Köhler and κ Köhler curves

1. Köhler curve

$$S(r, B_s, T) = \frac{e_s(r)}{e_s(\infty)} = exp\left(\frac{A(T)}{r} - \frac{B_s}{r^3}\right)$$

$$where \quad A(T) = \frac{2\sigma}{\rho_l R_v T}, \qquad B_s = \nu\phi_s r_d^3 \frac{\rho_s}{\rho_l}\frac{M_l}{M_s} = \nu\phi_s \frac{3m_s}{4\pi\rho_l}\frac{M_l}{M_s}$$

r_d is called the dry radius, used for a description of the amount of solute (CCN). Φ_s, the 'practical osmotic coefficient', or a fitting factor is usually less than one. Here we will assume: $\phi_s = 1$.

Plot the Köhler curves for the following parameters (see Figure 2):

- For NaCl $\nu = 2$
- M_l=18.01 g/mol
- M_s=58.44 g/mol
- $\rho_l = 10^3 kg/m^3$
- $\rho_s = 2.16 \cdot 10^3 kg/m^3$
- surface tension of water, σ=0.072 J/m^2; for atmospheric temperatures may be assumed constant (see Figure 1)
- $R_v = 461 J/kg/K$
- $r_d = 0.02, 0.05, 0.1 \mu m$
- $T = 0, 10, 20^o C$

Investigate if there is any difference between the full and approximate varsions of Köhler curve representations:

$$S(r, B_s, T) = exp\left(\frac{A(T)}{r} - \frac{B_s}{r^3}\right) \quad versus \quad S(r, B_s, T) = 1 + \frac{A(T)}{r} - \frac{B_s}{r^3}$$

2. κ-Köhler curve

$$S(r,\kappa,T) = a_w exp\left(\frac{A(T)}{r}\right)$$

$$where \quad \frac{1}{a_w} = 1 + \kappa\frac{V_s}{V_l}, \quad V_s = \frac{4}{3}\pi r_d^3, \qquad V_l = \frac{4}{3}\pi(r^3 - r_d^3)$$

V_s is the volume of solute, V_l is the volume of water. The volume of a drop is the sum of V_s and V_l.

$$S(r,\kappa,T) = \frac{r^3 - r_d^3}{r^3 - r_d^3(1-\kappa)} exp\left(\frac{A(T)}{r}\right)$$

Plot the κ-Köhler curves for the same parameters as above. Calculate κ values. Show that κ gets a constant value.

3. Critical supersaturation versus dry radius for different κ

Calculate the critical supersaturation values vesrus dry radius (see Figure 3).

Petters, M. D., and S. M. Kreidenweiss, 2007: A single parameter representation of hygroscopic growth and cloud condensation nucleus activity. Atmos. Chem. Phys., 7, 1961-1971. www.atmos-chem-phys.net/7/1961/2007.

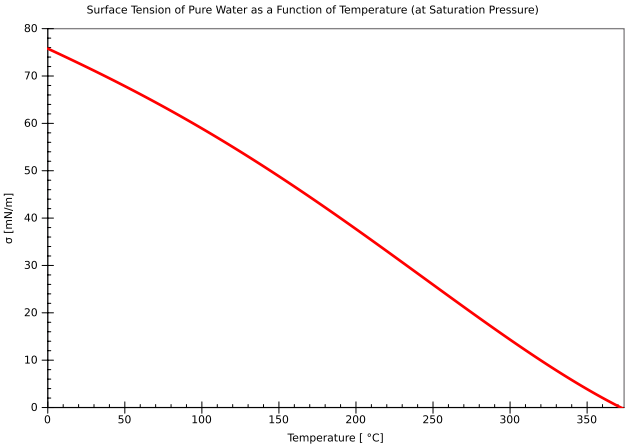

Figure 1: Surface tension for pure water

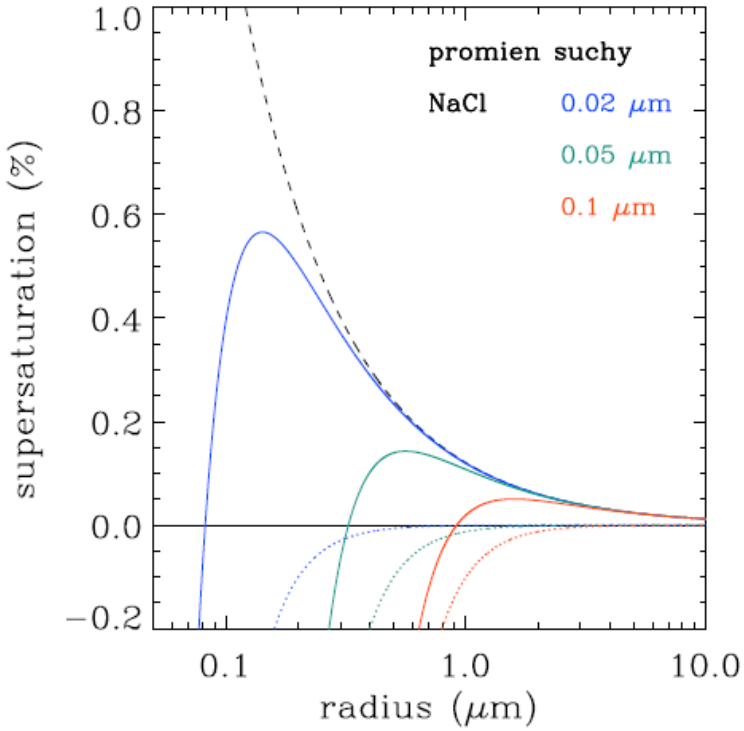

Figure 2: Köhler curve for NaCl

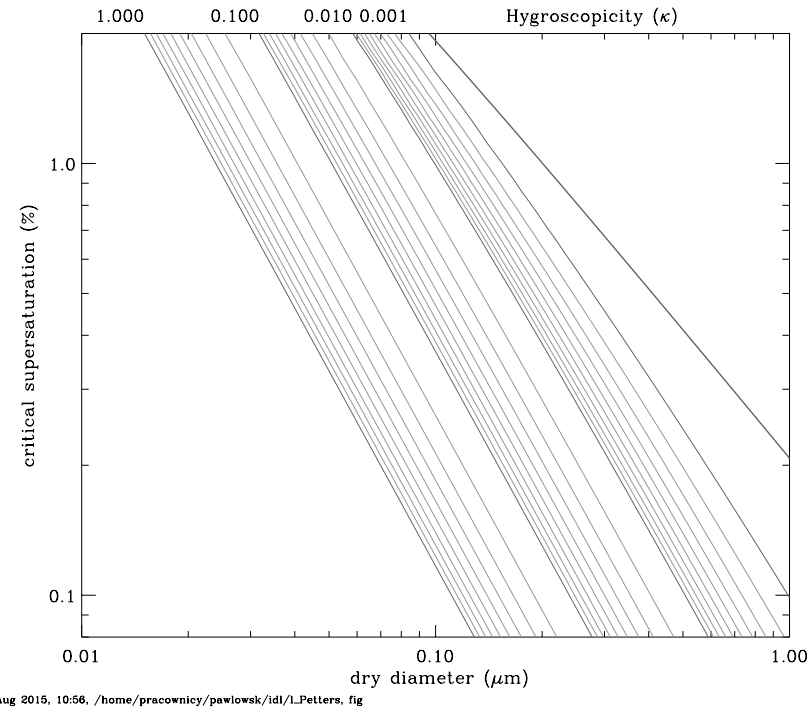

Figure 3: Calculated critical supersaturation for $0 \leq \kappa \leq 1$ computed for $\sigma = 72J/m^2$ and T=298.15K