

# Dynamics of the Atmosphere and the Ocean

## Lecture 1

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2023-2024 Fall



Acknowledgments to G.K. Vallis,  
A lot of material in this lecture comes from:  
Atmospheric and Oceanic Fluid Dynamics.  
Available from [www.princeton.edu/gkv/aofd](http://www.princeton.edu/gkv/aofd)  
(and later book published by Cambridge University Press)

James Holton's "An introduction to dynamic meteorology" is the must read textbook  
and should be a companion to this course





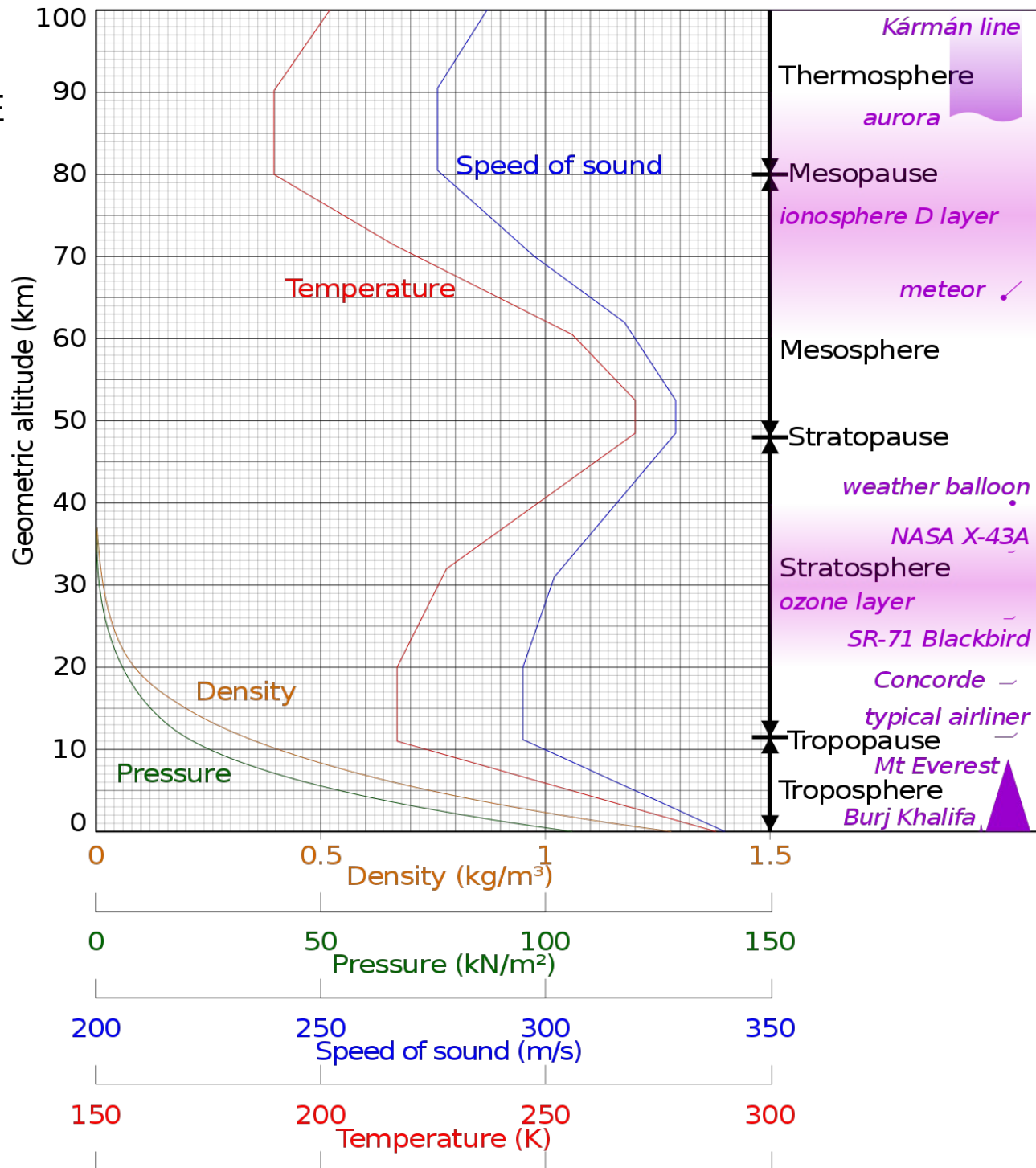
Geophysical fluid dynamics, in its broadest meaning, refers to the fluid dynamics of naturally occurring flows, such as lava flows, oceans, and planetary atmospheres, on Earth and other planets.

We will focus on the applications of geophysical fluid dynamics to the Earth's atmosphere and ocean.



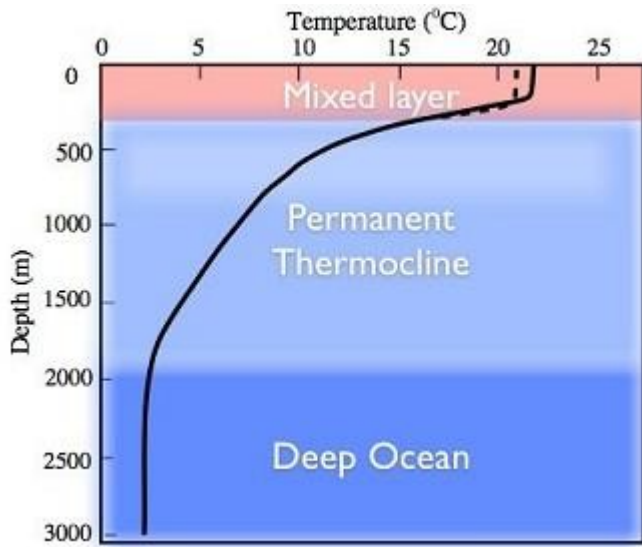


# ATMOSPHERE

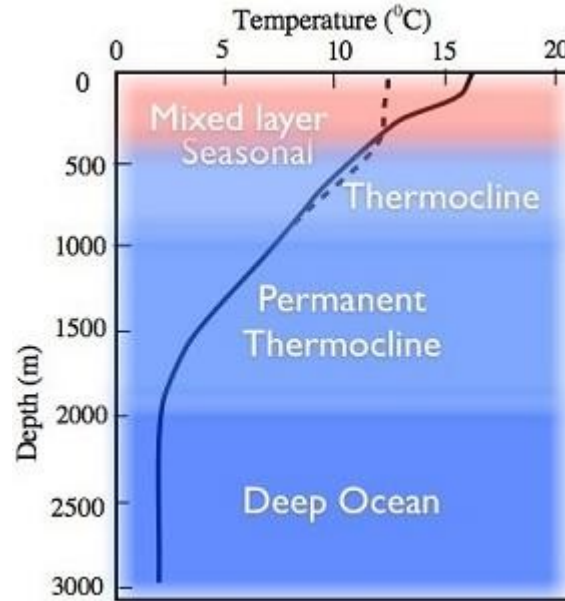


# OCEAN

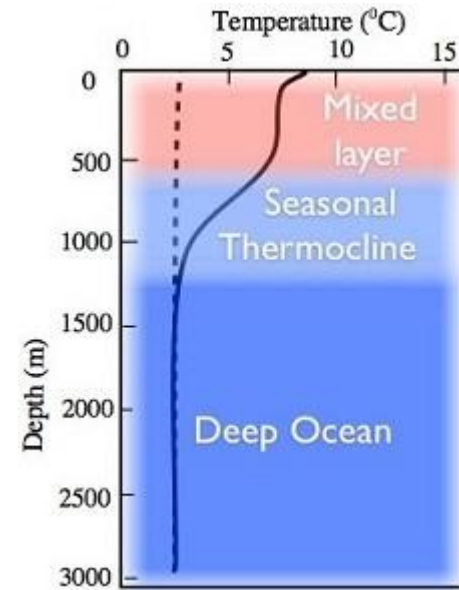
## Tropics



## Midlatitudes



## High latitudes

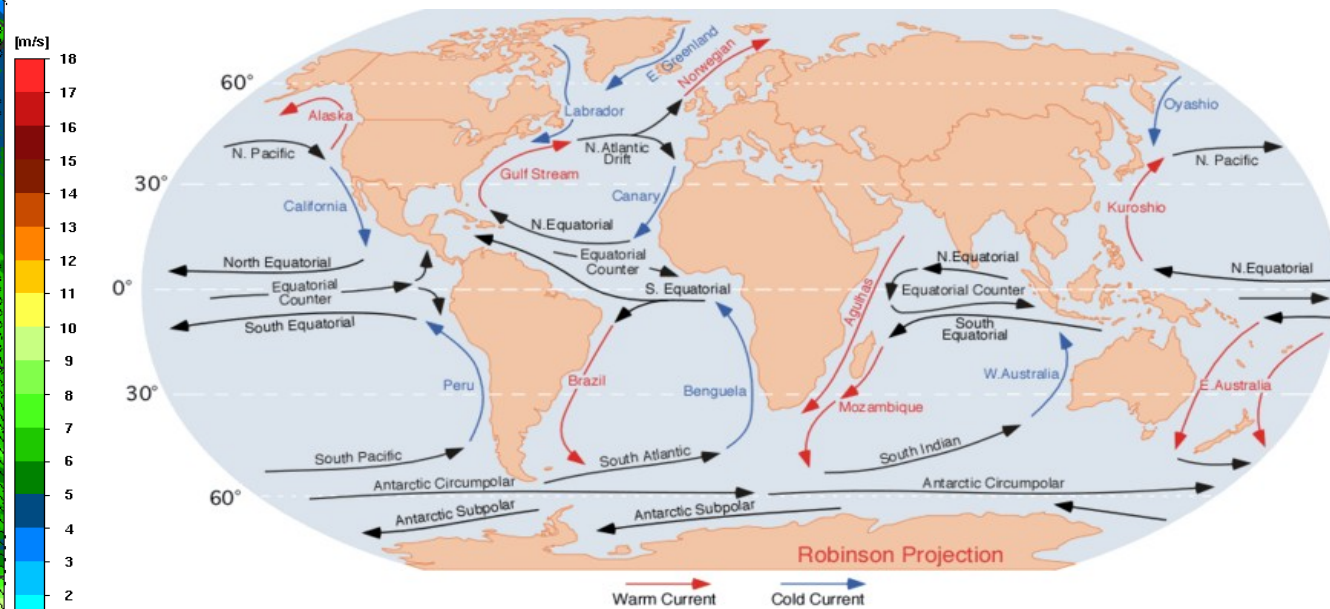
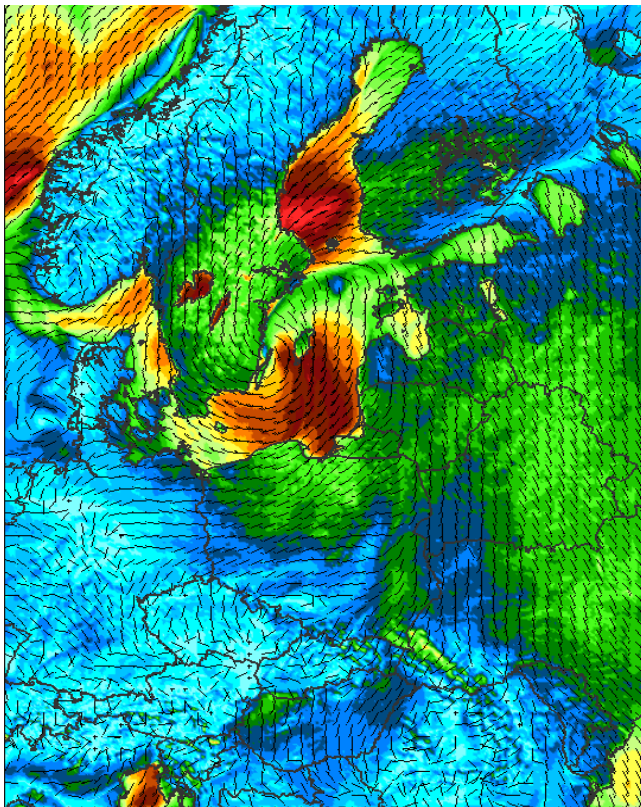


Not only temperature, but salinity as well



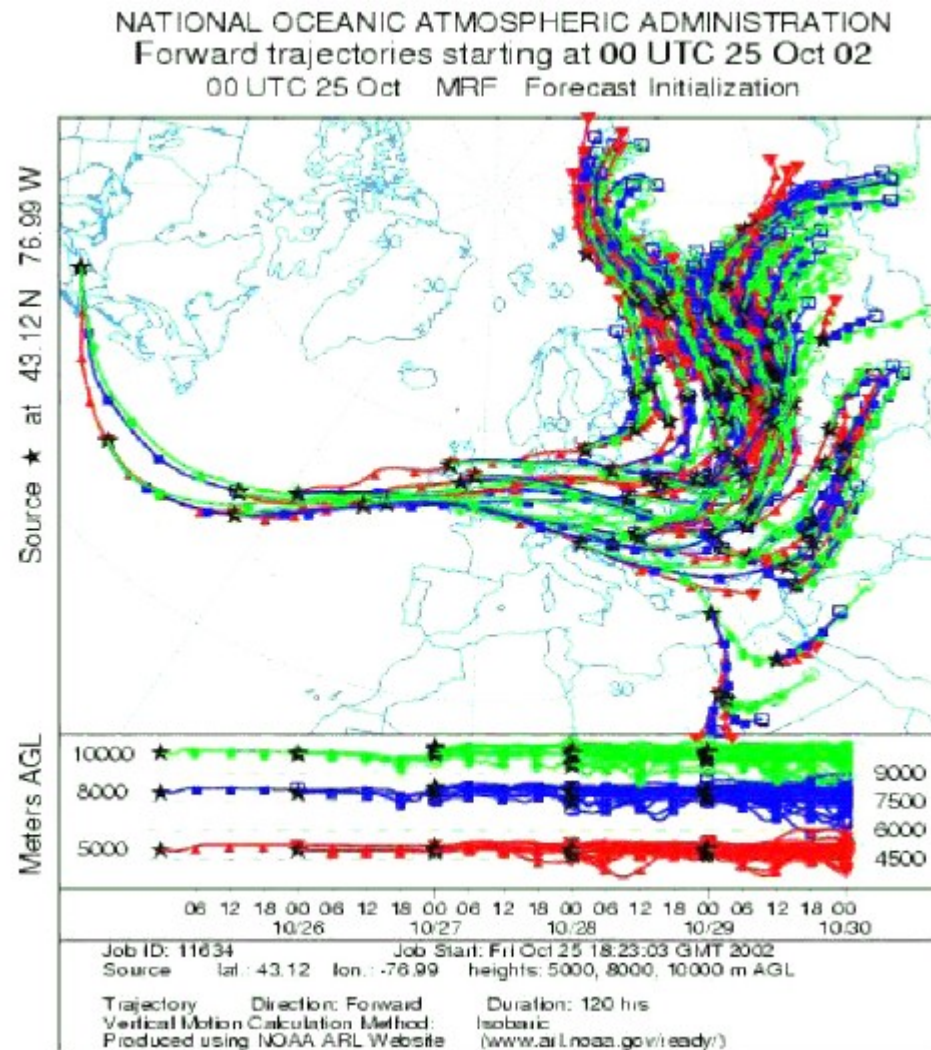
## Eulerian specification of the flow.

The **Eulerian** specification of the flow field is a way of looking at fluid motion that focuses on specific locations in the space ( $\mathbf{X}$ ) through which the fluid flows as time ( $t$ ) passes. Velocity  $\mathbf{V}(\mathbf{X},t)$  characterizes the flow.



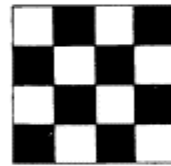
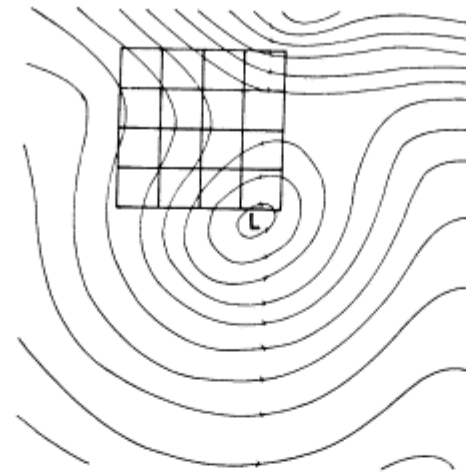
## Lagrangian specification of the flow

The Lagrangian specification of the flow field is a way of looking at fluid motion where the observer follows an individual fluid parcel as it moves through space and time. Position of the parcel is  $\mathbf{X}(t)$  and properties evolve with time.



Practical problem:

deformation of  
Lagrangian parcel  
in time...



a



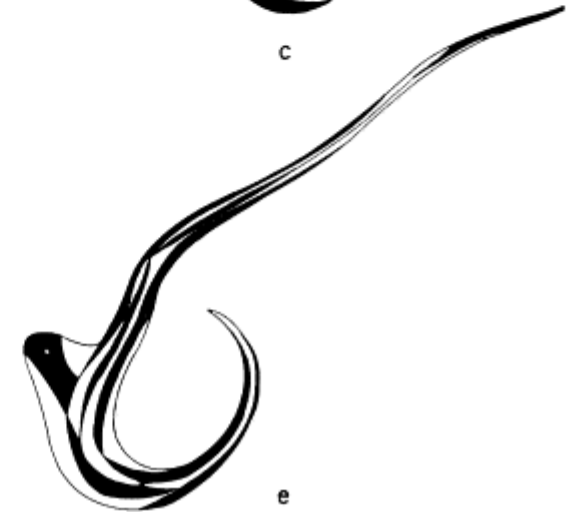
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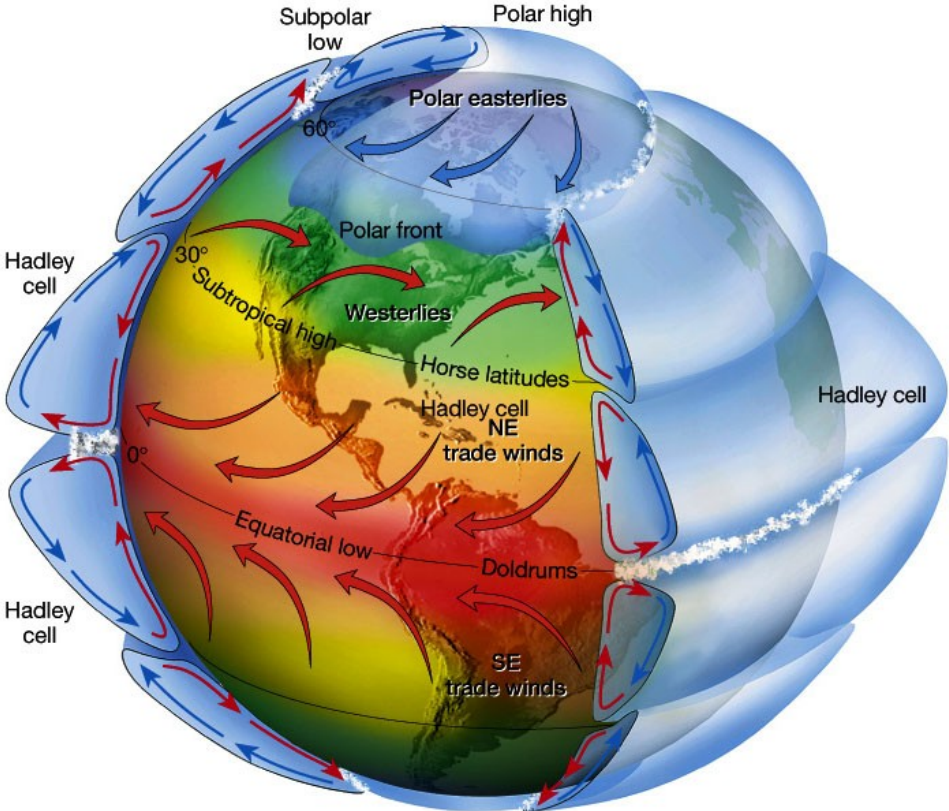
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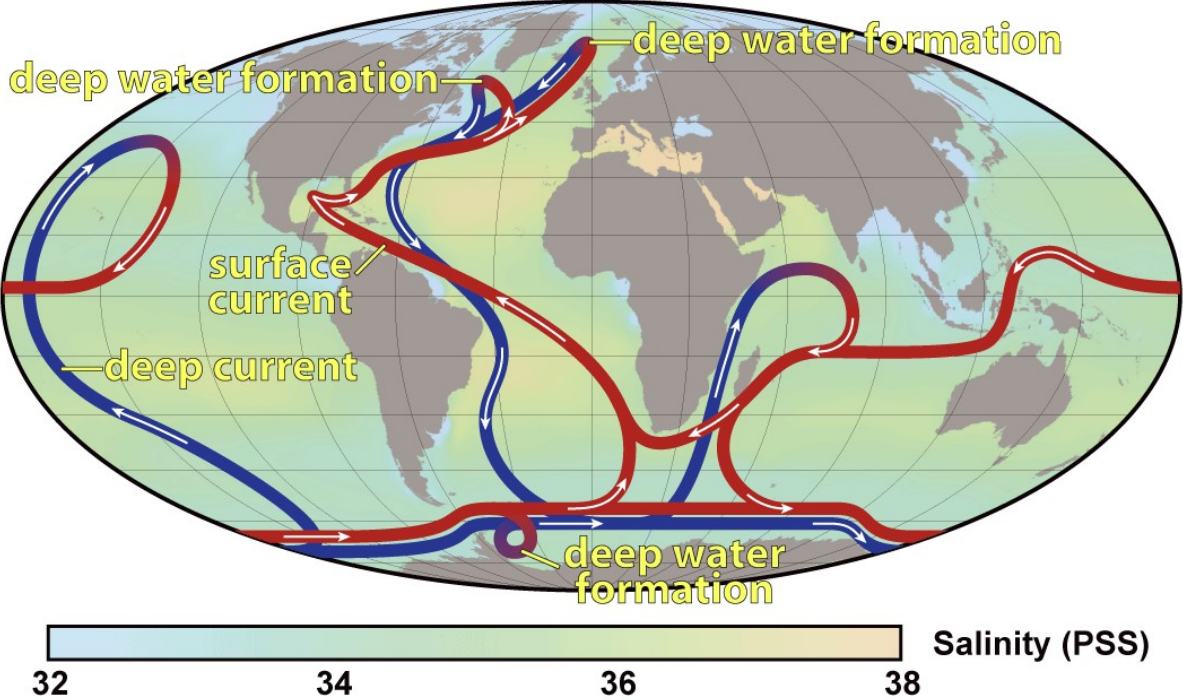
e



# Global-scale circulations



# Thermohaline Circulation



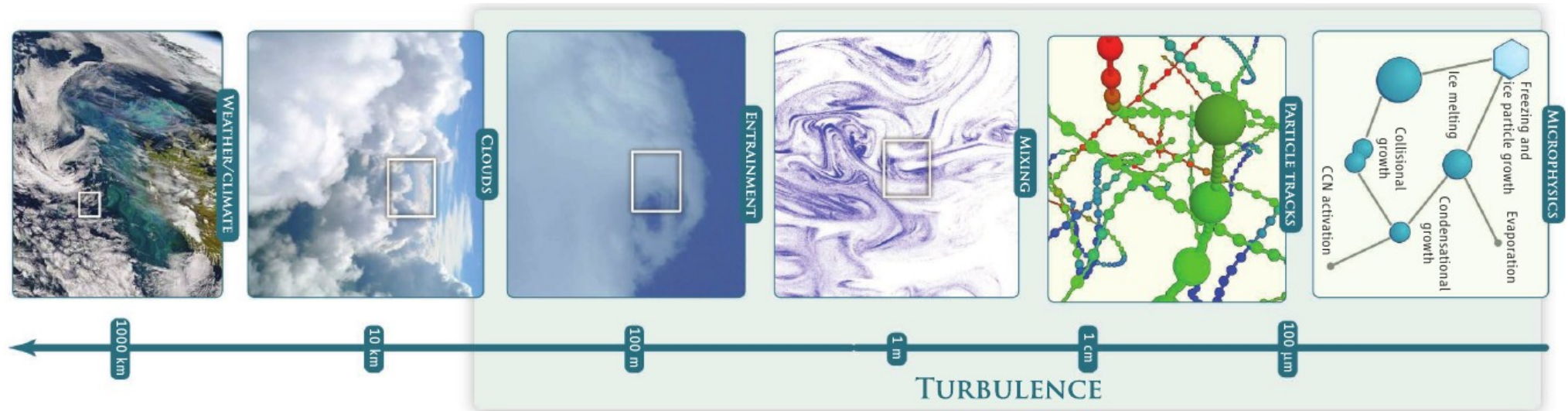
Both oceanic and atmospheric flows span over a wide range of scales: from global scale down to viscosity scale.

Flow type	Scale (m)	Scale name
	$10^{-7}$ m (0.1 $\mu$ m) or less	mean free path
viscous flow	$10^{-3}$ m (1mm)	
dissipation-scale eddies	$10^{-2}$ m (1cm)	turbulence
small eddies	$10^{-1}$ m (10cm)	
dust devils, eddies, vortices, surface waves	1-10m	
wind blows, surface waves	10-100m	
tornadoes,	100-1000m	
convective clouds, boundary layer eddies	$10^3$ - $10^4$ m (1-10km)	mesoscale
mesoscale convective systems, fronts, sea current loops	$10^4$ - $10^6$ m (10-1000km)	
hurricanes,	$10^5$ - $10^6$ m (100-1000km)	synoptic scale
low and high pressure systems	$10^6$ m (1000km)	
global circulation	$10^7$ m (10000km)	global scale

# Can We Understand Clouds Without Turbulence?

Advances at the interface between atmospheric and turbulence research are helping to elucidate fundamental properties of clouds.

E. Bodenschatz,<sup>1,2</sup> S. P. Malinowski,<sup>3</sup> R. A. Shaw,<sup>4</sup> F. Stratmann<sup>5</sup>





## Material and Eulerian Derivatives

The material derivative of a scalar ( $\phi$ ) and a vector ( $\mathbf{b}$ ) field are given by:

$$\frac{D\phi}{Dt} = \frac{\partial\phi}{\partial t} + \mathbf{v} \cdot \nabla\phi, \quad \frac{D\mathbf{b}}{Dt} = \frac{\partial\mathbf{b}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{b}. \quad (\text{D.1})$$

Various material derivatives of integrals are:

$$\frac{D}{Dt} \int_V \phi \, dV = \int_V \left( \frac{D\phi}{Dt} + \phi \nabla \cdot \mathbf{v} \right) dV = \int_V \left( \frac{\partial\phi}{\partial t} + \nabla \cdot (\phi \mathbf{v}) \right) dV, \quad (\text{D.2})$$

$$\frac{D}{Dt} \int_V dV = \int_V \nabla \cdot \mathbf{v} \, dV, \quad (\text{D.3})$$

$$\frac{D}{Dt} \int_V \rho \phi \, dV = \int_V \rho \frac{D\phi}{Dt} \, dV. \quad (\text{D.4})$$

These formulae also hold if  $\phi$  is a vector. The Eulerian derivative of an integral is:

$$\frac{d}{dt} \int_V \phi \, dV = \int_V \frac{\partial\phi}{\partial t} \, dV, \quad (\text{D.5})$$

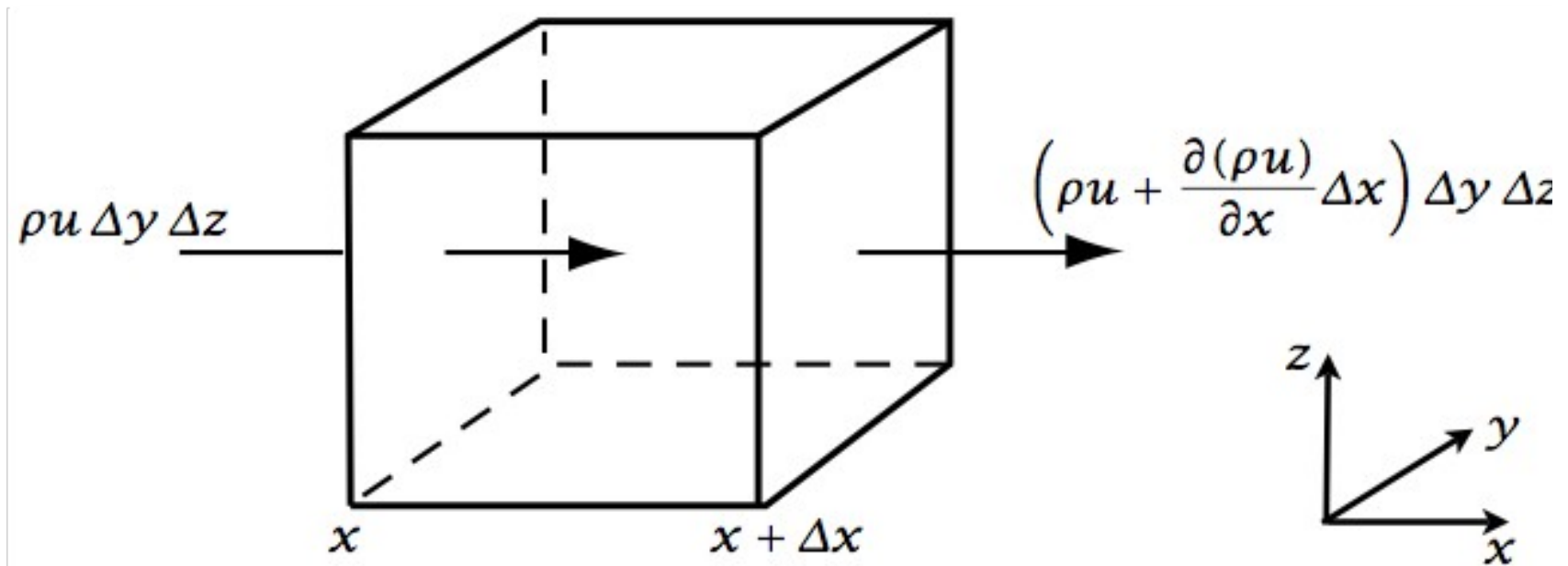
so that

$$\frac{d}{dt} \int_V dV = 0 \quad \text{and} \quad \frac{d}{dt} \int_V \rho \phi \, dV = \int_V \frac{\partial\rho\phi}{\partial t} \, dV. \quad (\text{D.6})$$

## Governing Equations

**Mass continuity** in Eulerian approach:

consider the flow of mass in and out of a control volume



**Fig. 1.1** Mass conservation in an Eulerian cuboid control volume.

$$\delta x \delta y \delta z \left[ \frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} \right] = 0.$$

$$\delta x \delta y \delta z \left[ \frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} \right] = 0.$$



This [in brackets] equals ZERO, hence....

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0.$$

**Continuity equation = conservation of mass**, which in classical mechanics is strictly fulfilled.

There are alternative derivations, consult Vallis book!!!

One example is by material derivative:

$$\frac{D}{Dt}(\rho \Delta V) = 0$$

Mass conserved



Similar conservation laws are valid for any PASSIVE SCALAR carried by the flow:


$$\frac{D\xi}{Dt} = \dot{\xi}.$$

**Any scalar with no sources or sinks (r.h.s. =0) is conserved!**

E.g. temperature with no heat sources or sinks, dye....

THE ABOVE FORM OF EQUATION IS CRUCIALLY IMPORTANT FOR ALL CONSERVATION LAWS IN FLUID DYNAMICS!

Notice in the l.h.s. the advection operator

$$(\mathbf{v} \cdot \nabla)$$


This stays for anything!!!

## Conservation of momentum:

$\mathbf{m}(x,y,z,t)$  - the momentum-density field (momentum per unit volume),

$$\mathbf{m} = \rho \mathbf{v}$$

The total momentum of a volume of fluid is given by the volume integral.

The rate of change of a momentum is given by the material derivative, and by Newton's second law is equal to the force acting on it:

$$\frac{D}{Dt} \int_V \rho \mathbf{v} dV = \int_V \mathbf{F} dV$$

$$\frac{D}{Dt} \int_V \rho \mathbf{v} dV = \int_V \rho \frac{D\mathbf{v}}{Dt} dV,$$

$$\int_V \left( \rho \frac{D\mathbf{v}}{Dt} - \mathbf{F} \right) dV = 0.$$

acceleration of  
fluid of density  $\rho$ ,  
integrated over a  
volume of the  
fluid parcel

$$\rho \frac{D\mathbf{v}}{Dt} = \mathbf{F}$$

NOTICE THE FORM OF THE ABOVE EQUATION!  
**NO FORCE = CONSERVATION OF MOMENTUM...**

The most common form of **momentum equation**:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = \frac{\mathbf{F}}{\rho}$$

In the r.h.s.  $\mathbf{F}$  stays for sum of ALL forces!

Which forces ???



## 1. Pressure force:

at the boundary of a fluid the pressure is the normal force per unit area

$$d\mathbf{F}_p = -p d\mathbf{S}.$$

pressure force

pressure

infinitesimal surface element

$$\mathbf{F}_p = - \int_S p d\mathbf{S}.$$

with application of divergence theorem one gets:

$$\mathbf{F}_p = - \int_V \nabla p dV$$

## 2. Viscosity force:

Many textbooks show that, for most Newtonian fluids, the viscous force per unit volume is equal to  $\mu\Delta v$ , where  $\mu$  is a coefficient of diffusivity. This is an extremely good approximation for most liquids and gases. With this term, the momentum equation becomes:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{v}$$

kinematic viscosity,  $\nu = \mu/\rho$ ,  
 $\nu \sim (\text{mean free path}) \cdot (\text{mean molecular velocity})$

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	$\mu$ (kg m <sup>-1</sup> s <sup>-1</sup> )	$\nu$ (m <sup>2</sup> s <sup>-1</sup> )
Air	$1.8 \cdot 10^{-5}$	$1.5 \cdot 10^{-5}$
Water	$1.1 \cdot 10^{-3}$	$1.1 \cdot 10^{-6}$
Mercury	$1.6 \cdot 10^{-3}$	$1.2 \cdot 10^{-7}$

---

### 3. Gravity force.

Let's consider situation with negligible viscosity and important gravity.

Let's focus on a vertical component of the momentum equation (along gravity acceleration).

vertical velocity component

gravity acceleration

$$\frac{Dw}{Dt} = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g$$

in static situation (no motion) vertical pressure gradient is balanced by gravity acceleration, this is condition for **hydrostatic balance**:

$$\frac{\partial p}{\partial z} = -\rho g.$$

Consider, that in many fluids density and pressure can be related. In general, equation which allows to calculate density with use of other properties of fluid is called a **CONSTITUTIVE EQUATION** (in a narrow sense), or EQUATION OF STATE

E.g. for air ideal gas equation can be used as a good approximation of equation of state:

$$p = \rho R T$$

gas constant for air                      temperature

For pure water we usually use:

$$\rho = \rho_0 [1 - \beta_T (T - T_0)]$$

thermal expansion coefficient  $\longrightarrow$

For ocean water relation is more complicated, salinity and pressure should be accounted for.

**Energy equation** or the first principle of thermodynamics:

$$dI = T d\eta - p d\alpha + \mu dS$$

internal energy per unit mass

entropy

specific volume

chemical potential

salinity

In fact this is conservation equation for internal energy and can be expressed in the form similar to these discussed earlier.

E.g. assuming that air is the ideal gas we can write:

$$dQ = dI + p d\alpha$$

$$dQ = c_v dT + p d\alpha$$

$$dQ = c_p dT - \alpha dp$$

$$\alpha = RT/p \text{ and } c_p - c_v = R$$



Let's consider the material derivative of internal energy:

$$\frac{DI}{Dt} + p \frac{D\alpha}{Dt} = \dot{Q}.$$

$$c_v \frac{DT}{Dt} + p \frac{D\alpha}{Dt} = \dot{Q}, \quad \text{or} \quad c_p \frac{DT}{Dt} - \frac{RT}{p} \frac{Dp}{Dt} = \dot{Q}.$$

Finally we get **energy equation** in form:

$$c_v \frac{DT}{Dt} + p\alpha \nabla \cdot \mathbf{v} = \dot{Q}.$$

## Adiabatic processes:

In adiabatic processes there are no sources and sinks of heat:

$$c_p dT = \alpha dp$$

The **potential temperature**,  $\Theta$  is the temperature that a fluid would have if moved adiabatically to some reference pressure (often 1000 hPa, close to the pressure at the earth's surface). In adiabatic flow the potential temperature of a fluid parcel is conserved, by definition:

$$\frac{D\theta}{Dt} = 0$$

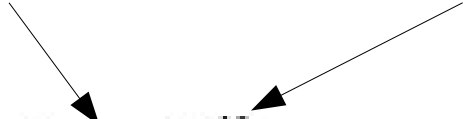
For an ideal gas:

$$d\eta = c_p d \ln T - R d \ln p$$

$$c_p d \ln \theta = c_p d \ln T - R d \ln p$$

Consequently:

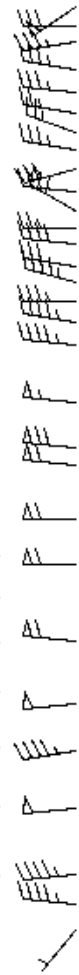
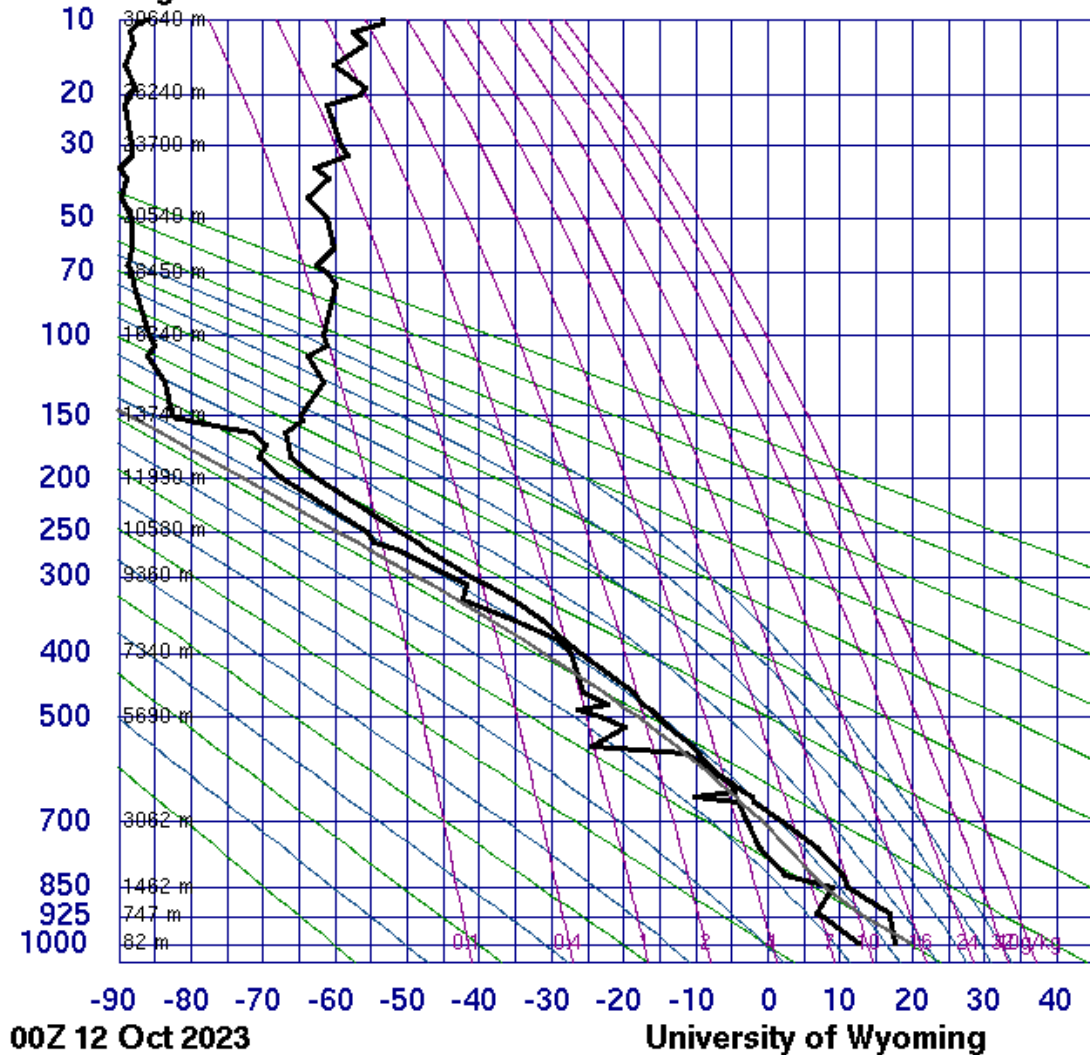
reference pressure                       $\kappa=R/c_p$


$$\theta = T \left( \frac{p_R}{p} \right)^\kappa$$

Finally we may write **a compact form of the energy equation:**

$$c_p \frac{D\theta}{Dt} = \frac{\theta}{T} \dot{Q}$$

**12374 Legionowo**



SLAT	52.40
SLON	20.96
SELV	96.00
SHOW	-0.47
LIFT	2.64
LFTV	2.53
SWET	278.0
KINX	30.50
CTOT	24.30
VTOT	26.50
TOTL	50.80
CAPE	0.00
CAPV	0.00
CINS	0.00
CINV	0.00
EQLV	-9999
EQTV	-9999
LFCT	-9999
LFCV	-9999
BRCH	0.00
BRCV	0.00
LCLT	282.4
LCLP	879.4
LCLE	317.3
MLTH	293.0
MLMR	8.45
THCK	5608.
PWAT	28.16

Potential temperature:  
an application

Vertical profile of the  
atmosphere over Warsaw  
from ballon  
measurement.

Black lines: measured  
temperature (right) and  
dew point temperature -  
right.

Adiabats ( $\theta = \text{const}$ ) -  
green lines.

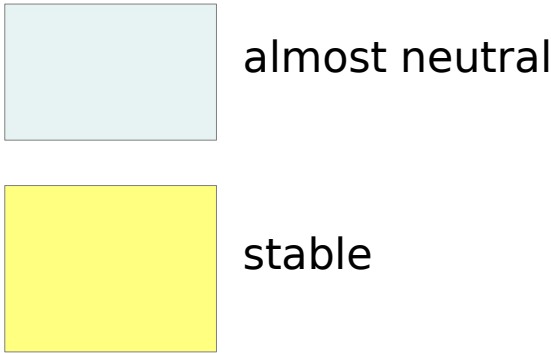


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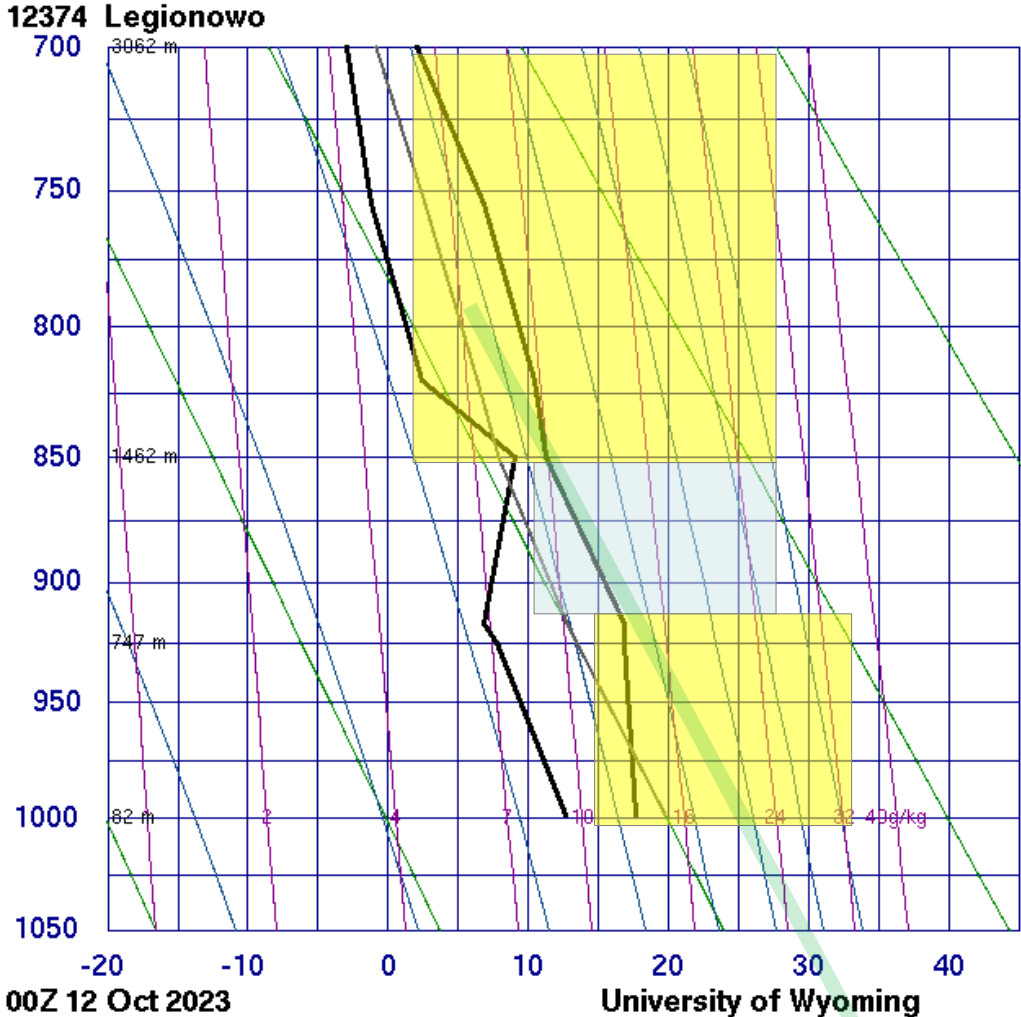
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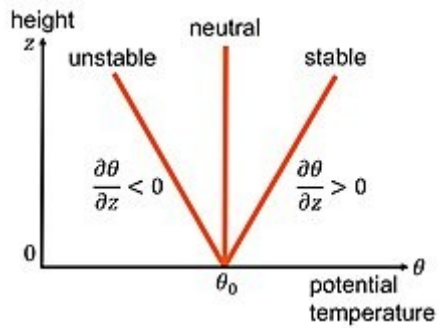
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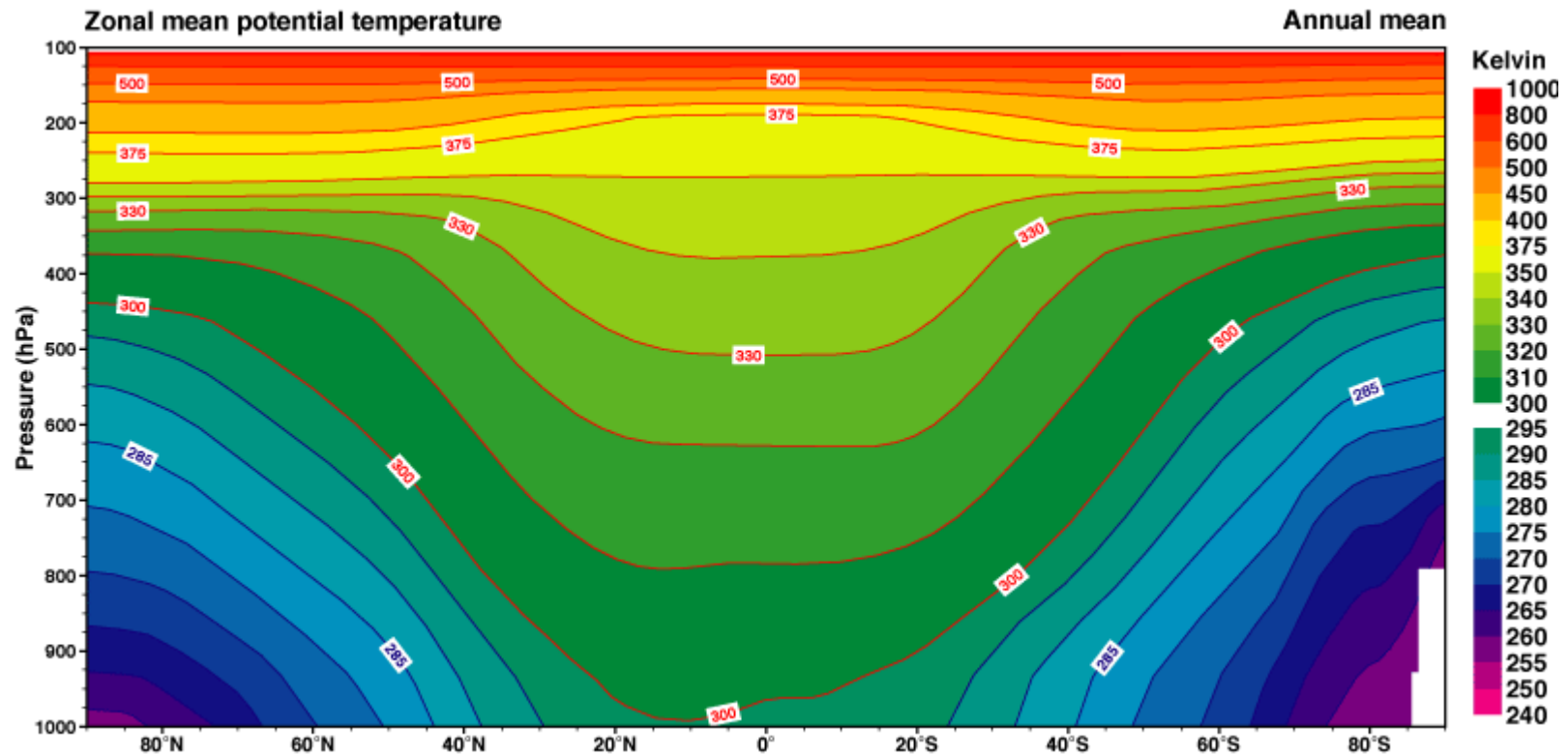
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$$\frac{D\theta}{Dt} = 0$$



## Atmospheric stability and meridional potential temperature distribution



## Forms of the Thermodynamic Equation

### *General form*

For a parcel of constant composition the thermodynamic equation is

$$T \frac{D\eta}{Dt} = \dot{Q} \quad \text{or} \quad c_p \frac{D \ln \theta}{Dt} = \frac{1}{T} \dot{Q} \quad (\text{T.1})$$

where  $\eta$  is the entropy,  $\theta$  is the potential temperature,  $c_p \ln \theta = \eta$  and  $\dot{Q}$  is the heating rate. Applying the first law of thermodynamics  $T d\eta = dI + p d\alpha$  gives:

$$\frac{DI}{Dt} + p \frac{D\alpha}{Dt} = \dot{Q} \quad \text{or} \quad \frac{DI}{Dt} + RT \nabla \cdot \mathbf{v} = \dot{Q} \quad (\text{T.2})$$

where  $I$  is the internal energy.

## The Equations of Motion of a Fluid

For dry air, or for a salt-free liquid, the complete set of equations of motion may be written as follows:

The *mass continuity equation*:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0. \quad (\text{EOM.1})$$

If density is constant this reduces to  $\nabla \cdot \mathbf{v} = 0$ .

The *momentum equation*:

$$\frac{D\mathbf{v}}{Dt} = -\frac{\nabla p}{\rho} + \nu \nabla^2 \mathbf{v} + \mathbf{F}, \quad (\text{EOM.2})$$

The *thermodynamic equation*:

$$\frac{D\theta}{Dt} = \frac{1}{c_p} \left( \frac{\theta}{T} \right) \dot{Q}. \quad (\text{EOM.3})$$

where  $\dot{Q}$  represents external heating and diffusion, the latter being  $\kappa \nabla^2 \theta$  where  $\kappa$  is the diffusivity.

The *equation of state*:

$$\rho = g(\theta, p) \quad (\text{EOM.4})$$

where  $g$  is a given function. For example, for an ideal gas,  $\rho = p_R^\kappa / (R\theta p^{\kappa-1})$ .

## **Numerical Simulation of Cloud–Clear Air Interfacial Mixing**

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(Manuscript received 25 October 2002, in final form 2 February 2004)

$$D/Dt \equiv \partial/\partial t + \mathbf{v} \cdot \nabla$$

$$B \equiv g \left[ \frac{T - T_0}{T_0} + \varepsilon (q_v - q_{v_0}) - q_c \right], \quad (2)$$

$$\frac{D\mathbf{v}}{Dt} = -\nabla\pi + \mathbf{k}B + \nu\nabla^2\mathbf{v}, \quad (1a)$$

$$\nabla \cdot \mathbf{v} = 0, \quad (1b)$$

$$\frac{DT}{Dt} = \frac{L}{c_p} C_d + \mu_T \nabla^2 T, \quad (1c)$$

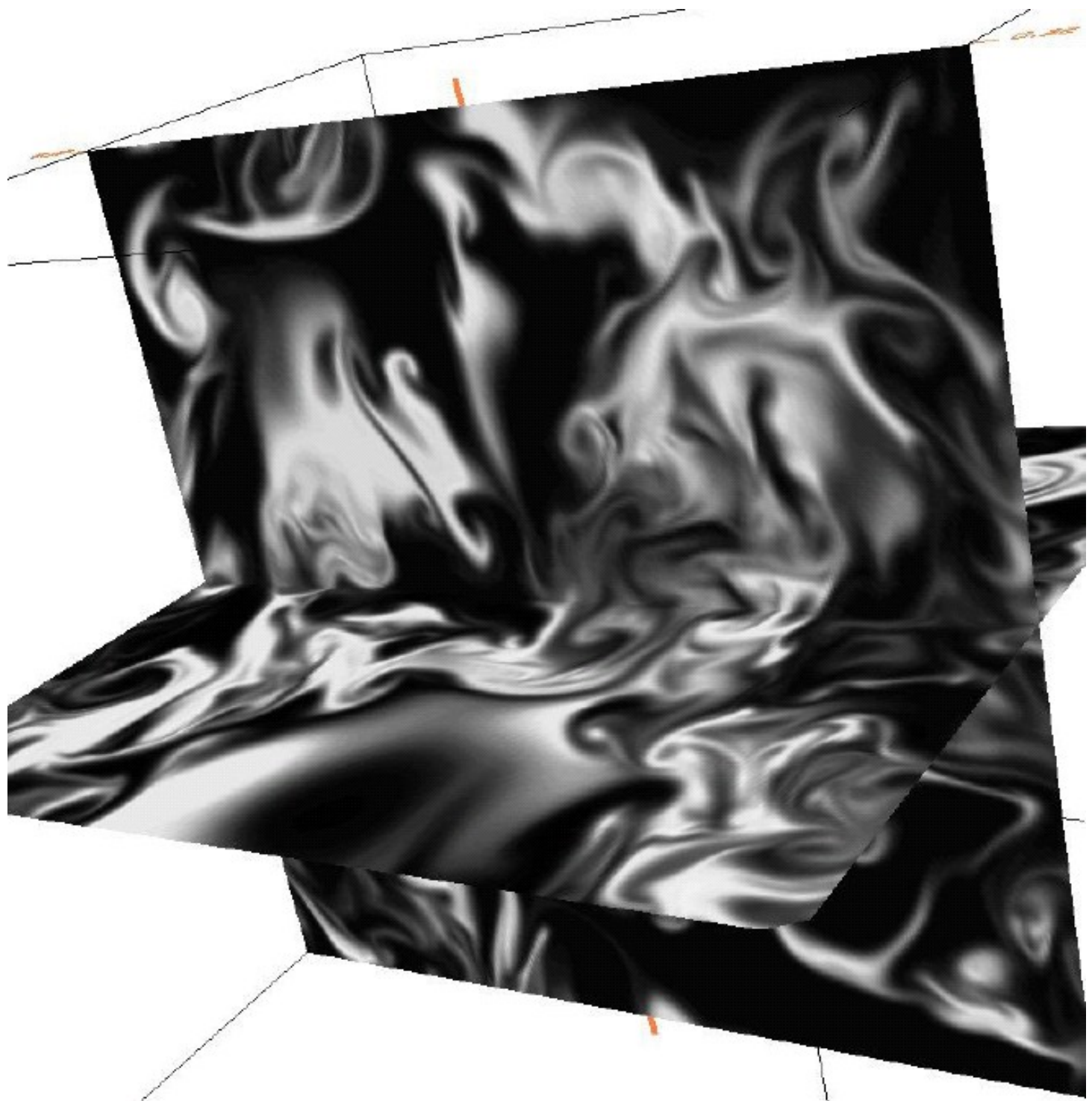
$$\frac{Dq_v}{Dt} = -C_d + \mu_v \nabla^2 q_v, \quad (1d)$$

$$\frac{Dq_c}{Dt} = C_d \quad C_d = \int f \frac{dm}{dt} dr, \quad \frac{D^* f}{D^* t} = -\frac{\partial}{\partial r} \left( f \frac{dr}{dt} \right) + \eta,$$

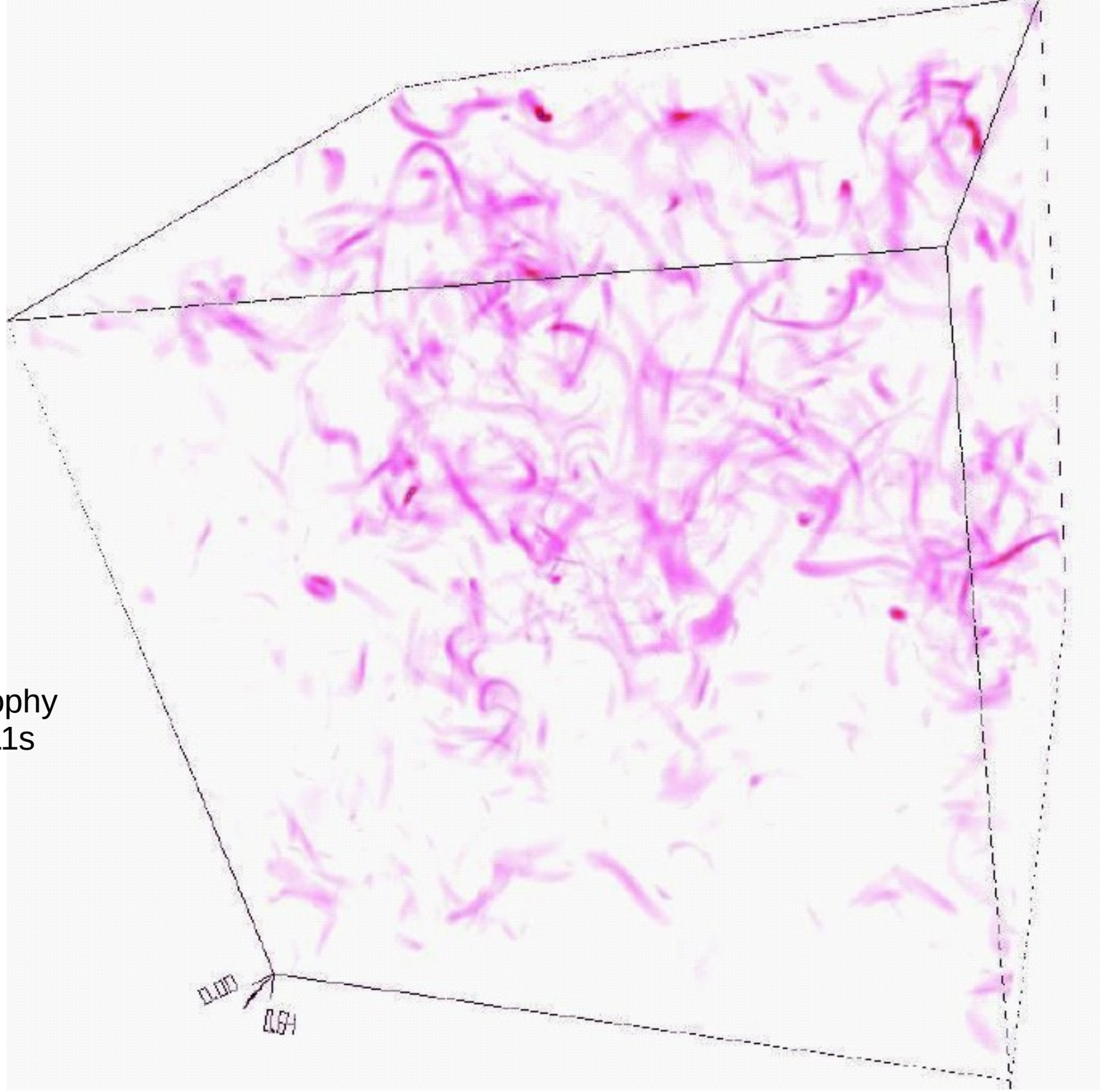
$$D^*/D^*t \equiv \partial/\partial t + (\mathbf{v} - \mathbf{k}v_t) \cdot \nabla$$



Cloud water  
after 11s



Enstrophy  
after 11s





Similar equations,  
another application

