Terminal Velocity Adjustment for Cloud and Precipitation Drops Aloft

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ABSTRACT

The velocities of cloud and precipitation drops aloft are obtained from the sea level velocity by multiplication with an adjustment factor. For cloud drops (1–40 μm diameter) the adjustment factor is found from the Stokes-Cunningham equation, and depends upon the Knudsen number and dynamic viscosity. For larger drops (40 μm–6 mm diameter) the adjustment factor is obtained from a semi-empirical fit to the data of Beard (1976) and depends upon the drop diameter, air density and dynamic viscosity. The adjustment factor for each size range is reduced to a simple function of drop size, air temperature and pressure. The velocities aloft using the adjustment method are found to be within 1% of the more precise values of Beard (1976) for reasonable atmospheric conditions. Polynomial formulas are included for calculating the sea level velocities.

1. Introduction

The calculation of the velocity of drops falling aloft is necessary for interpretation of Doppler radar data and for computation of cloud models involving detailed microphysics. The literature on terminal velocity of cloud and precipitation drops has been covered in a previous article (Beard, 1976) wherein the most recent data were used to develop expressions for cloud and precipitation velocities as a function of drop diameter and atmospheric properties. In Beard (1976) the common method of adjusting the velocity aloft using the change in the air density was found to be inapplicable except for the largest raindrops. Thus for even the first-order approximation to drop velocities aloft a more sophisticated approach is required. However, the formulas given in Beard (1976) are, in general, too cumbersome for repeated calculations of velocity as they require the solution of high-order logarithmic polynomials: It is the purpose of this paper to present a method in which the results of Beard (1976) are used to obtain an adjustment factor for the sea level velocities depending in a simple way on the pressure, temperature and drop diameter.

2. Method

a. Empirical velocity deviation

A velocity adjustment factor is defined by the equation[1]

$$V = V_0 f, \tag{1}$$

where the velocity aloft is determined by the product of a basic state velocity (e.g., sea level) and the velocity adjustment factor f. In order to examine the nature of the adjustment factor, a velocity deviation defined as

$$\epsilon = f - 1 = (V/V_0) - 1 \tag{2}$$

has been plotted in Fig. 1 using the values of velocity obtained from the equations given in Beard (1976) for the basic state of Gunn and Kinzer (1949), i.e., V_0 (1 atm, 20°C, 50% relative humidity), and for the conditions aloft of Foote and du Toit (1969), i.e., V (500 mb, −10°C, 100% relative humidity). In this case the velocity deviation is seen to be 20% for the smallest cloud drops decreasing to 10% for large cloud drops and then increasing to 35% for the largest raindrops. The velocity adjustment factor has a range in this example of 1.10 to 1.37.

The behavior of the velocity deviation is understood by examining the change in the drag force aloft for two flow regimes for falling drops of diameter d, *viz.*: 1) slip flow about a rigid sphere at negligible Reynolds numbers ($1 \lesssim d \lesssim 40$ μm), and 2) continuum flow around a noncirculating water drop of equilibrium shape at low to high Reynolds numbers ($40\ \mu\text{m} \lesssim d \lesssim 6$ mm).

In the first regime the alteration of the drag force aloft and the resultant velocity deviation in Fig. 1 are due to a decrease in the dynamic viscosity of 10% and also to an increase in the slip correction factor aloft which varies from 10% at 1 μm to 0% at 40 μm. The combined change in the "slip viscosity" factor

[1] A list of symbols is found in the Appendix.

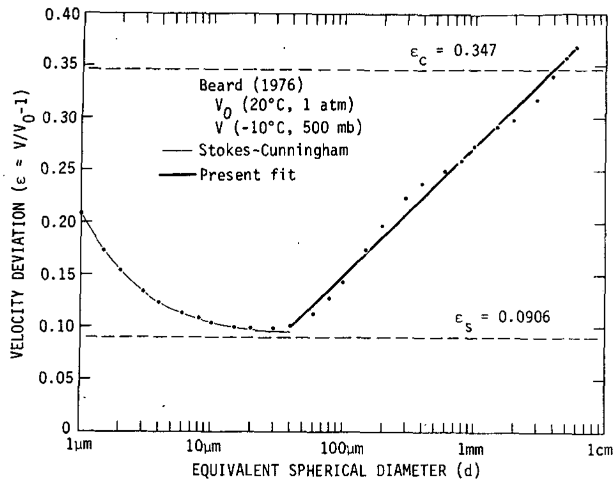

FIG. 1. Velocity deviation as a function of equivalent drop diameter for saturated air at 500 mb and $-10°C$. Shown also are the Stokes, Stokes-Cunningham and constant C_D deviations.

is responsible for the decrease in the velocity deviation from $\epsilon \approx 0.20$ at 1 μm to $\epsilon \approx 0.10$ at 40 μm.

In the second regime the alteration of the drag force aloft and the resultant velocity deviation are due to the change in the air density as well as the dynamic viscosity. At $d \approx 40$ μm the Reynolds number is sufficiently small that the drag force can be obtained approximately from Stokes flow yielding a velocity adjustment that is only a function of the change in the dynamic viscosity aloft. For larger Reynolds numbers the drag force must be obtained from the Navier-Stokes equation, and consequently for $d > 40$ μm the change in the air density aloft becomes increasingly important. Thus the increase in ϵ in Fig. 1 from 0.10 at 40 μm to 0.35 at 4 mm is due to the increasing influence of the change in air density aloft.

In the next few subsections the relative effects of the slip correction, dynamic viscosity and air density will be clarified by a comparison of the theoretical and empirical velocity deviations for cloud and precipitation drops to the data in Fig. 1.

b. Velocity deviation for cloud drops ($1 \lesssim d \lesssim 40$ μm)

A theoretical velocity deviation based on Stokes drag can be calculated from the ratio of the Stokes velocity aloft to that at sea level:

$$\epsilon_s = (V/V_0) - 1 = (\eta_0 \Delta\rho / \eta \Delta\rho_0) - 1 = (\eta_0/\eta) - 1, \quad (3)$$

where η is the dynamic viscosity and $\Delta\rho$ the difference between the densities of water and air ($\Delta\rho = \rho_w - \rho$). The above equation is simplified by the fact that $\Delta\rho/\Delta\rho_0 = 1$ to within 0.1% for reasonable atmospheric conditions. This result yields an approximate lower limit for ϵ in Fig. 1 and is given by the dashed line labeled $\epsilon_s = 0.0906$. A far better estimate for ϵ is obtained from the well-known Stokes-Cunningham formula for the velocity which occurs for reduction in drag from the first-order effects of slip, i.e.,

$$\epsilon = (\eta_0/\eta)[(1+2.51l/d)/(1+2.51l_0/d)] - 1, \quad (4)$$

where the mean free path l is given by

$$l = l_0(\eta/\eta_0)(p_0\rho_0/p\rho)^{\frac{1}{2}} \quad (5)$$

and $l_0 = 6.62 \times 10^{-8}$ m, $\eta_0 = 1.818 \times 10^{-5}$ kg m^{-1} s^{-1}, $p_0 = 1$ atm and $\rho_0 = 1.20$ kg m^{-3}. The prediction given by (4), indicated as the Stokes-Cunningham curve in Fig. 1, results in a precise fit for $d \lesssim 20$ μm since the Stokes-Cunningham equation was used in this range to compute the velocities V_0 and V. The use of the Stokes-Cunningham velocity deviation is seen to be inapplicable for $d \gtrsim 40$ μm since the actual value of the ϵ increases with drop size in Fig. 1, whereas the Stokes-Cunningham result decreases asymptotically to ϵ_s.

It is helpful to think in terms of a "slip viscosity" given by $\eta/(1+2.51\ N_{Kn})$ where the Knudson number is defined as $N_{Kn} = l/d$. As N_{Kn} approaches zero the slip viscosity approaches η, and ϵ given by (4) approaches ϵ_s. Thus the behavior of the velocity deviation in the diameter range 1–40 μm is due to the change in the slip viscosity aloft which depends on both η and N_{Kn} at $d = 1$ μm but only on η at $d = 40$ μm.

c. Velocity deviation for larger drops ($40\ \mu m \lesssim d \lesssim 6$ mm)

The velocity deviation for raindrops is often estimated from the empirical fact that the drag coefficient for a spheroid at large Reynolds numbers does not change appreciably with Reynolds number (Spilhaus, 1948; Kessler, 1969). The drag coefficient for a falling body is found by using a drag force equivalent to its net weight so that

$$C_D = 8M\Delta\rho g / \rho_w \pi d^2 \rho V^2, \quad (6)$$

where M is the drop mass and g the gravitational acceleration. For a change in altitude with both $M\Delta\rho g/\rho_w$ and d fixed the assumption of a constant C_D leads to

$$\rho V^2 = \rho_0 V_0^2. \quad (7)$$

Thus the velocity deviation for raindrops may be estimated from

$$\epsilon_c = (V/V_0) - 1 = (\rho_0/\rho)^{\frac{1}{2}} - 1. \quad (8)$$

For the change from 1 atm to 500 mb shown in Fig. 1 the actual velocity deviation for raindrops is found to lie approximately between ϵ_s and ϵ_c. Obviously the use of the air density alone, i.e., ϵ_c, does not provide a sufficient basis for calculating velocities aloft for most drop sizes $d \gtrsim 40$ μm. The assumption of a constant drag coefficient leads to an estimate for only the largest raindrops where C_D is nearly constant.

In Beard (1976), precipitation drops ($d \gtrsim 40$ μm) were found to behave as noncirculating spheroids with an axis ratio that depends only on drop size (see also Green, 1975). Because of the invariance of axis ratio with a change in altitude, the task of finding the velocity for a particular drop size is equivalent to determining the Reynolds number from the drag coefficient curve with the appropriate axis ratio. The significant variables in the terminal velocity problem for precipitation drops are found simply by inspection of C_D and the Reynolds number, $N_{Re}=\rho Vd/\eta$ resulting in $V=V(d,\rho,\eta)$. The solution for V given in Beard (1976) is too complex to yield a convenient expression for the velocity adjustment factor. However, it is apparent from the above discussion that f and ϵ are not just functions of the air density but also have a dependence on d and η.

An inspection of Fig. 1 for 40 μm$\leqslant d\leqslant$6 mm shows that the velocity deviation is given as a first approximation by a straight (semi-logarithmic) line. A reasonable hypothesis based on these data is that the equation of the line be determined by the values of $\epsilon_s(\eta)$ and $\epsilon_c(\eta,\rho)$ which will result in an expression for ϵ that is a function of d, ρ and η.

The equation for the line was calculated using the two-point formula taking one point as ϵ_1 (40 μm) and the other as ϵ_2 (6 mm) and assuming that $\epsilon_1\propto\epsilon_s$ and $\epsilon_2\propto\epsilon_c$ with the proportionality constants evaluated from Fig. 1:

$$\epsilon_1/\epsilon_s=0.100/0.0906=1.104, \quad (9)$$

$$\epsilon_2/\epsilon_c=0.367/0.347=1.058. \quad (10)$$

The two-point formula was then used to obtain a general equation for the velocity deviation:

$$\epsilon=\epsilon_1+\frac{\epsilon_2-\epsilon_1}{\ln d_2-\ln d_1}(\ln d-\ln d_1)=1.104\epsilon_s + (1.058\epsilon_c-1.104\epsilon_s)(\ln d+5.52)/5.01. \quad (11)$$

3. Results

The general validity of the velocity deviation formula for precipitation drops given by Eq. (11) was checked for various atmospheric conditions. First the basic velocities V_0 were calculated from formulas given in Beard (1976) for saturated air at 1 atm and 20°C. Next the velocity adjustment factor $f=\epsilon+1$ was calculated from (11) and multiplied by V_0 to obtain the adjusted velocity V'. The comparison of the adjusted velocity aloft with the velocity calculated directly from formulas in Beard (1976) was made by means of the fractional deviation

$$\sigma=|V-V'|/V \quad (12)$$

and the root-mean-square (rms) deviation

$$\sigma_{rms}=[(1/n)\Sigma\sigma^2]^{\frac{1}{2}}. \quad (13)$$

This comparison is shown in Table 1 for five levels of the summer atmosphere given by Foote and du Toit (1969), and for two additional test cases.

The deviations (σ) for the summer atmosphere in the diameter range 40 μm–6 mm are seen to be quite small as the maximum fractional deviations are $\sigma_{max}\leqslant 1.3\%$ and the mean deviations are $\sigma_{rms}\leqslant 0.7\%$. This indicates that the formula given by Eq. (11) works very well for the summer atmosphere of Foote and du Toit.

The special atmospheres in the last two columns were designed to test the velocity adjustment formula for extreme combinations of ϵ_s and ϵ_c. For the case at 903 mb and −10°C, the air density is the same as for sea level pressure and 20°C so that $\epsilon_c=0$, whereas $\epsilon_s=9.1\%$. A plot of the velocity adjustment formula (11) for this case would contrast strikingly with Fig. 1 since the value of ϵ would *decrease* to zero as the diameter increases from 40 μm to ~6 mm. For the case at 500 mb and 20°C where $\epsilon_c=42.3\%$ and $\epsilon_s=0$, a calculation of (11) would yield a velocity deviation that is even more exaggerated than that plotted in Fig. 1 since the spread between ϵ_c and ϵ_s is even larger. The deviations (σ) for the special atmospheres in the diameter range 40 μm–6 mm are $\sigma_{max}=2.2$, 2.9% and $\sigma_{rms}=1.4$, 1.6%. Even for these hypothetical extremes, designed especially to test the range of applicability of (11), the result in Table 1 shows that the velocity adjustment scheme still provides a close approximation to the method of Beard

TABLE 1. Deviation (σ) of the adjusted velocity aloft from the velocity of Beard (1976). Also shown are the velocity deviations from sea level based on Stokes drag (ϵ_s) and on a constant drag coefficient (ϵ_c).

Pressure (mb)	900	800	700	600	500	903	500
Temperature (°C)	20	15	10	0	−10	−10	20
Density (kg m^{-3})	1.060	0.960	0.856	0.763	0.661	1.194	0.584
40 μm−6 mm							
σ_{max} (%)	0.6	0.9	1.2	1.3	1.2	2.2	2.9
σ_{rms} (%)	0.4	0.5	0.6	0.6	0.7	1.4	1.6
1−40 μm							
σ_{max} (%)	0.1	0.2	0.4	0.4	0.3	0.2	0.6
σ_{rms} (%)	0.03	0.1	0.2	0.2	0.1	0.1	0.2
ϵ_s (%)	0	1.4	2.8	5.8	9.1	9.1	0
ϵ_c (%)	6.1	11.5	18.1	25.1	34.4	0	42.3

(1976). For reasonable atmospheric conditions aloft the results in Table 1 for the summer atmosphere indicate that the adjustment formula (11) should give values within 1% of the method of Beard (1976).

The fractional and rms deviations in Table 1 for the diameter range 1–40 μm are even smaller than those for larger drops, since both V' and V are based on the Stokes-Cunningham velocity for $d \lesssim 20$ μm. The maximum deviation originates from error in the use of (4) near $d=40$ μm, because the change in velocity due to the change in air density is completely ignored. This approximation does not result in a significant error at such small Reynolds numbers ($N_{Re} \leqslant 0.12$ for $d \leqslant 40$ μm). For reasonable atmospheric conditions the results shown in Table 1 for 1–40 μm demonstrate that the velocity adjustment formula for cloud drops [Eq. (4)] gives velocities that are typically accurate to within 0.2% with a maximum error of 0.4%.

4. Discussion

Calculations of velocities aloft are readily made with the formulas in the two size ranges compiled in Table 2. If the values of either η or ρ are not directly available then they may be obtained from the temperature and pressure by the following approximate formulas given also in Table 2:

$$\eta \approx 1.832 \times 10^{-5}\{1+0.00266[T(\mathrm{K})-296]\} \ [\mathrm{kg\ m^{-1}\ s^{-1}}], \tag{14}$$

$$\rho \approx 0.348 p(\mathrm{mb})/T(\mathrm{K}) \ [\mathrm{kg\ m^{-3}}]. \tag{15}$$

The dynamic viscosity has been approximated from Sutherland's formula (List, 1949) by a linear function of the temperature accurate to $\lesssim 0.3\%$ in the range −10 to +50°C. In the formula for the air density, the virtual temperature (T_v) has been replaced by T resulting in an error for the density of saturated air of $\leqslant 0.8\%$ for $T \leqslant 20$°C. Since the air density enters into the velocity adjustment for both size ranges as $(\rho_0/\rho)^{\frac{1}{2}}$ the error in the use of (13) is $\lesssim 0.4\%$. Thus with a small additional sacrifice in accuracy ($\lesssim 0.4\%$) the velocity adjustments for the two size ranges may be calculated with only the knowledge of d, p and T. There probably is not much point in trying for higher accuracies (i.e., using more precise values of η and ρ) in calculating raindrop velocities aloft ($d \gtrsim 1$ mm), because the velocity adjustment is based on formulas in Beard (1976) that are already of an approximate nature.

TABLE 2. Velocity adjustment formulas for two diameter ranges. Also included are formulas for the dynamic viscosity $\eta(T)$ and air density $\rho(p,T)$.

1 μm – 40 μm

$V = V_0 f$

$f = (\eta_0/\eta)(1+2.51l/d)/(1+2.51l_0/d)$

$l = l_0(\eta/\eta_0)(p_0\rho_0/p\rho)^{\frac{1}{2}}$

$l_0 = 6.62 \times 10^{-8}$ m

$\eta_0 = 1.818 \times 10^{-5}$ kg m^{-1} s^{-1}

$p_0 = 1013.25$ mb

$\rho_0 = 1.204$ kg m^{-3}

40 μm – 6 mm

$V = V_0 f$

$f = 1.104\epsilon_s + [(1.058\epsilon_c - 1.104\epsilon_s)(5.52+\ln d)/5.01] - 1$

$\epsilon_s = (\eta_0/\eta) - 1$

$\epsilon_c = (\rho_0/\rho)^{\frac{1}{2}} - 1$

$\eta \approx 1.832 \times 10^{-5}\{1+0.00266[T(\mathrm{K})-296]\}$ [kg m^{-1} s^{-1}]

$\rho \approx 0.348 p(\mathrm{mb})/T(\mathrm{K})$ [kg m^{-3}]

TABLE 3. Values of sea level velocities V_0 for saturated air ($\rho = 1.194$ kg m^{-3}) at 1 atm and 20°C.

d (μm)	V_0 (cm s^{-1})	d (μm)	V_0 (cm s^{-1})	d (μm)	V_0 (cm s^{-1})	d (mm)	V_0 (cm s^{-1})
1.0	3.48E-3	10	0.304	100	25.0	1.0	402
1.5	7.47E-3	15	0.670	150	46.7	1.5	542
2.0	1.294E-2	20	1.201	200	69.4	2.0	653
3.0	2.84E-2	30	2.68	300	114.7	3.0	808
4.0	4.98E-2	40	4.70	400	158.8	4.0	885
6.0	1.105E-1	60	10.18	600	244	5.0	912
8.0	1.951E-1	80	17.08	800	326	6.0	914

The equation for the diameter range 40 μm–6 mm can be written in the compact form

$$V = V_0(a+bX), \tag{16}$$

where $a = 1+\epsilon_s(T)$, $b = [1.058\epsilon_c(\rho) - 1.104\epsilon_s(T)]/5.01$ and $X = \ln(d) + 5.52$. At any level aloft the values of a and b may be easily calculated from the simple expressions for $\epsilon_s[\eta(T)]$ and $\epsilon_c[\rho(p,T)]$. When using a computer to calculate a *set* of velocities aloft $V(d)$, the values $X(d)$ and $V_0(d)$ can first be initialized in an array. The set of velocities aloft is then found by combining $X(d)$, $V_0(d)$ as given by (16) with the altitude-dependent values of a and b.

The basic state velocities V_0 are furnished in Table 3 for seven drop size categories per decade (e.g., 1, 1.5, 2, 3, 4, 6, 8 μm) for saturated air at 1 atm and 20°C using the formulas given in Beard (1976). For other drop sizes the velocities may be approximated by interpolation or evaluated graphically using these data. Alternatively, V_0 may be calculated to the desired accuracy from the formulas in Table 4. These expressions for V_0 were obtained from a fit to the data in Table 3 in terms of $x = \ln(d)$ and $y = \ln(V_0)$. In order to reduce the order of the polynomial to a manageable size the diameter range was divided into the ranges 1–40 μm and 40 μm–6 mm. The closeness of fit to the data in Table 3 was determined from the fractional and rms deviations given by Eqs. (12) and (13) with V and V' replaced by V_0 and V_0'.

In the range 1–40 μm, a third-order polynomial is entirely adequate since σ_{max} and σ_{rms} are negligible ($\lesssim 0.07\%$). For the range 40 μm–6 mm there are several flow regimes which are responsible for a more complex variation of V_0 with d. The choice of the particular formula for each range depends on the desired accuracy and also on the compatability of the

TABLE 4. Polynomial formulas and coefficients for computing sea level velocities V_0 for saturated air ($\rho = 1.194$ kg m^{-3}) at 1 atm and 20°C. Also included are the fractional and rms deviations from values given in Table 3.

$$V_0 = \exp(y)$$

$$y = c_0 + c_1 x + \ldots + c_m x^m = \sum_{j=1}^{m} c_j x^j$$

$$x = \ln(d)$$

Table of coefficients (c_j)

	$1 \leqslant d \leqslant 40$ µm			40 µm $\leqslant d \leqslant 6$ mm	
j	$m=2$	$m=3$	$m=4$	$m=7$	$m=9$
0	0.12914E2	0.105035E2	0.67122E1	0.65639E1	0.706037E1
1	0.21170E1	0.108750E1	−0.43101E0	−0.10391E1	0.174951E1
2	0.10735E-1	−0.133245E0	−0.44511E0	−0.14001E1	0.486324E1
3		−0.659969E-2	−0.64352E-1	−0.82736E0	0.660631E1
4			−0.51751E-2	−0.34277E0	0.484606E1
5				−0.83072E-1	0.214922E1
6				−0.10583E-1	0.587140E-0
7				−0.54208E-3	0.963480E-1
8					0.869209E-2
9					0.330890E-3
$\sigma_{\rm rms}$	0.76%	0.04%	0.82%	0.41%	0.29%
$\sigma_{\max}$	1.42%	0.07%	1.87%	1.07%	0.80%
σ(40 µm)	0.46%	0.01%	0.27%	0.08%	0.05%

formulas at the crossover point which can be assessed from the value of σ(40 µm). Polynomial formulas have also been generated by Foote and du Toit (1969), Wobus *et al.* (1971) and Dingle and Lee (1972); however, these previous results were based on the data of Gunn and Kinzer (1949) and have considerable deviations ($\sigma_{\max} \sim 8\%$) from more recent data (see Beard, 1976, Table 3). The present fit, although not quite as accurate as the formulation of Beard (1976), is more straightforward for calculating V_0. The best fits in Table 4 have coefficients with only six significant figures and result in typical errors of $\lesssim 0.3\%$.

5. Conclusions

The velocity aloft may be calculated by using an adjustment factor applied to a known velocity (e.g., the sea level velocity). In the range of cloud drop diameters (1–40 µm) the adjustment factor is found from the Stokes-Cunningham equation and depends upon the Knudsen number and dynamic viscosity. For larger drop diameters (40 µm–6 mm) the adjustment factor is obtained from a semi-empirical fit to the data of Beard (1976) and depends upon the drop diameter, air density and dynamic viscosity.

The adjustment method is compared to the more precise values of Beard (1976) for the summer atmosphere of Foote and du Toit and found to have an rms velocity deviation $\leqslant 0.7\%$. For a similar comparison, but using extreme combinations of temperature and pressure, the rms velocity deviations are found to be $\leqslant 1.6\%$.

The dependency of the adjustment factor for both size ranges is reduced to only three variables (*viz.*, d,T,p) by approximating ρ by the density of dry air with an additional error in velocity $\leqslant 0.4\%$. Use of the polynomials given in Table 4 for obtaining V_0 results in no significant error for cloud drops and only $\sim 0.3\%$ error for the ninth-degree formula for larger drops.

The velocity aloft for larger drops is conveniently given by Eq. (16), since $V_0(d)$ and $X(d)$ need be evaluated only once and then combined later with the altitude-dependent values of the coefficients $a(T)$ and $b(T,p)$.

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APPENDIX

List of Symbols

a	Stokes adjustment factor $[=1+\epsilon_s]$
b	$(1.058\epsilon_c - 1.104\epsilon_s)/5.01$
c	polynomial coefficient
C_D	drag coefficient $[=8M\Delta\rho g/\rho_w \pi d^2 \rho V^2]$
d	equivalent spherical diameter
f	adjustment factor $[=V/V_0]$
g	gravitational acceleration (9.8 m s^{-1})
j	index for polynomial term
l	mean free path of air molecules
M	drop mass
m	polynomial degree
$N_{\rm Kn}$	Knudsen number $[=l/d]$
$N_{\rm Re}$	Reynolds number $[=\rho V d/\eta]$

p static air pressure
T air temperature
T_v virtual temperature
V, V' terminal velocity aloft
V_0 terminal velocity at basic state
X $5.51+\ln(d)$
x $\ln(d)$
y $\ln(V_0)$
$\Delta\rho$ $\rho_w-\rho$
ϵ velocity deviation $[=(V/V_0)-1=f-1]$
ϵ_c velocity deviation for a constant drag coefficient $[=(\rho_0/\rho)^{\frac{1}{2}}-1]$
ϵ_s velocity deviation for Stokes drag $[=(\eta_0/\eta)-1]$
η dynamic viscosity
ρ air density
ρ_w water density
σ fractional deviation $[=|V-V'|/V]$
$\sigma_{\rm rms}$ root-mean-square deviation $\{=[(1/n)\Sigma\sigma^2]^{\frac{1}{2}}\}$
Subscript 0 indicates basic state (1 atm, 20°C).

REFERENCES

Beard, K. V., 1976: Terminal velocity and shape of cloud and precipitation drops aloft. *J. Atmos. Sci.*, **23**, 851–864.

Dingle, A. N., and Y. Lee, 1972: Terminal fallspeeds of raindrops. *J. Appl. Meteor.*, **11**, 877–879.

Foote, G. B., and P. S. du Toit, 1969: Terminal velocities of raindrops aloft. *J. Appl. Meteor.*, **8**, 249–253.

Green, A. W., 1975: An approximation for the shapes of large raindrops. *J. Appl. Meteor.*, **14**, 1578–1583.

Gunn, R., and G. D. Kinzer, 1949: The terminal velocity of fall for water drops in stagnant air. *J. Meteor.*, **6**, 243–248.

Kessler, E., 1969: On the distribution and continuity of water substance in atmospheric circulations. *Meteor. Monogr.*, No. 32, Amer. Meteor. Soc., p. 20.

List, R. J., 1949: *Smithsonian Meteorological Tables*, 6th ed. The Smithsonian Institution, Washington, D. C., 527 pp. (see p. 394).

Spilhaus, A. F., 1948: Raindrop size, shape and falling speed. *J. Meteor.*, **5**, 108–110.

Wobus, H. B., F. W. Murray and L. R. Koenig, 1971: Calculation of the terminal velocity of water drops. *J. Appl. Meteor.*, **10**, 751–754.