

CLOUD PHYSICS - tutorial 4

Köhler and κ -Köhler curves

1. Kelvin effect

The exact form of equilibrium saturation over a water droplet is given by:

$$S_{ex} = \exp\left(\frac{A(T)}{r}\right) \quad \text{where} \quad A(T) = \frac{2\sigma}{\rho_l R_v T}$$

The approximate form of equilibrium saturation over a water droplet is:

$$S_{approx} = 1 + \frac{A(T)}{r}$$

Plot $S_{ex}(r)$ and $S_{approx}(r)$ for a given temperature (e.g. $T = 0^\circ C$). Calculate the relative error $(S_{ex} - S_{approx})/S_{ex}$. For which value of r the relative error is smaller than 1%, 0.1%?

Assume that the surface tension of water is independent of temperature.

2. Raoult effect

The equilibrium saturation ratio over an aqueous solution droplet is given by:

$$S = a_w \exp\left(\frac{A(T)}{r}\right)$$

where a_w is the activity of water in solution (the Raoult effect) and is given by:

$$a_w = 1 - \frac{B_s}{r^3} \quad \text{where} \quad B_s = \nu \phi_s r_d^3 \frac{\rho_s}{\rho_l} \frac{M_l}{M_s} = \nu \phi_s \frac{3m_s}{4\pi \rho_l} \frac{M_l}{M_s}$$

r is the radius of a droplet, r_d is the dry radius that describes the amount of solute (CCN). ν is the dissociativity (in the case of NaCl $\nu=2$). Φ_s , the 'practical osmotic coefficient', or a fitting factor is usually less than one.

The κ -Köhler form of the activity is given by:

$$a_w = \frac{r^3 - r_d^3}{r^3 - r_d^3(1 - \kappa)}$$

Show that for weak solutions (i.e. when $r \gg r_d$) and a single chemical compound $B_s = \kappa r_d^3$.

Plot $a_w(r)$ showing the Raoult effect for NaCl ($\kappa = 1.28$) and NH_4NO_3 (ammonium nitrate, azotan amonu, $\kappa = 0.67$), and different values of dry radius ($r_d = 0.02, 0.05, 0.1 \mu m$).

3. Köhler curve

In the κ -Köhler form:

$$S(r, \kappa, r_d, T) = \frac{r^3 - r_d^3}{r^3 - r_d^3(1 - \kappa)} \exp\left(\frac{A(T)}{r}\right)$$

Plot the Köhler curves for NaCl and NH_4NO_3 :

- $r_d = 0.02, 0.05, 0.1 \mu\text{m}$
- $T = 0, 10, 20^\circ \text{C}$

For sufficiently big droplets (let's call them R) the equilibrium saturation converges to 100%. It means that the curvature and solute terms of Köhler equation becomes unimportant. For given κ values plot a relation between dry radii and R .