Terminal fall speeds of drops and droplets

To describe many cloud microphysical processes, an accurate knowledge of the terminal fall velocity of cloud droplets and raindrops is needed. This information is also necessary when attempting to infer particle sizes, and thus rain rates, from Doppler radar data. For large raindrops it is also important to know how their shape is modified by pressure forces on their surface as they move through the air.
A complete basis for the calculation of terminal velocity can be obtained from the Navier-Stokes equations of motion for the air flowing past the drops and the motion of the water inside the drop. The terminal velocity of a particle is reached when the drag forces exerted on a drop by the surrounding air become sufficiently large to balance the gravitational force. With such a balance, the terminal velocity, u_∞, can be expressed as

$$u_\infty = \frac{2}{9}\frac{r^2 g \rho_l}{(C_D Re/24)\,\rho\nu}, \tag{1}$$

where Re is Reynolds number, C_D is the drag coefficient, ρ_l is water density, ρ is air density and ν is kinematic viscosity of air.
Generally speaking, since the Reynolds number is itself a function of the fall speed of the particle, Eq. (1) represents an implicit equation for u_∞ and needs to be solved iteratively once information about C_D is known. However, at very low Reynolds numbers ($Re \ll 1$), in the Stokes regime, the drag is proportional to the speed (rather than square of speed) and so $C_D Re/24 = 1$. In this situation the terminal velocity can be derived exactly and it takes a form

$$u_\infty = \frac{2}{9}\frac{r^2 g \rho_l}{\nu\rho}. \tag{2}$$

Some effort has been made to extend the exact solution for the regime with Reynolds number close to unity. However, most descriptions are based on the fitting of functional forms to experimental laboratory data. Several functions can be found in the literature, all proposing different formulae for different regimes of Reynolds numbers, that are conveniently translated into drop sizes. For small cloud droplets, typically smaller than 20 μm in diameter, the Stokes solution presented above provides a good approximation. For

Table 1: Analytic formulae for droplet fall velocity in three different Reynolds (Re) numbers regimes. Droplet radius is in cm and terminal velocity is in cm/s.

Stokes regime, low Re; r $< 30\mu$m	$u_\infty = 1.19 \cdot 10^6 \mathrm{r}^2$
intermediate Re; 40 μm $<$ r $< 600\mu$m	$\mathrm{u}_\infty = 8.00 \cdot 10^3 r$
high Re; 0.6 mm $<$ r $<$2 mm	$\mathrm{u}_\infty = 2.01 \cdot 10^3 \mathrm{r}^{1/2}$

intermediate drop sizes from a few tens of microns to about a millimetre, the fall velocity is proportional to r. For very large drops, where the movement can induce an internal circulation within a drop, a much weaker dependence on the drop's radius is expected (proportional to $\sqrt{r}$). Analytical formulae for terminal fall speed are given in Table 1.
To accommodate solutions to Eq. (1) for higher Reynolds numbers (i.e. larger droplets) series solutions can be developed to fit laboratory data for the drag coefficient. A good fit to the laboratory data is described by the following expression:

$$u_\infty(r,T,p) = f(T,p) \cdot \exp\left[\sum_{j=0}^{j_{\max}} c_j (\ln(2r))^j\right]. \tag{3}$$

where here u_∞ is the terminal drop velocity in cm/s, r is the drop radius in cm. The c_j as well as the correction factor, f, for ambient pressure and temperature, depends on the size of the drop and are given in Table 2. Figure 1 presents terminal velocity as given by Eq. (3) in atmospheric conditions p=1000 hPa and T=20 $^\circ C$, and aloft i.e. for p=500 hPa and T=-10 $^\circ C$. Velocities based on the analytic formulae as given in Table 1 are also shown.

References

Beard, K. V., 1977: Terminal velocity adjustment for cloud and precipitation drops aloft. *J. Atmos. Sci.*, **34**, 1293-1298.
Gunn, R., and Kinzer, G. D., 1949: The terminal velocity of fall for water droplets in stagnant air. *J. Meteor.*, **6**, 243-248.

TODO Plot Terminal velocity as a function of drop's radius for different values of temperature and pressure. Check how it compares with analytical expressions. Be inspired by Fig.1.

Table 2: Empirical terminal velocity parameters for the expression in Eq. (3). Here l is mean free path of air molecules, with a reference value of $l_0 = 6.62 \cdot 10^{-6}$cm and η is dynamic viscosity with reference value of $\eta_0 = 1.818 \cdot 10^{-5}$kgm^{-1}s^{-1}. Reference pressure and density are $p_0 = 1013.25$ hPa, and $\rho_0 = 1.204$ kgm^{-3} All quantities taken from Table 2 of Beard (1977) and Beard corrigendum (1977).

$1\mu\text{m} \leq \text{r} \leq 20\mu\text{m}$
$j_{\max} = 4$
$c_{1,4} = \{10.5035, 1.08750, -0.133245, -0.00659969\}$
$f = (\eta_0/\eta)[1 + 1.255\, l/r(\text{cm})]/[1 + 1.255\, l_0/r(\text{cm})]$
$l = l_0(\eta/\eta_0)(p_0\, \rho_0/p\, \rho)^{\frac{1}{2}}$
$20\mu\text{m} \leq \text{r} \leq 3\text{mm}$
$j_{\max} = 8$
$c_{1,4} = \{6.5639, -1.0391, -1.4001, -0.82736\}$
$c_{5,8} = \{-0.34277, -0.083072, -0.010583, -0.00054208\}$
$f = 1.104\epsilon_s + [1.058\epsilon_c - 1.104\epsilon_s] \cdot [6.21 + \ln r(cm)]/5.01 + 1$
$\epsilon_s = (\eta_0/\eta) - 1$
$\epsilon_c = (\rho_0/\rho)^{\frac{1}{2}} - 1$
$\eta \approx 1.832 \cdot 10^{-5}\{1 + 0.00266[T(K) - 296]\}\, [\text{kg}\, \text{m}^{-1}\, \text{s}^{-1}]$
$\rho \approx 0.348\, p(\text{hPa})/T(\text{K})[\text{kg}\, \text{m}^{-3}]$

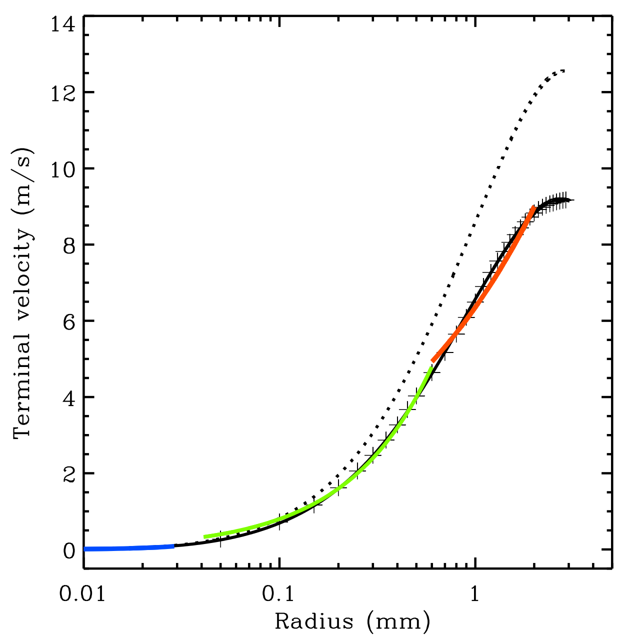

Figure 1: Terminal velocity. Black continuous lines show velocities calculated using Eq. (3) with coefficient from Table 2 and with adjustment coefficients as formulated in Table 2. The black continuous line is for temperature 20°C and pressure 1000hPa; the black dotted line (bigger velocities) is for -10°C and 500 hPa. Blue, green and red dashed lines show velocities calculated using analytic formulae given in Table 1. The plus signs overlying the black continuous line show terminal fall speed from observational data given by Gunn and Kinzer (1949).