Adiabatic Liquid Water Content


The amount of water vapor condensed \((q_l = m_l/m, \text{ where } m_l \text{ is the mass of liquid water, and } m \text{ is the mass of the cloudy air})\) in adiabatic process is given by:

\[
dq_l = \frac{c_p}{L_v} (\Gamma_d - \Gamma_s) \, dz
\]

where \(q_l\) is the specific mass of liquid water, \(c_p\) is the specific heat at constant pressure, and \(L_v\) is the latent heat of vaporization. \(\Gamma_d = g/c_p\) is the dry adiabatic lapse rate, and \(\Gamma_s = \Gamma_d \left(1 + \frac{L_v r_s}{c_p R_d T} \right)\) is the saturated moist adiabatic lapse rate.

\(r_s = \epsilon (e_s/p)\) is the saturation water vapor mixing ratio, \(e_s\) is the saturation water vapor pressure (can be expressed as: \(e_s = e_{s0} \exp \frac{17.27(T-273.15)}{(T-273.15)+237.7} \), where \(e_{s0} = 611 Pa\)).

Eq. 1 can be expressed as \(dq_l = c_q dz\), where \(c_q = \frac{c_p}{L_v} (\Gamma_d - \Gamma_s)\) is called the condensation rate. The condensation rate is a function of temperature, \(T\), and pressure, \(p\).

For shallow clouds (up to 500 m thick) the condensation rate, \(c_q\) is approximately constant and takes the same value as at the cloud base, \(c_q(T_0, p_0)\). Eq.1 can be integrated:

\[q_l(z) = c_q(T_0, p_0) (z - z_0), \text{ where } z_0 \text{ is the cloud base height.}\]

The liquid water content (LWC) is:

\[LWC = \frac{m_l}{V} = \frac{m_l}{m} \frac{m}{V} = \rho q_l \text{ where } \rho \text{ is the air density.}\]

Eq.1 can be written in a form:

\[
d \left( \frac{LWC}{\rho} \right) = \frac{c_p}{L_v} (\Gamma_d - \Gamma_s) \, dz
\]

As in the case of the specific mass of liquid water, \(q_l\), the liquid water content can be approximated by a linear function:

\[LWC(z) = c_{LWC} (z - z_0)\]

where \(c_{LWC} = \rho_0 c_q(T_0, p_0)\), and \(\rho_0\) is the density of the air at the cloud base.