

Water condensed in adiabatic (and pseudo-adiabatic) process

$$h_\ell = c_\ell - q_l L_v \quad c_\ell = c_d + q_t(c_v - c_d) = q_d c_d + q_t c_v$$

$$dh_\ell = c_\ell dT - q_l dL_v - L_v dq_l \quad dL_v = (c_v - c_l) dT$$

$$c_\ell dT - q_l dL_v = (q_d c_d + q_s c_v + q_l c_v - q_l c_v + q_l c_l) dT =$$

$$(q_d c_d + q_s c_v + q_l c_l) dT = c_p dT$$

$$c_p dT - L_v dq_l - v dp = 0$$

$$c_p \frac{dT}{dz} dz - L_v dq_l + g dz = 0$$

$$dq_l = \frac{c_p}{L_v} \left(\frac{dT}{dz} + \frac{g}{c_p} \right) dz$$

$$\Gamma_d = \frac{g}{c_p} \quad \Gamma_s = -\frac{dT}{dz}$$

$$dq_l = \frac{c_p}{L_v} (\Gamma_d - \Gamma_s) dz$$

$$q_l = \frac{m_l}{m}$$

$$LWC = \frac{m_l}{V} = \frac{m_l}{m} \cdot \frac{m}{V} = \rho q_l$$

Show that $LWC = c_w(z - z_0)$ provides a good approximation. $c_w [g/m^4]$ is called the condensation rate. z_0 is the height of the cloud base.