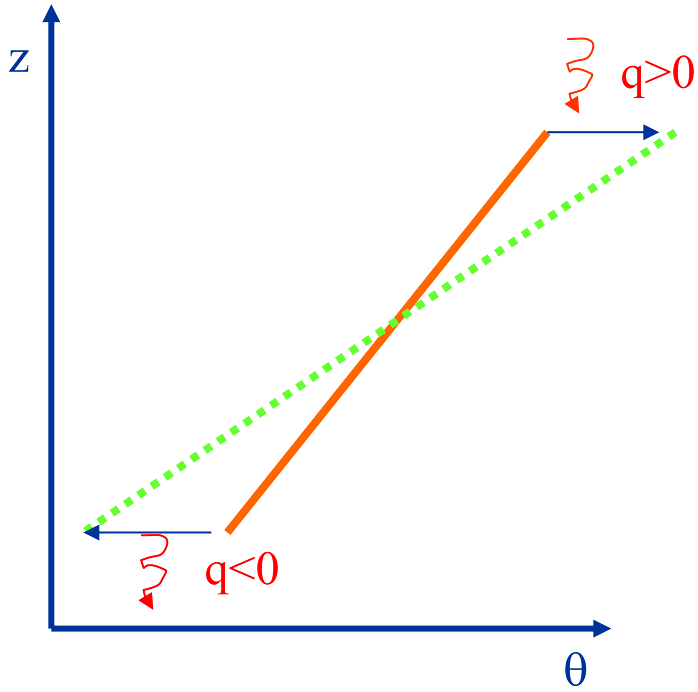
Absorption of longwave radiation and transfers of latent and sensible heat from the Earth's surface warm the troposphere from below.

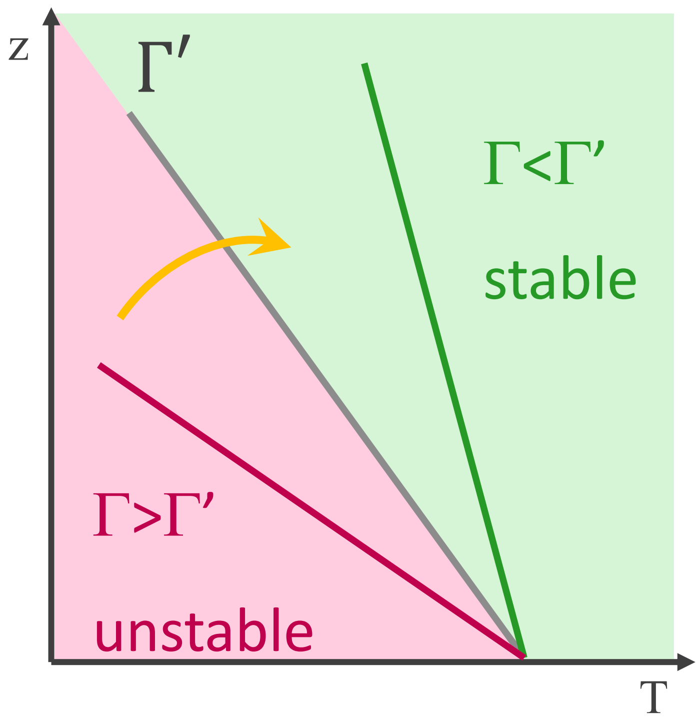


Heating from below and cooling from above act to swing the profile of potential temperature counterclockwise.

It destabilizes the layer.

Stabilizing influences





Cooling from below and heating from above act to swing the profile of potential temperature clockwise.

It stabilizes the layer.

Factors modyfying the stability