

Condensational growth of cloud droplet - different approach

Starting with the "Western" formula we have:

$$\frac{dr}{dt} = \frac{A \cdot (S - 1)}{r} \quad (1)$$

where $S = e/e_s \cong q_v/q_s$ and

$$A = \left[\frac{\rho_w \cdot R_v \cdot T}{e_s \cdot D} + \frac{\rho_w L}{K \cdot T} \left(\frac{L}{R_v \cdot T} - 1 \right) \right]^{-1} \quad (2)$$

This is like in most textbooks.

First we work on the right term. The "-1" in this term is very small compared to $L/(R_v \cdot T)$, about 5% (**show that it is OK**), so we can safely neglect this. Then we have:

$$A = \left(\frac{\rho_w \cdot R_v \cdot T}{e_s \cdot D} + \frac{\rho_w \cdot L^2}{K \cdot R_v \cdot T^2} \right)^{-1} \quad (3)$$

Using the Clausius-Clapeyron relation we can approximate $L/(R_v \cdot T^2) \cong \partial q_s / \partial T \cdot (1/q_s)$. Then we can approximate $K \cong D \cdot c_p \cdot \rho_a$ which is reasonable (accurate to about 10-15 %) under normal atmospheric conditions, ρ_a here is the air density. This then gives

$$A = \left(\frac{\rho_w \cdot R_v \cdot T}{e_s \cdot D} + \frac{\partial q_s}{\partial T} \frac{1}{q_s} \frac{L \cdot \rho_w}{D \cdot c_p \cdot \rho_a} \right)^{-1} \quad (4)$$

Now we work on the left term. We can approximate $q_s = \epsilon \cdot e_s / (p - e_s) \cong \epsilon \cdot e_s / p$ since $p \gg e_s$ for almost all conditions. Then we can use $\epsilon = R_d / R_v$ and $\rho_a = p / (R_d \cdot T)$ from the ideal gas law. This gives

$$A = \left(\frac{\rho_w}{q_s \cdot \rho \cdot D} + \frac{\partial q_s}{\partial T} \frac{1}{q_s} \frac{L \cdot \rho_w}{D \cdot c_p \cdot \rho_a} \right)^{-1} \quad (5)$$

OR

$$A = \frac{q_s \rho_a D}{\rho_w \left(1 + \frac{L}{c_p} \frac{\partial q_s}{\partial T} \right)} \quad (6)$$

Show that formula (2) and (6)-(7) give similar results and that approximation that have been made have a small effect (order 10 % or less).