

## CLOUD PHYSICS - tutorial 3 Activation

### Köhler and $\kappa$ -Köhler curves

#### 1. Kelvin effect

The exact form of equilibrium saturation over a water droplet is given by:

$$S_{ex} = \exp\left(\frac{A(T)}{r}\right) \quad \text{where} \quad A(T) = \frac{2\sigma}{\rho_l R_v T}$$

The approximate form of equilibrium saturation over a water droplet is:

$$S_{approx} = 1 + \frac{A(T)}{r}$$

Plot  $S_{ex}(r)$  and  $S_{approx}(r)$  for a given temperature (e.g.  $T = 0^\circ C$ ). Calculate the relative error  $(S_{ex} - S_{approx})/S_{ex}$ . For which values of  $r$  the relative error is smaller than 1%, 0.1%?

Assume that the surface tension of water is independent of temperature.

#### 2. Raoult effect

The equilibrium saturation ratio over an aqueous solution droplet is given by:

$$S = a_w \exp\left(\frac{A(T)}{r}\right)$$

where  $a_w$  is the activity of water in solution (the Raoult effect) and is given by:

$$a_w = \frac{r^3 - r_d^3}{r^3 - r_d^3(1 - \kappa)}$$

or in approximative form as:

$$a_w = 1 - \kappa \frac{r_d^3}{r^3}$$

$r$  is the radius of a droplet,  $r_d$  is the dry radius that describes the amount of solute (CCN),  $\kappa$  is a hygroscopicity parameter.

Plot  $a_w(r)$  showing the Raoult effect for NaCl ( $\kappa = 1.28$ ) and  $\text{NH}_4\text{NO}_3$  (ammonium nitrate, *azotan amonu*,  $\kappa = 0.67$ ), and different values of dry radii ( $r_d = 0.02, 0.05, 0.1 \mu\text{m}$ ).

### 3. Köhler curve

The  $\kappa$ -Köhler form:

$$S(r, \kappa, r_d, T) = \frac{r^3 - r_d^3}{r^3 - r_d^3(1 - \kappa)} \exp\left(\frac{A(T)}{r}\right)$$

Plot the Köhler curves for NaCl and  $\text{NH}_4\text{NO}_3$ :

- $r_d = 0.01, 0.03, 0.1 \mu\text{m}$
- $T = 0, 20^\circ \text{C}$

For sufficiently big droplets (let's call them  $R$ ) the equilibrium saturation converges to 100%. It means that the curvature and solute terms of Köhler equation become unimportant (assume two scenarios: the equilibrium saturation is less than 0.1% or 0.01%) For  $\kappa = 1.28$  and  $\kappa = 0.2$  plot relations between dry radii and  $R$ .

### 4. Critical radius and critical saturation

Show how critical radii and critical supersaturations (the maximum of the Köhler curve) depend on dry radii,  $r_d$ , and hygroscopicity parameter,  $\kappa$ . Assume a constant value of temperature.