Session 4

1. Show that geostrophic balance in isothermal coordinates may be written

$$f\mathbf{U_g} = \mathbf{k} \times \nabla_T (TR\ln p + \Phi)$$

2. Derive the thermal wind equation

$$\bar{f} \times \frac{\partial \bar{u}_g}{\partial p} = \frac{R}{p} \nabla_p T,$$

from the geostrophic balance in pressure coordinates.

   (a) Write the equation above in cartesian components using $\bar{f} = f\hat{k}$. Use this result to explain the observations that, if the temperature falls in the poleward direction, then the eastward wind will increase with height.

   (b) A **barotropic** atmosphere is one in which the density depends only on the pressure, $\rho = \rho(p)$. An atmosphere in which density depends on both the temperature and the pressure, $\rho = \rho(T, p)$, is referred to as a **baroclinic** atmosphere. Show that the geostrophic wind is independent of height in a barotropic atmosphere.

3. The mean temperature in the layer between 750 and 500 hPa decreases eastward by 3°C per 100 km. If the 750 hPa geostrophic wind is from the southeast at 20 m/s, what is the geostrophic wind speed and direction at 500 hPa? What is mean temperature advection in the 750-500 hPa layer? Let $f = 10^{-4}$ 1/s.