Extra exercises for midterm exam

1. Derive the hypsometric equation. Derive the thermal wind equation. Derive the equation for geostrophic balance in isothermal coordinates.

2. Find the geopotential profile $\Phi(p)$ in a hydrostatic atmosphere of uniform temperature. What is the geopotential height at the $p = 300$ hPa surface for the temperature $T = 250$ K and surface pressure $p_s = 1000$ hPa? Find the geopotential profile $\Phi(p)$ in a hydrostatic atmosphere of constant potential temperature θ.

3. Assuming that the geostrophic wind is purely zonal ($v_g = 0$) and independent of height, the wind profile in the atmospheric Ekman layer reads

$$\begin{aligned} u &= u_g[1 - e^{-\gamma z} \cos{(\gamma z)}], \\ v &= u_g e^{-\gamma z} \sin{(\gamma z)}, \end{aligned}$$

where $\gamma = \sqrt{f/(2A)}$ and A is the eddy diffusivity. Evaluate the frictionally induced **ageostrophic transport** (U, V), defined as

$$U = \int_0^\infty (u - u_g)\,\mathrm{d}z, \quad V = \int_0^\infty (v - v_g)\,\mathrm{d}z.$$

How is the ageostrophic transport oriented with respect to the friction force (τ_x, τ_y) at the gradient wind level?

4. The planet Venus rotates about its axis so slowly, that to a reasonable approximation the Coriolis parameter may be set equal to zero. For steady, frictionless motion parallel to latitude circles the momentum equation then reduces to a type of cyclostrophic balance

$$\frac{u^2 \mathrm{tan}\varphi}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial y}$$

By transforming this expression to isobaric coordinates, show that the thermal wind equation in this case can be expressed in the form

$$\omega_r^2(p_2) - \omega_r^2(p_1) = \frac{R \mathrm{ln}(p_2/p_1)}{a \mathrm{sin}\varphi \mathrm{cos}\varphi}\frac{\partial \langle T \rangle}{\partial y}$$

where R is the gas constant, a the radius of the planet, and $\omega_r = u/(a\cos\varphi)$ is the relative angular velocity. How must $\langle T \rangle$ (vertically averaged temperature) vary with respect to latitude in order that ω_r be a function of pressure only? If the zonal velocity at about 60 km height above the equator ($p = 2.9 \times 10^5$ Pa) is 100 m/s and the zonal velocity vanishes at the surface of the planet ($p_0 = 9.5 \times 10^6$ Pa), what is the vertically averaged temperature difference between the equator and pole assuming that ω_r depends only on pressure? The planetary radius is $a = 6100$ km, and the gas constant is $R = 187\ \frac{\mathrm{J}}{\mathrm{kgK}}$.