CLOUD PHYSICS - tutorial 3 Activation

Köhler and κ-Köhler curves

1. **Kelvin effect**

   The exact form of equilibrium saturation over a water droplet is given by:

   $$S_{ex} = exp\left(\frac{A(T)}{r}\right) \qquad \text{where} \quad A(T) = \frac{2\sigma}{\rho_l R_v T}$$

   The approximate form of equilibrium saturation over a water droplet is:

   $$S_{approx} = 1 + \frac{A(T)}{r}$$

   Plot $S_{ex}(r)$ and $S_{approx}(r)$ for a given temperature (e.g. $T = 0°C$). Calculate the relative error $(S_{ex} - S_{approx})/S_{ex}$. For which values of r the relative error is smaller than 1%, 0.1%?
   Assume that the surface tension of water is independent of temperature.

2. **Raoult effect**

   The equilibrium saturation ratio over an aqueous solution droplet is given by:

   $$S = a_w exp\left(\frac{A(T)}{r}\right)$$

   where a_w is the activity of water in solution (the Raoult effect) and is given by:

   $$a_w = 1 - \frac{B_s}{r^3} \quad \text{where} \quad B_s = \nu\phi_s r_d^3 \frac{\rho_s}{\rho_l}\frac{M_l}{M_s} = \nu\phi_s \frac{3m_s}{4\pi\rho_l}\frac{M_l}{M_s}$$

   r is the radius of a droplet, r_d is the dry radius that describes the of the amount of solute (CCN). ν is the dissociativity (in the case of NaCl ν=2). Φ_s, the 'practical osmotic coefficient', or a fitting factor is usually less than one.

   The κ-Köhler form of the activity is given by:

   $$a_w = \frac{r^3 - r_d^3}{r^3 - r_d^3(1-\kappa)}$$

   Show that for weak solutions (i.e. when $r >> r_d$) and a single chemical compound $B_s = \kappa r_d^3$.

   Plot $a_w(r)$ showing the Raoult effect for NaCl ($\kappa = 1.28$) and NH_4NO_3 (ammonium nitrate, azotan amonu, $\kappa = 0.67$), and different values of dry radii ($r_d = 0.02,\ 0.05,\ 0.1\ \mu m$).

3. **Köhler curve**

   The κ-Köhler form:

$$S(r, \kappa, r_d, T) = \frac{r^3 - r_d^3}{r^3 - r_d^3(1-\kappa)} exp\left(\frac{A(T)}{r}\right)$$

   Plot the Köhler curves for NaCl and NH_4NO_3:

   - $r_d = 0.02,\ 0.05,\ 0.1\ \mu m$
   - $T = 0,\ 10,\ 20^o\ C$

   For sufficiently big droplets (let's call them R) the equilibrium saturation converges to 100%. It means that the curvature and solute terms of Köhler equation become unimportant. For given κ values plot a relation between dry radii and R.

4. **Critical radius and critical saturation**

   Show how critical radii and critical supersaturations (the maximum of the Köhler curve) depend on dry radii, r_d, and hygroscopicity iparameter, κ. Assume a constant value of temperature.