

## Entrainment and mixing

Turbulent mixing dilutes the concentration of, thereby reducing the competition for the available vapor, and enhancing the growth rate of favored droplets. Turbulent mixing processes are also thought of in terms of competing timescales. If we let  $t_m$  denote a mixing timescale, and  $t_e$  an evaporation timescale, then if  $t_e \ll t_m$  one speaks of the mixing as being **homogeneous**, while if  $t_e \gg t_m$  the mixing is described as **inhomogeneous**. In both cases the picture is one of mixing between saturated cloudy air, and unsaturated air. Thermodynamically the mixing of sub-saturated air requires net evaporation, either by reducing the number of droplets in the saturated air after the mixing, or by reducing the size of the droplets.

In inhomogeneous mixing, one imagines the dry air saturating through the rapid evaporation of droplets on the edge of an air parcel, before the dry air that is being mixed throughout the parcel has time to mix throughout the parcel. In the homogeneous mixing limit unsaturated air can be mixed throughout the parcel before the droplets have time to adjust. So mixing precedes evaporation and all of the droplets within the mixing volume adjust homogeneously to the changing thermodynamic conditions that accompany the mixing. In the inhomogeneous limit only a fraction of the droplets feel the intrusion of dry air, and these react disproportionately, for instance by complete evaporation. Hence for homogeneous mixing the required evaporation is carried disproportionately by the reduction in the average drop size, while for inhomogeneous mixing it is carried by a reduction in drop number. For the case of inhomogeneous mixing, the air in which all the droplets have been evaporated is just saturated. This condition follows by definition, as otherwise further mixing would require further evaporation. If subsequent homogenization within the otherwise undilute air is not rapid enough, further upward motion will result in the activation of additional cloud droplets, thus broadening the droplet spectrum overall. Otherwise the condensation occurs on the depleted remaining droplets, thereby reducing the competition for the available vapor, and favoring the growth of larger droplets.

**TODO** For the case of homogeneous mixing calculate the mean droplet concentration and mean volume effective radius of droplets after a mixing event (Fig. 1).

Assume that a volume  $V_c$  of cloudy saturated air is homogeneously mixed with a volume  $V_e$  of environmental dry air. The mixing process is isobaric, i.e.  $p = \text{const}$ . The cloudy air is initially characterized by:

- $n$  - the total number number of droplets having all the same initial mean volume radius  $r_{v0}$
- the initial cloud droplet number concentration is  $N_{c0} = n/V_c$
- the initial liquid water content is:  $LWC_0 = 4/3\pi\rho_l N_{c0}r_{v0}^3$
- $q_{c0}$  is the initial water vapor specific humidity, which is assumed to be saturated  $q_{c0} = q_s(T_{c0}, p)$
- $T_{c0}$  - temperature

The environmental air is characterized by:

- $n = 0$  - no droplets
- the initial cloud droplet number concentration is  $N_e = 0$
- $q_e$  is the initial water vapor specific humidity
- $T_e$  - temperature

After the mixing event, that consist in homogeneous mixing of volume  $V_c$  of cloudy air and a volume  $V_e$  of environmental dry air (the 'mixing coefficient' is defined as  $\chi = V_c/(V_c + V_e)$ ) the air is characterized by:

- $n$  - the total number of droplets
- $N = \chi N_{c0}$  - cloud droplet number concentration
- $q = q_{c0}\chi + (1 - \chi)q_e$  - water vapor specific humidity. Because the originally saturated cloudy air is mixed with the environmental dry air the mixture is not saturated. Cloud droplets have to be evaporated until the saturation is reached in temperature  $T$
- the amount of water to be evaporated until the saturation is reached is:  $\delta LWC = (q_s(T, p) - q)\rho$ , where  $T$  is the temperature after the mixing event, and  $\rho$  is the density of the air

- $T$  - temperature of the mixture; can be assumed at the beginning that it is a temperature of the mixture before the evaporation event, i.e. a weighted average of the initial temperature of the cloudy air and the temperature of the environmental air. *For those who like a challenge, calculate the temperature that results of mixing and evaporation.*

The liquid water content of the mixture is:

$$LWC = LWC_o - \delta LWC \qquad LWC = \frac{4}{3}\pi\rho_l N r_v^3$$

To plot a figure like in Fig. 1 assume:

- $p = 800hPa$
- $T_{c0} = 10^\circ C, T_e = 8^\circ C$
- $N_{c0} = 40cm^{-3}$
- $LWC_0 = 1.1g/m^3$
- different values of relative humidity in the environmental air; like in Fig. 1

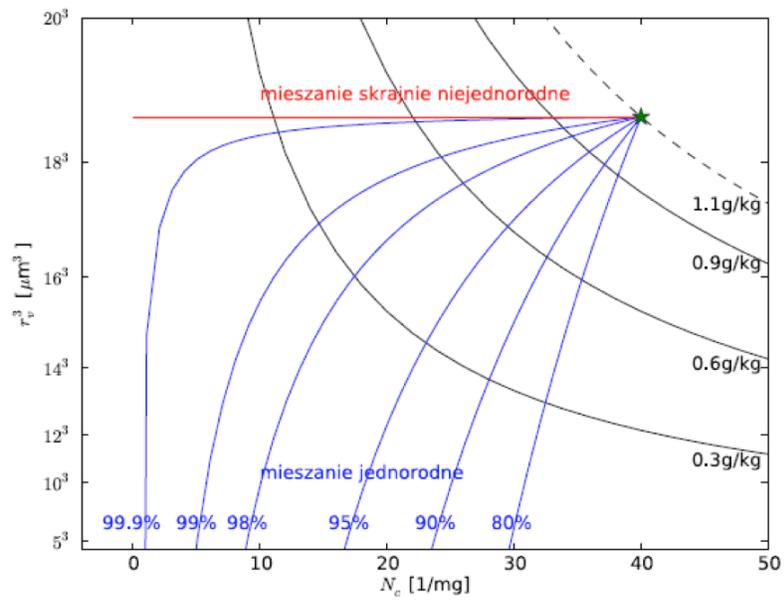


Figure 1: Entrainment diagram. *Mieszanie skrajnie niejednorodne* means *Extremely inhomogeneous mixing*, *Mieszanie jednorodne* means *Homogeneous mixing*.