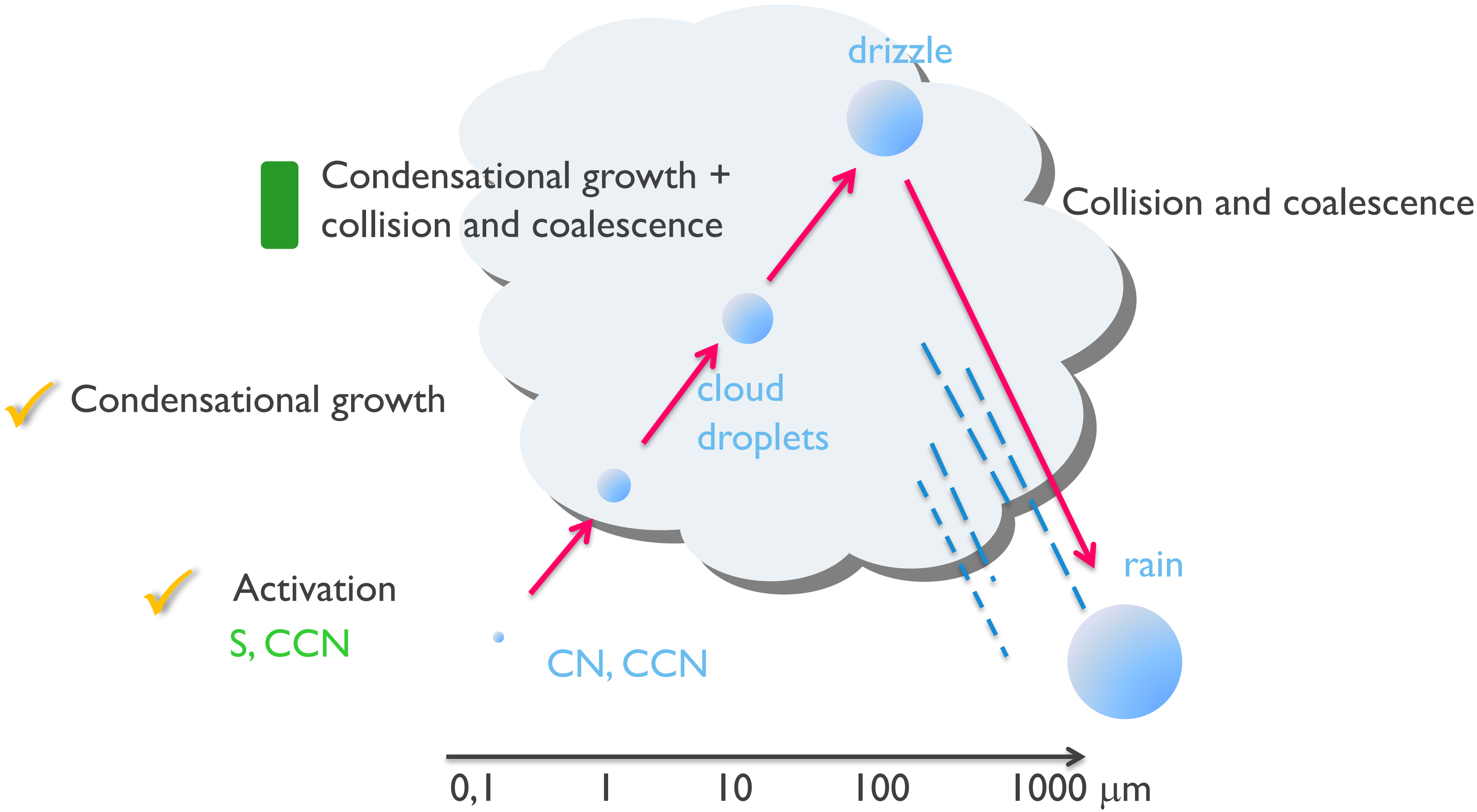
COALESCENCE

UNIVERSITY
OF WARSAW



Condensational growth +
collision and coalescence
Collision and coalescence
Condensational growth
Activation
S, CCN
drizzle
cloud
droplets
rain
CN, CCN
0,1
1
10
100
1000 μm

It is not possible to grow the rain-drops by condensation alone

THEORITICALLY

Given a concentration 100cm $^{-3}$ a cloud whose maximum water content is 2 g/m^3 would consist of drops of only about 30μm in diameter.

$$N = 100cm^{-3}\ ,\ \ LWC = 2gm^{-3}$$

$$LWC = \frac{\pi}{6}\rho_l N d_v^3 \quad \longrightarrow \quad d_v = \left(\frac{LWC}{\frac{\pi}{6}\rho_l N}\right)^{1/3} = 33\mu\text{m}$$

$$d_v > d \ \longrightarrow \ d \approx 30\mu\text{m}$$

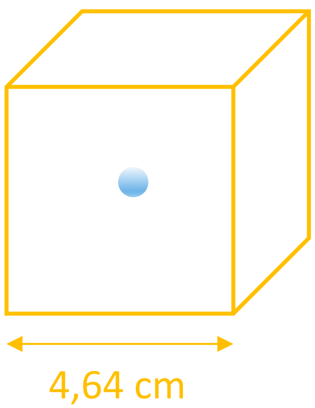
Given that the maximum water content of most clouds is less than 2g/m^3 , and the droplet concentration somewhat higher, the chance of growing precipitation size droplets is even further reduced.

$$LWC < 2gm^{-3}\ ,\ N > 100cm^{-3} \quad \longrightarrow \quad d < 30\mu m$$

Even if random process favored a very small percentage of the droplets to grow very large by condensation,

a 1mm drop requires a 100 cm^3 volume of air whose adiabatic liquid water content is 5 gm^{-3} to be void of any other drops,

an effectively impossible requirement.



$$d_v = 1mm \longrightarrow m = 0.5 \cdot 10^{-3} g$$

$$V = 100cm^3 \longrightarrow LWC = \frac{m}{V} = \frac{5 \cdot 10^{-4} g}{10^{-4} m^3} = 5gm^{-3}$$

OR if LWC=2 gm^{-3} a 1mm drop requires a 250 cm^3 volume of air to be void of any other drops.

EMPIRICALLY

it is also readily apparent that rain-drops do not form by condensation alone.

Observed size distribution of rain drops suggests that the number concentration of raindrops is less than 1 per litre (N_r=10^{-3} cm^{-3}), which is 100 000 fold less than the concentration of cloud droplets (N_c=100 cm^{-3}).

$$\frac{N_r}{N_c} = \frac{10^{-3}}{100} = 10^{-5}$$

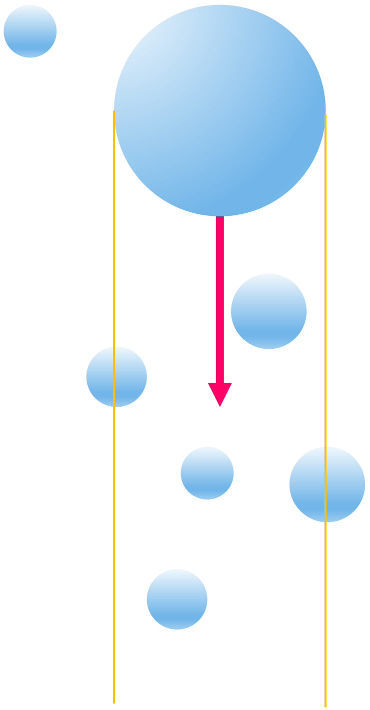
Both (theoretical and empirical) lines of thought suggest that rain, at least in warm clouds, requires cloud droplets to become aggregated into larger rain drops.

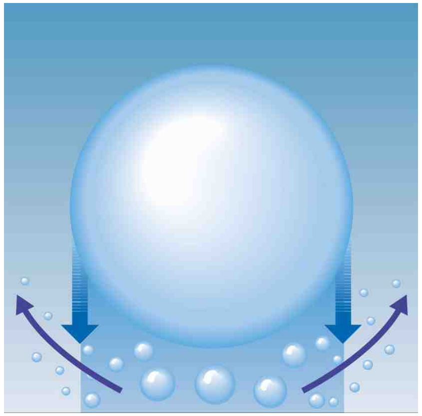
This process of aggregation involves two steps:

- collisions between droplets and
- their subsequent coalescence.

Collisions

- Collisions may occur through differential response of the droplets to gravitational, electrical, or aerodynamics forces
  - Gravitational effects dominate in clouds: large droplets fall faster then smaller ones, overtaking and capturing a fraction of those lying in their path
  - For this mechanism to be efficient the differential fall spead has to be large…..





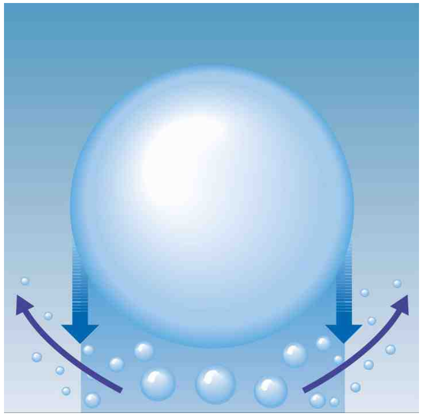
Small droplets can also be swept aside

If drops have the same size, no overtaking or collision

Collision efficiency

- Collision efficiency (E_{coll}) is equal to the fraction of those droplets with radius r in the path swept out by the collector drop that actually collide with it.

$$E_{coll} = \frac{\text{number of collisions}}{\text{number of droplets with radius r in the path swept out by the collector drop}}$$



Coalescence efficiency

- Collision does not guarantee coalescence
- For sizes smaller than 100 μm in radius, the most important interactions are:
  - the drops bounce apart
  - the drops coalesce and remain permanently united
- The ratio of the number of coalescences to the number of collisions is called the coalescence efficiency (E_{coal}).

$$E_{\text{coal}} = \frac{\text{number of coalescences}}{\text{numer of collisions}}$$

Collision efficiency

- The growth of a drop by the collision-coalescence process is governed by the collection efficiency (E), which is the product of collision efficiency (E_{coll}) and coalescence efficiency (E_{coal}).

$$E = E_{coll} \cdot E_{coal}$$

$$E = \frac{\text{number of coalescences}}{\text{number of droplets with radius r in the path swept out by the collector drop}}$$

A large collector drop of radius R and terminal velocity V(R) falls through a volume containing many smaller drops of radius r and terminal velocity V(r).
In some time interval Δt the large-drop will meet all droplets in the cylinder whose volume is given by:

$$\pi(R+r)^2[V(R)-V(r)]$$

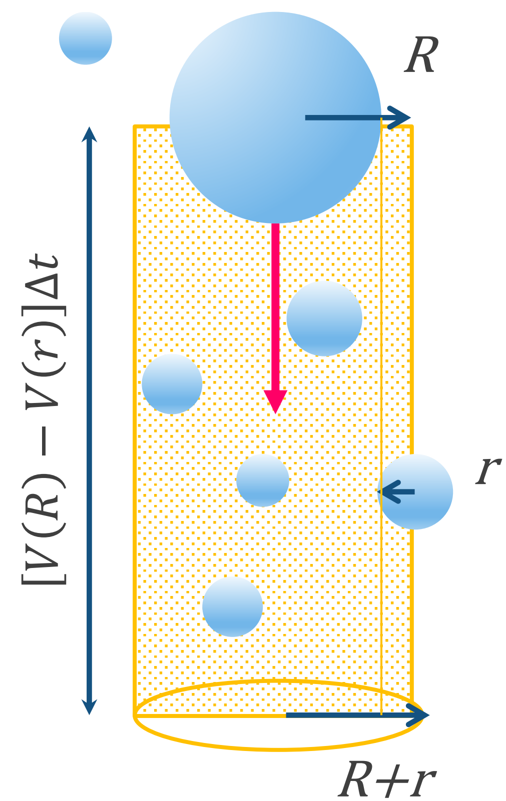
For simplicity we assume that all collisions result in coalescence.

The mass of the large-drop will increase by the mass of all the small droplets it assimilates:

$$\Delta M=\pi(R+r)^2[V(R)-V(r)]\Delta t N_r\left(\frac{4}{3}\pi r^3\rho_l\right)$$

Taking the limit as Δt goes to zero yields the gravitational collection equation:

$$\frac{dM}{dt}=N_r\pi(R+r)^2[V(R)-V(r)]\left(\frac{4}{3}\pi r^3\rho_l\right)$$



The gravitational collection equation can be generalize to allow for a population of cloud droplets

$$N_r = \int n(r)dr$$

$$\frac{dM}{dt} = \int \left(\frac{4}{3}\pi r^3 \rho_l\right) K(R,r) n(r) dr$$

$$K(R,r) = \pi(R+r)^2[V(R) - V(r)]E(R,r)$$

collision kernel

Collection efficiency

The theory of gravitational collection presumes the existence of two populations of droplets. The smaller droplets may be described by a distribution, but it is presumed that the largest of these smaller droplets is always much smaller than the collector drops.

The separation into collector, and collected drops, turns out to be an unnecessary idealization. In principle any two drops can collide with some probability.

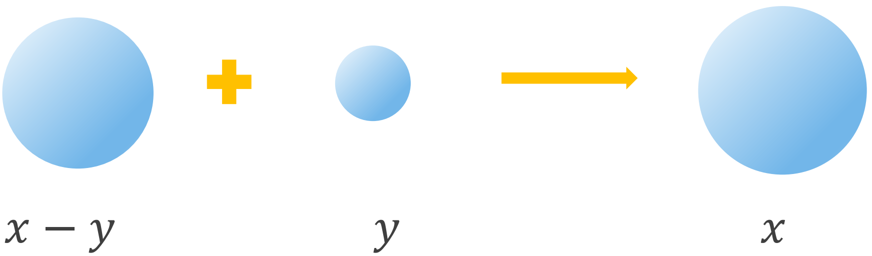
How does a population of droplets evolve as a result of collection process?

$n(x,t)dx$– the number of drops in some vanishingly small size interval $(x, x+dx)$, at some time t. x stands for the drop volume.

We are interested in the temporal evolution of the distribution:

$$\frac{\partial}{\partial t}[n(x,t)dx]$$

Drops with a volume x can be created by binary interactions between two smaller drops. The coalescence between a drop of mass $x-y$ and a drop of mass y will create a drop of mass x.



If we associated $x-y$ with the mass of the larger drop, then y can vary between 0 and $x/2$.

$$(x-y>y \longrightarrow y<x/2).$$

How does a population of droplets evolve as a result of collection process?

The change of number of drops of mass x can be express similarly to the change of a mass od drop of size R

$$\Delta M = \Delta t \int \left(\frac{4}{3}\pi r^3 \rho_l\right) K(R,r) n(r) dr$$

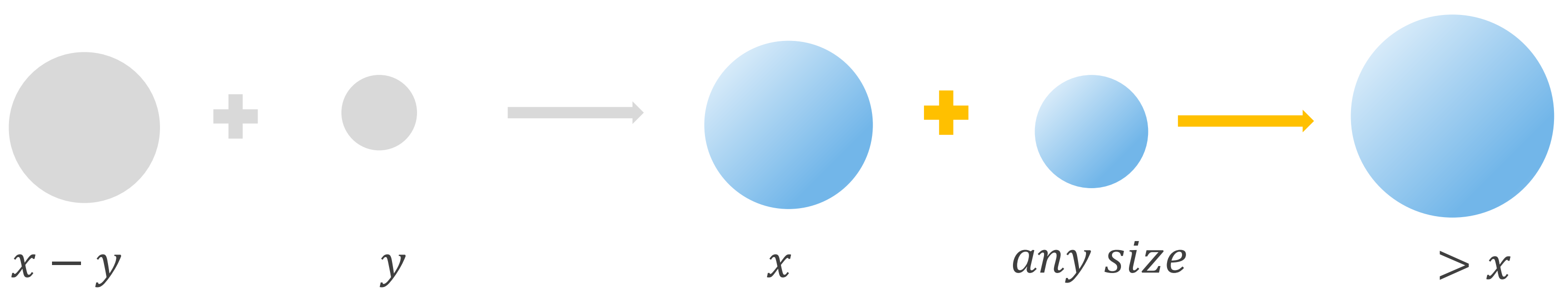
$$\Delta[n(x,t)dx] = \Delta t \left[dx \int_0^{x/2} n(x-y,t) n(y,t) K(x-y,y) dy \right]$$

$K(x,y)n(y)dy$ the probability that a drop of mass x will collect a drop of mass y in the time interval Δt.

The number of drops of size x will be reduced each time a drop of size x collect another drop.

From the definition of the collection kernel K the reduction of drops of size x in a time interval Δt is:

$$\Delta[n(x,t)dx] = -\Delta t n(x,t)dx \int_0^\infty n(y,t)K(x,y)dy$$



The net change in drops of size x is:

$$\Delta[n(x,t)dx] = \Delta t \Delta x \left[\int_0^{x/2} n(x-y,t)n(y,t)K(x-y,y)dy - n(x,t)\int_0^{\infty} n(y,t)K(x,y)dy \right]$$

Dividing both size by $dx\Delta t$ and taking the limit as $\Delta t \to 0$ yields an integro-differential equation that describes the evolution of the number distribution of drops as a result of binary interactions:

$$\frac{\partial}{\partial t}n(x,t) = \int_0^{x/2} n(x-y,t)n(y,t)K(x-y,y)dy - n(x,t)\int_0^{\infty} n(y,t)K(x,y)dy$$

This equation is sometimes called the Smoluchowski coalescence equation after Marian Smoluchowski who first derived it.

By symmetry: If we associated $x-y$ with the mass of the smaller drop, then y, can vary between $x/2$ and x $(x-y>y \longrightarrow y<x/2)$.

$$\int_0^{x/2} n(x-y,t)n(y,t)K(x-y,y)dy = \int_{x/2}^{x} n(x-y,t)n(y,t)K(x-y,y)dy$$

Smoluchowski equation

$$\frac{\partial}{\partial t}n(x,t) = \frac{1}{2}\int_0^x n(x-y,t)n(y,t)K(x-y,y)dy - n(x,t)\int_0^\infty n(y,t)K(x,y)dy$$

The first term on the RHS desribes the rate of increase of number of drops having a mass $m=\rho_l x$ due to the collision and coalesence.

The second term on the RHS secribes the rate of reduction of drops of mas m due to the coalescence of drops having a mass m with the other drops.

In the cloud-physics literature the above equation if often called the Stochastic_Collection Equation, although it is a purely deterministic equation, i.e. it containes no stochastic elements.

The reason for this is because this equation can be interpreted in as the mean-field representation of a stochastic process, analogous to the way diffusion is the mean field representation of Brownian motion; the latter being a stochastic process whose net effect can be described deterministically.

Marian Smoluchowski (1872 – 1917)

- Polish physicist.
- He was a pioneer of statistical physics and an avid mountaineer.
- Smoluchowski's scientific output included fundamental work on the kinetic theory of matter. His investigations concerned also an explanation of Brownian motion of particles. He introduced equations which presently bear his name.
- In 1906, independently of Albert Einstein, he described Brownian motion. Smoluchowski presented an equation which become an important basis of the theory of stochastic processes.
- In 1916 he proposed the equation of diffusion in an external potential field. This equation bears his name.

The kernel, $K(x, y)$, encapsulates the physics of binary interactions among a population of droplets. In principle it encapsulates all factors wheather drops will collide and coalesce, thereby collecting on another.

For warm-rain formation, hydrodynamic interactions in the presence of a gravitational field are thought to underly the collection process.

$$K(x,y) = \pi\left(r_x + r_y\right)^2\left[V(r_x) - V\left(r_y\right)\right]E(x,y)$$

$E(x, y)$ denotes the collection efficiency, which is the product of collision efficiency and coalescence efficiency

$$E(x,y) = E_{coll}(x,y) \cdot E_{coal}(x,y)$$

$$E(x,y) = E_{coll}(x,y) \cdot E_{coal}(x,y)$$

In cloud physics we assume that the coalescence efficiency is unity ($E_{coal} = 1$).

The coalescence efficiency is uncertain to within perhaps a factor of two, and only qualitative information about the latter is known. Hence it is often assumed that uncertainities in the collision efficiency subsume the effects that might cause the coalescence efficiency to depart from unity.

Indeed present research suggests that the largest gap in our understanding of the collection efficiency is not due to a poor understanding of coalescence efficiencies, but rather in how turbulence effects modify the collision efficiencies.

Gravitational collection kernel

Important properties of the gravitational kernel are that:

- it forbids drops of equal size collecting each other (they have the same fall velocity), and
- it increases markedly with the size of the interacting drops.

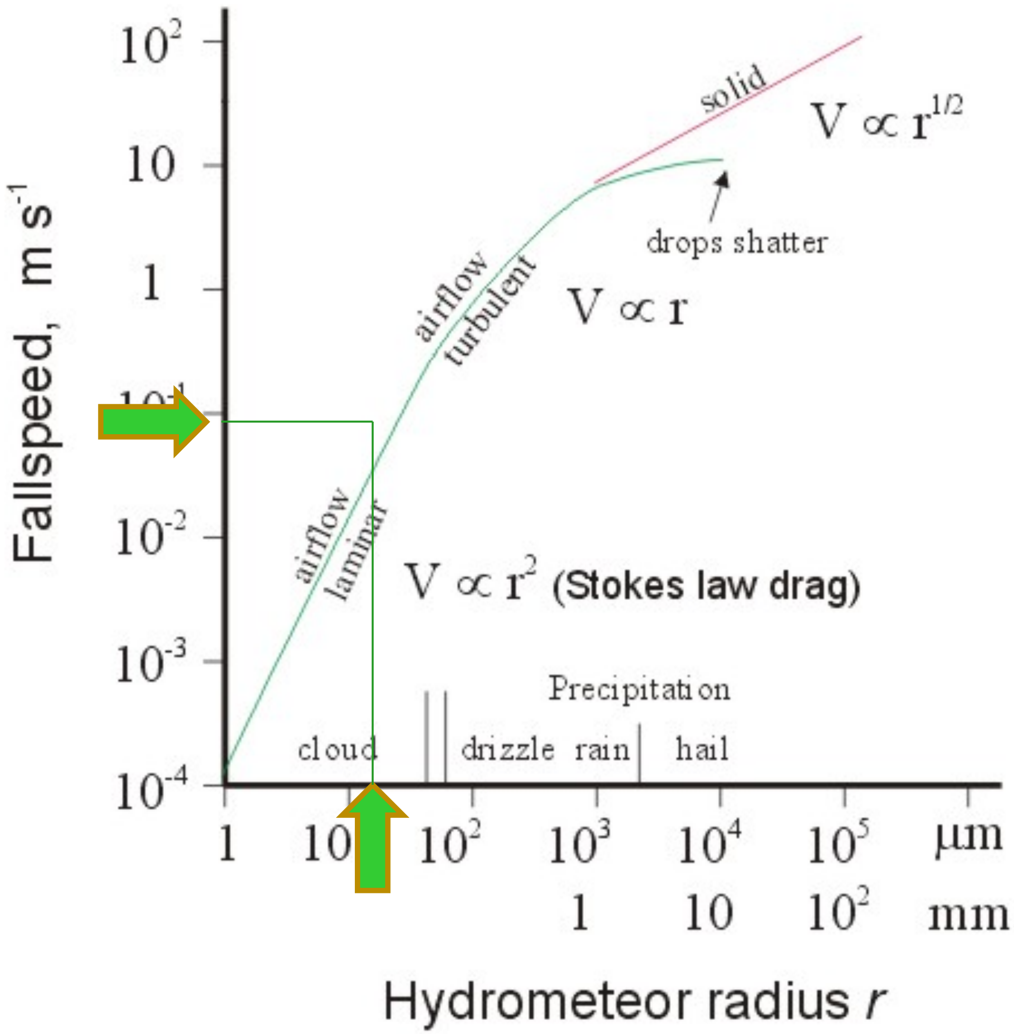
For small droplets the terminal velocity is proportional to r^2. Hence, taking $r_x > r_y$:

$$K(x,y) \propto \pi\left(r_x + r_y\right)^2 \left[V(r_x) - V\left(r_y\right)\right]$$

$$K(x,y) \propto \left(r_x + r_y\right)^2 \left(r_x^2 - r_y^2\right) = \left(1 + \frac{r_y}{r_x}\right)^2 \left[1 - \left(\frac{r_y}{r_x}\right)^2\right] r_x^4$$

The collection rate increases not only with the difference in size of the interacting drops, but also with the absolute size of the collecting drops – the latter being more important. Larger drops sweep out a larger volume and hence are more efficient collectors.

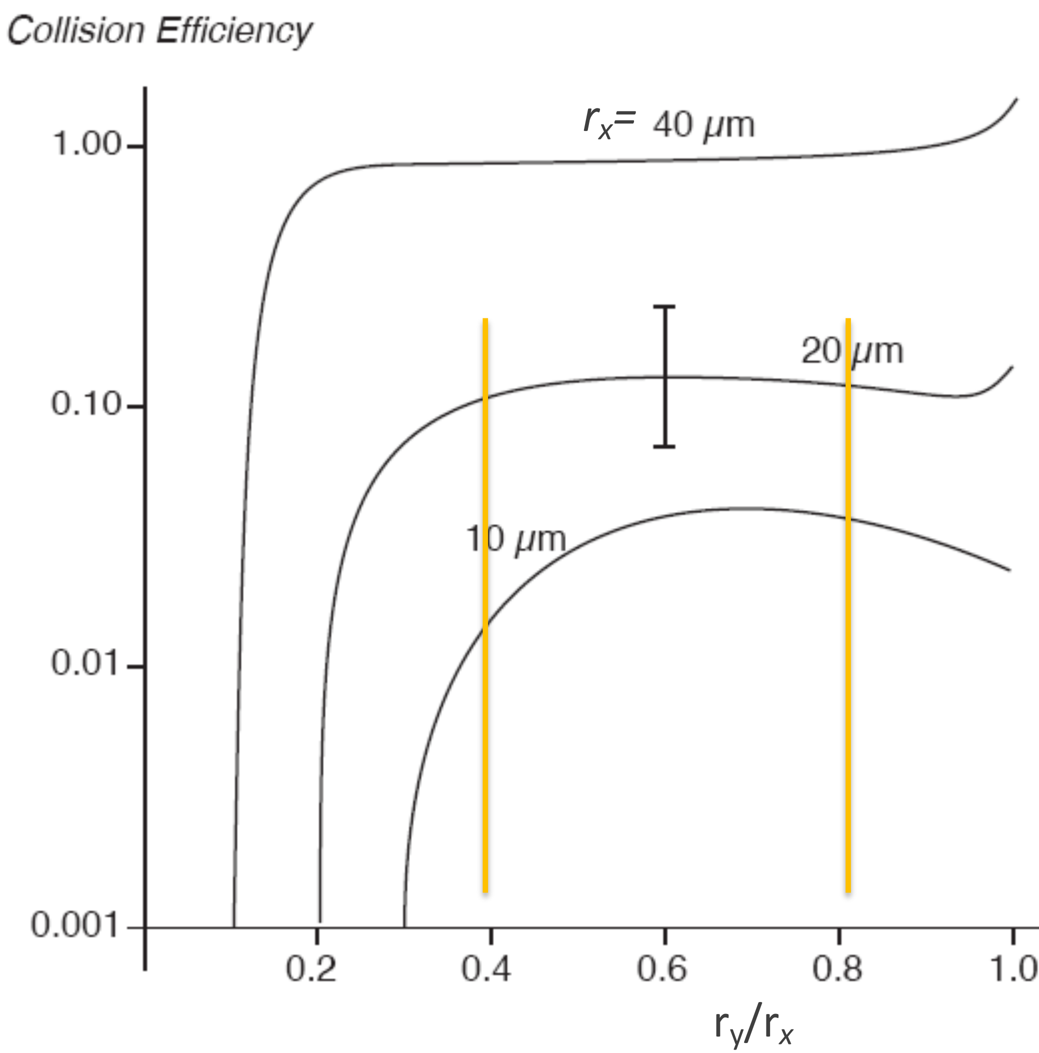
FALL SPEED



Adapted from MacIlveen (1992)

Collision efficiency

$$K(x,y) \propto (r_x + r_y)^2 (r_x^2 - r_y^2) = \left(1 + \frac{r_y}{r_x}\right)^2 \left[1 - \left(\frac{r_y}{r_x}\right)^2\right] r_x^4$$

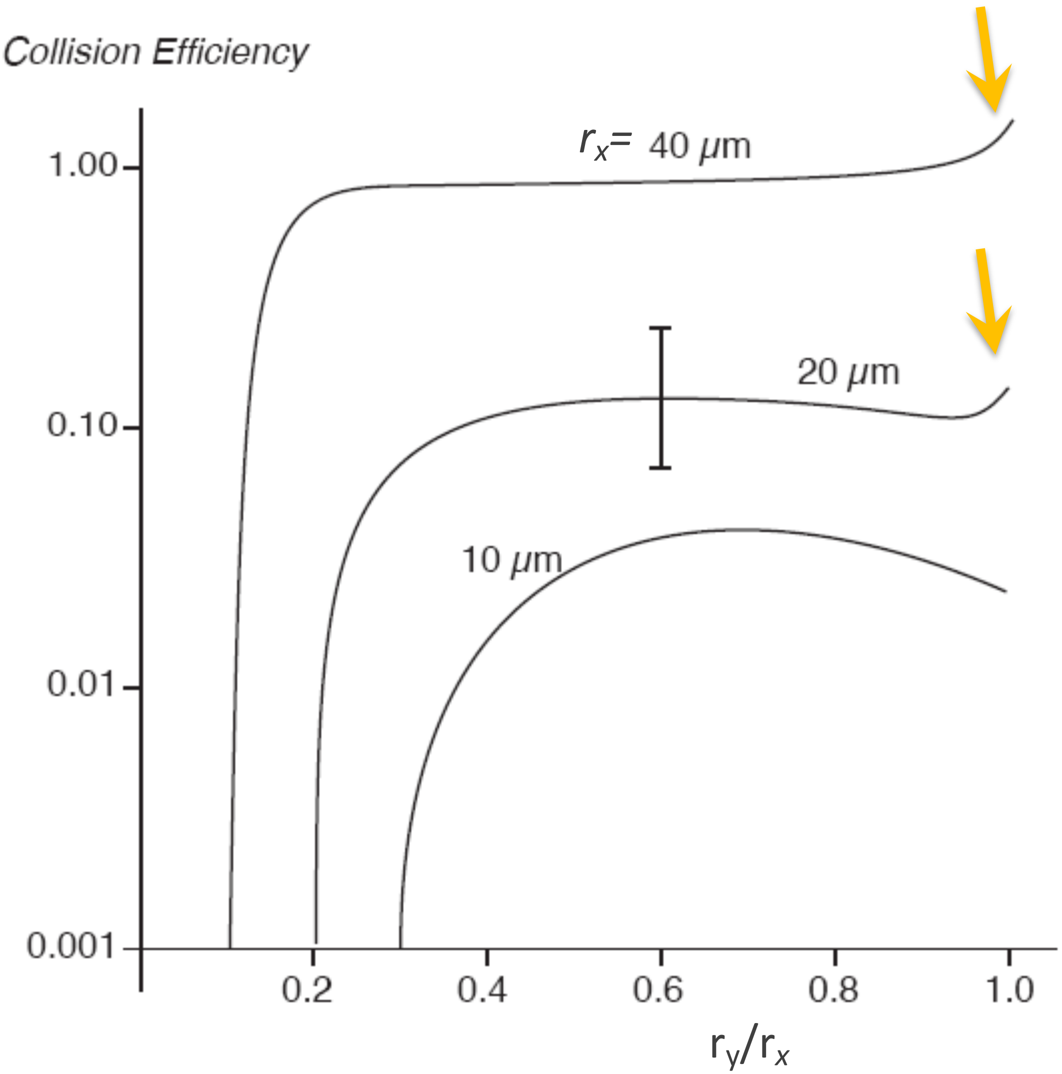


The above equation does not account for how drops, when they fall, modify the flow field around them.

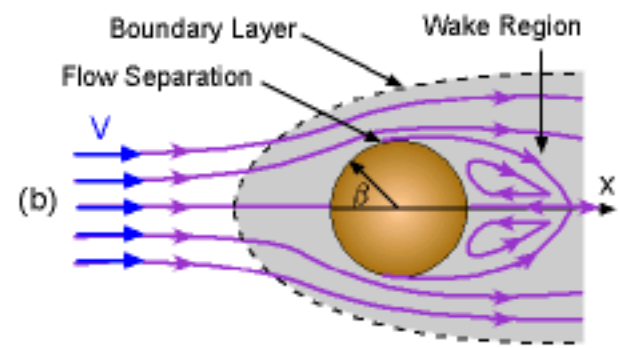
The figure illustrates the collision efficiency as understood based on theoretical/numerical studies.

For small droplets (radius smaller than 40 μm) the collision efficiency is largest for size ratios of about 0.4 to 0.8, and the collision efficiency increases dramatically with size, roughly in proportion to the square or cube of the collector droplet radius.

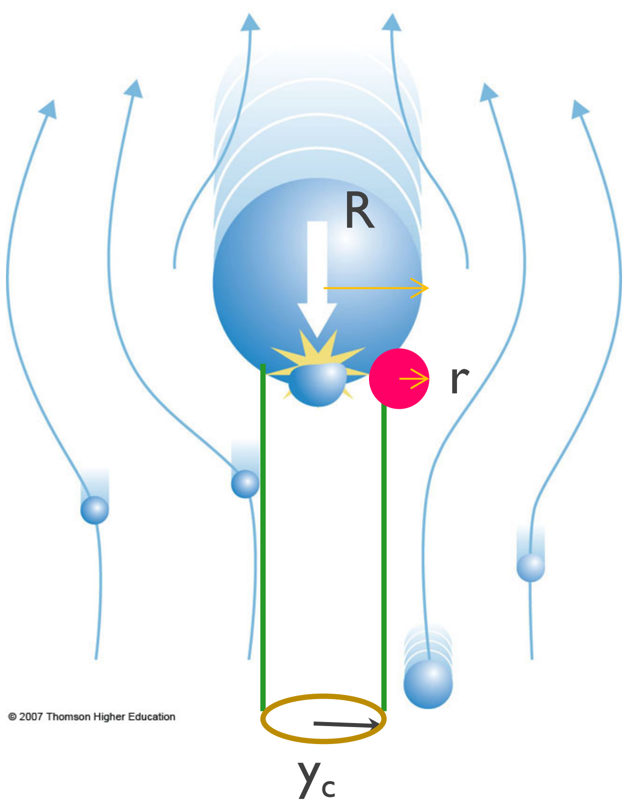
Collision efficiency

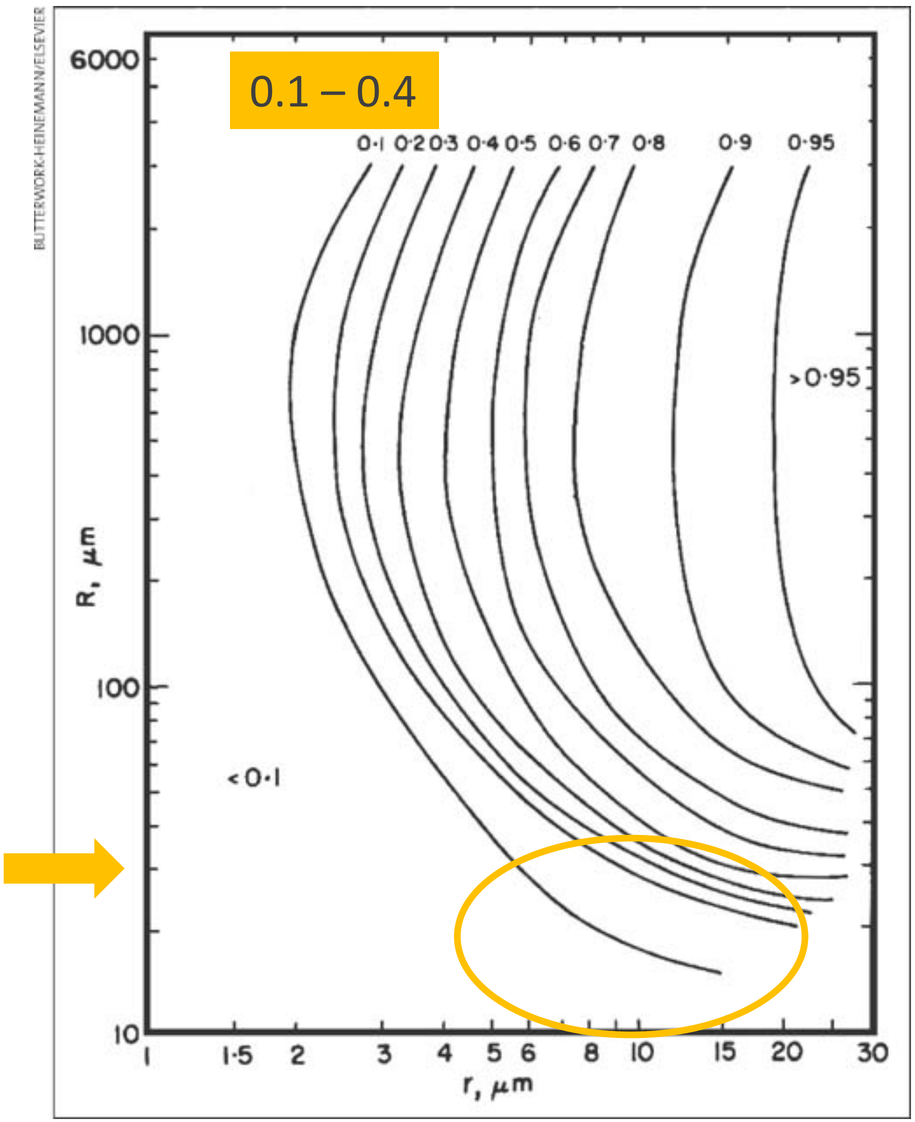


The data shows some tendency of like-sized drops to be super-efficient, which is associated with wake-capture effects, wherein the interaction of the droplet wakes induces a collision that would not have otherwise been expected



Gravitational collision efficiency





$$E_c = \frac{y_c^2}{(R+r)^2}$$

E_c is generally an increasing function of R and r, but for R greater than about 100 μm E_c depends largely on r.

Long collection kernel

In practice the collisional kernel is constructed by interpolating between fixed points given by experimental data. It has been found that a reasonable fit to this data can be obtained using polynomial functions.
Long suggested the following approximation to the kernel:

$$K(x,y) = \begin{cases} 9.44 \cdot 10^9 (x^2 + y^2), & \max(x,y) \leq 50\mu m \\ 5.78 \cdot 10^3 (x+y), & \max(x,y) > 50\mu m \end{cases}$$

x and y are droplets volume in cm^3, K is in cm^3/s.
In Long's fit to the experimental data we see that the collision efficiency increases with the square of the droplet mass for small cloud droplets.

An interesting aspect of the Long kernel is that it does not retain the property of the gravitational kerne wherein a mono-disperse droplet spectrum will not grow by collection (i.e. the kernel does not vanish when x=y).
Instead the Long kernel emphasizes the role of larger drops in accelerating the collection process, through its dependence of the droplet mass.

Collection growth equation

$$\frac{dM}{dt} = \int \left(\frac{4}{3}\pi r^3 \rho_l\right) K(R,r) n(r) dr \qquad K(x,y) = \pi\left(r_x + r_y\right)^2 \left[V(r_x) - V\left(r_y\right)\right] E(x,y)$$

$$M = \frac{4}{3}\pi \rho_l R^3$$

$$\frac{dR}{dt} = \frac{1}{4\pi\rho_l R^2}\frac{dM}{dt} = \frac{1}{3}\int \left(\frac{R+r}{R}\right)^2 \left[V(R) - V(r)\right] E(R,r) r^3 n(r) dr$$

$$R \gg r\,, \qquad V(R) \gg V(r)$$

$$\frac{dR}{dt} = \frac{\rho_d \bar{E}}{4\rho_l} r_l V(R) \qquad r_l = \frac{\rho_l}{\rho_d}\int_0^\infty \frac{4}{3}\pi\, n(r) r^3 dr$$

Drop growth rate by condensation and accretion

Gravitational collisions between cloud droplets are effective when the droplet radius reaches approximately 40 μm

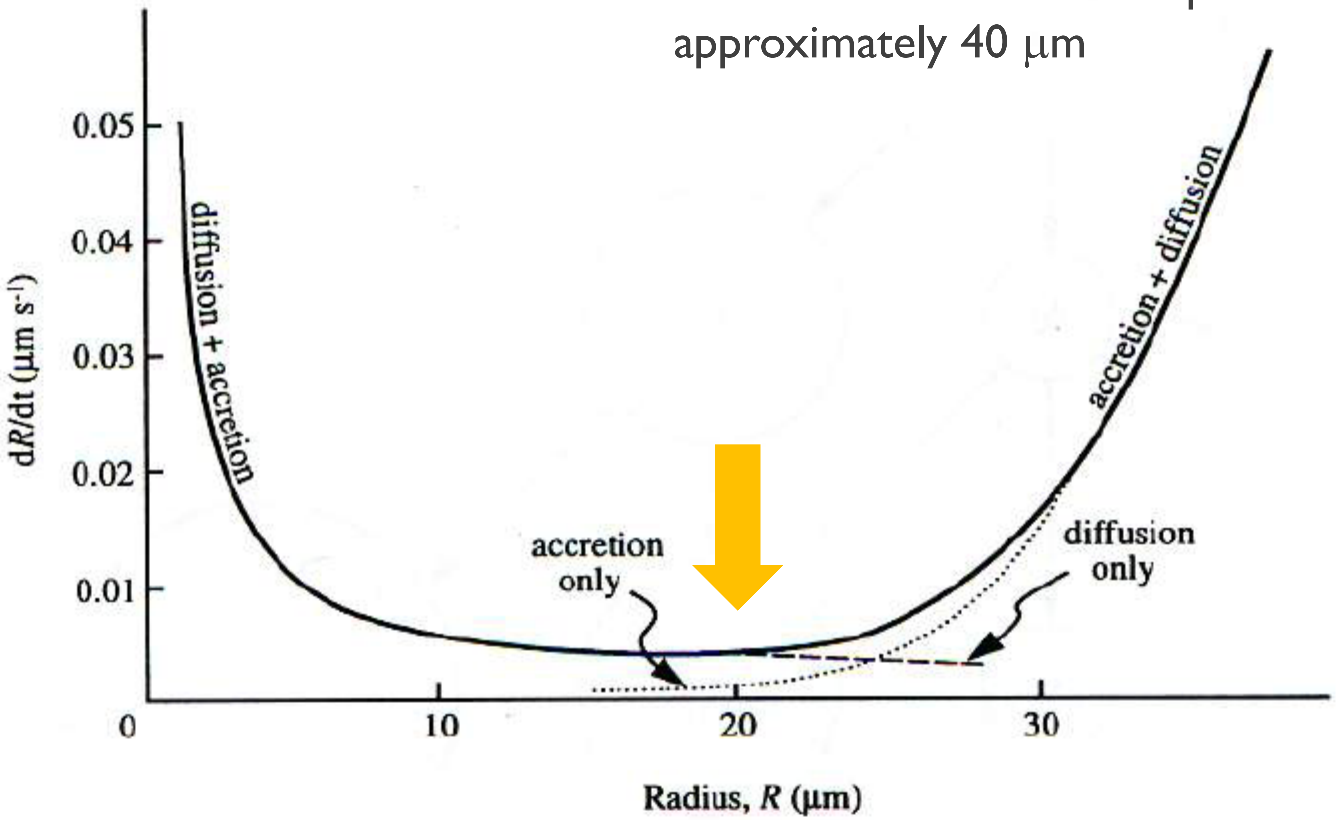


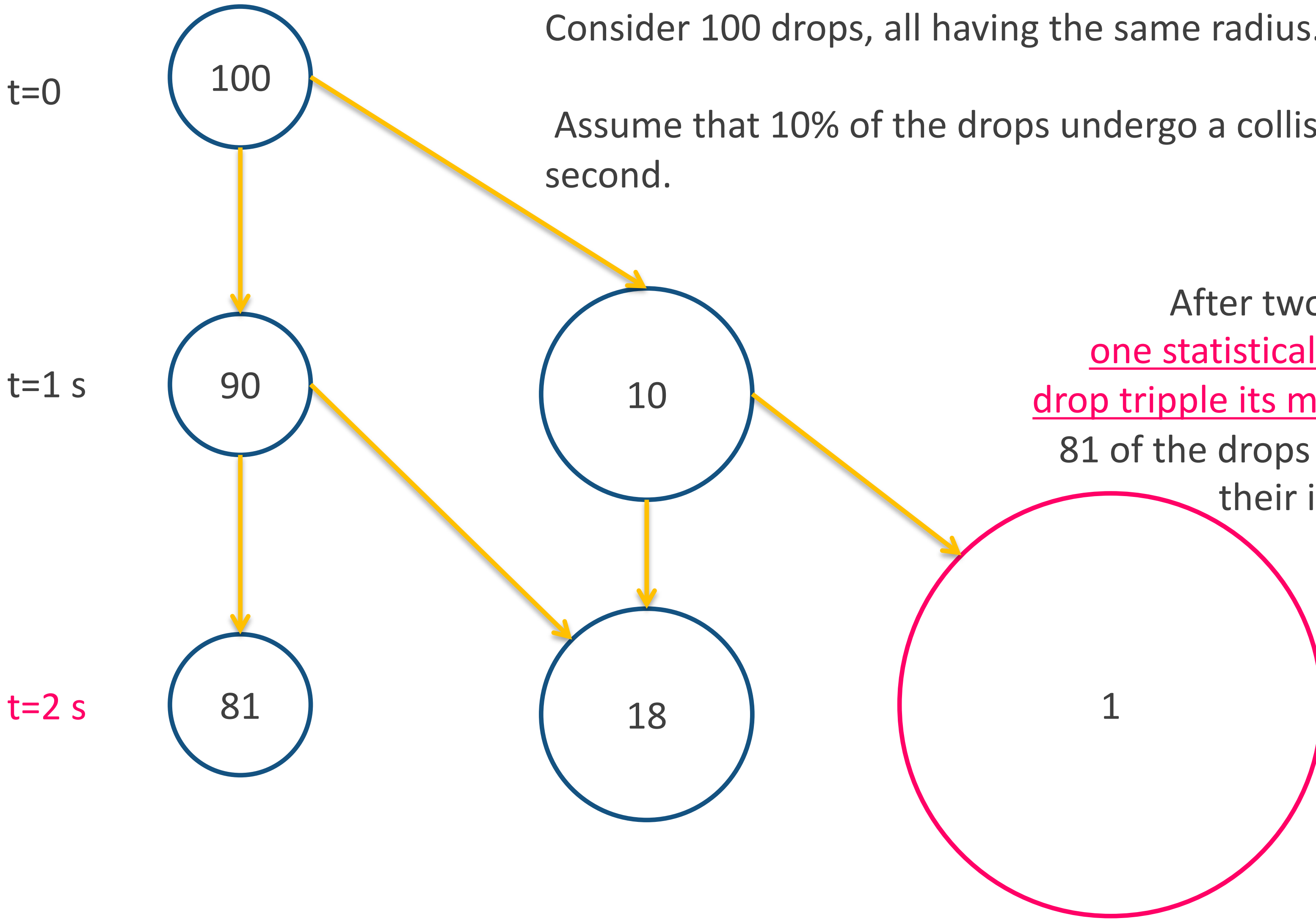
Figure 8.6 Drop growth rate by condensation and accretion. The dashed line represents growth by diffusion only, and the dotted line represents growth by accretion only, while the solid curve represents the combined growth rate. Condensational growth rate decreases with increasing radius, while accretional growth rate increases with increasing radius.

Stochastic collection model

- In the continuous collection model it is assumed that the collector particle collides in a continuous and uniform fashion with smaller cloud particles that are distributed uniformly in space.
- In reality, collisions are individual events that are statistically distributed in space and time.
- This has given rise to the stochastic collection model that accounts for the probabilistic aspects of collision and coalescence.
- Using the stochastic model, some drops are „statisticly favored" for rapid growth.

Consider 100 drops, all having the same radius.

Assume that 10% of the drops undergo a collision in one second.



After two seconds, one statistically favored drop tripple its mass, while 81 of the drops remain at their initial size.

Stochastic processes are of particular importance for the first 20 collisions, or so, because they allow the largest drops to get past the ‘gap’. After this point, there is a sufficient number of large drops and the collection becomes essentially continuous.

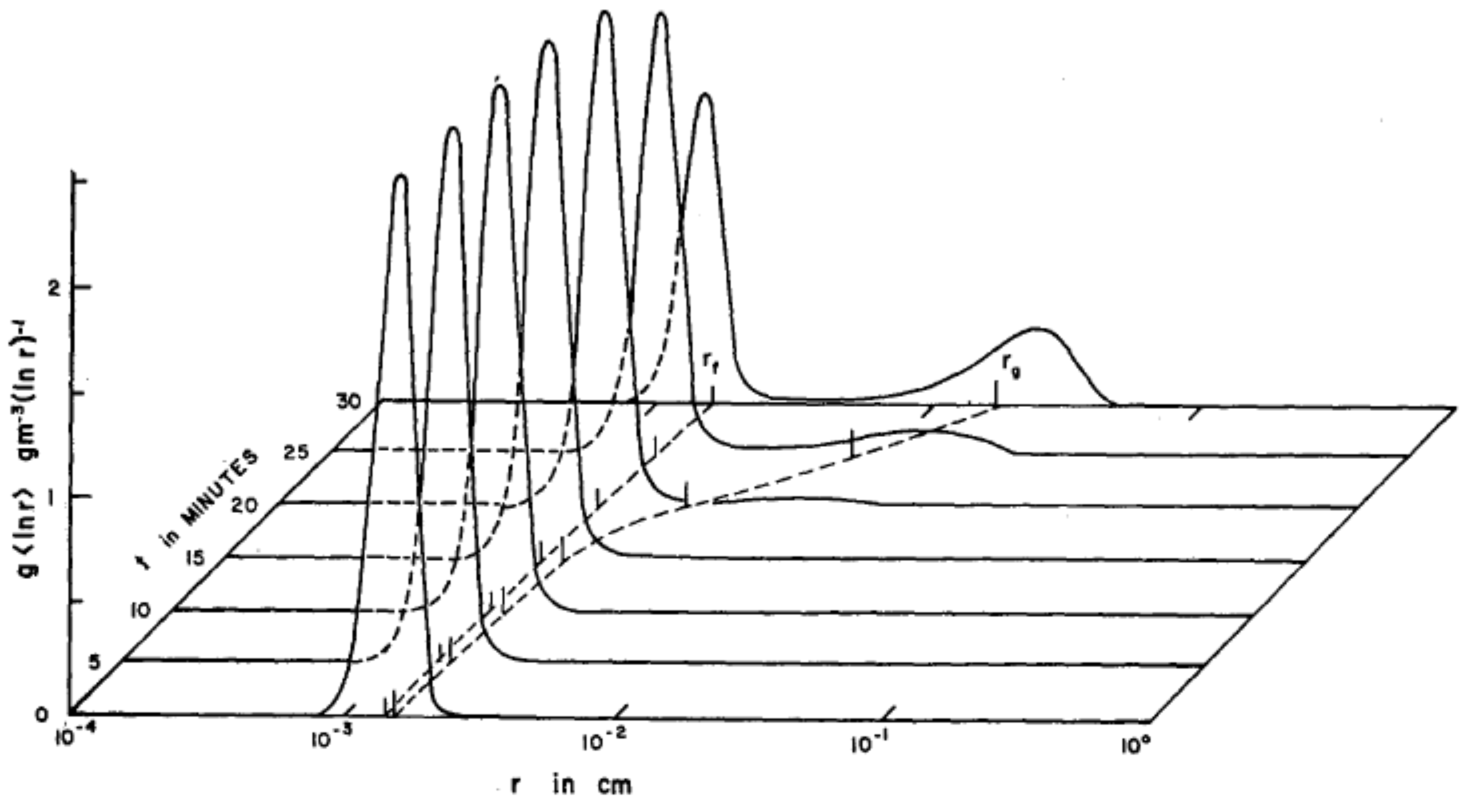


FIG. 2. Time evolution of the initial spectrum for $r_f{}^0 = 14$ μm, var $x = 0.25$.

Berry and Reinhardt, 1974

Collisions between all droplets are possible

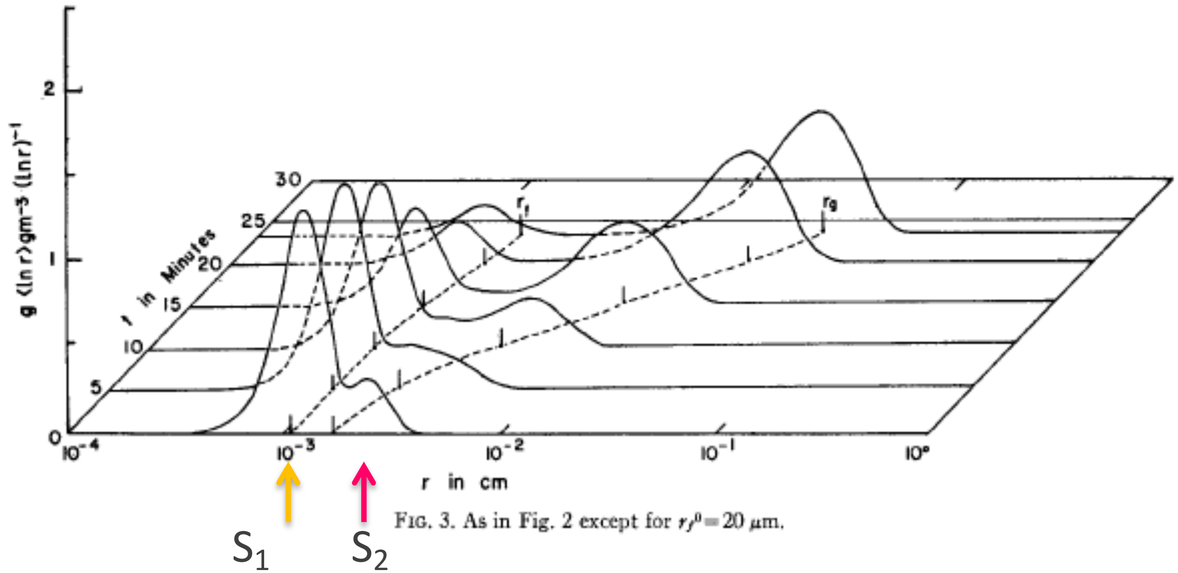


FIG. 3. As in Fig. 2 except for $r_f^0 = 20$ μm.

Collisions between S_1 droplets are allowed

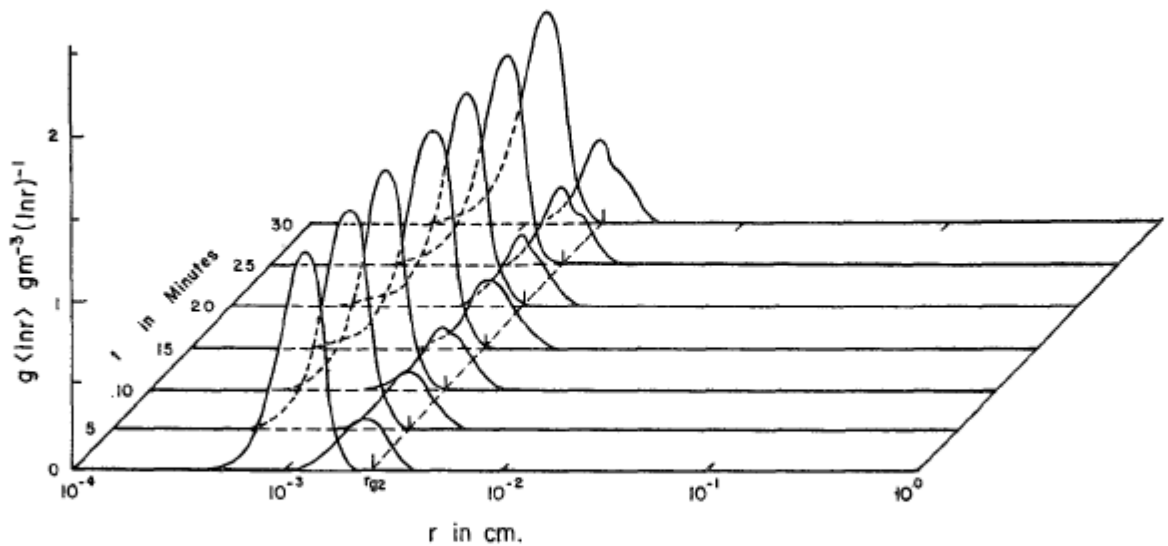


FIG. 5. Time evolution of the initial spectrum given in Fig. 3, with only S1-S1 interactions being allowed.

Collisions between S_1 and S_2 drops are possible

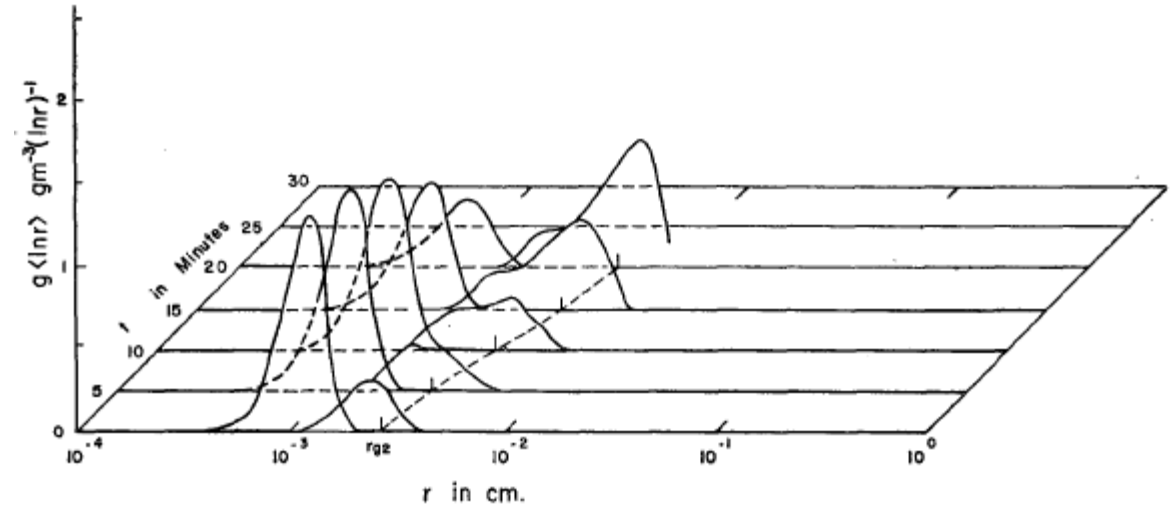


FIG. 6. Time evolution of the initial spectrum given in Fig. 3, with only S2-S1 interactions being allowed.

Collisions between S_2 drops are possible

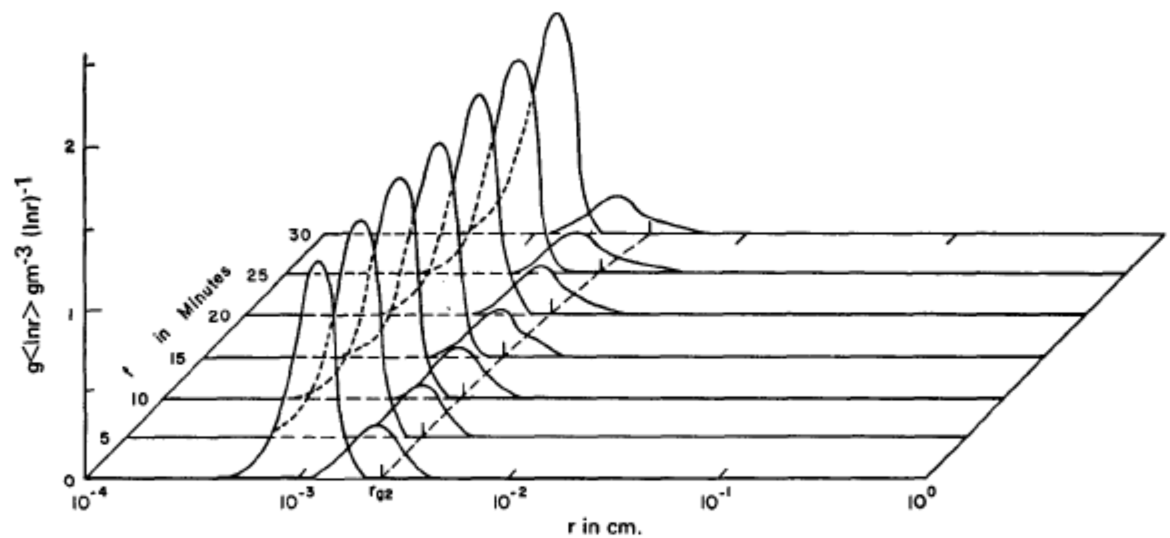
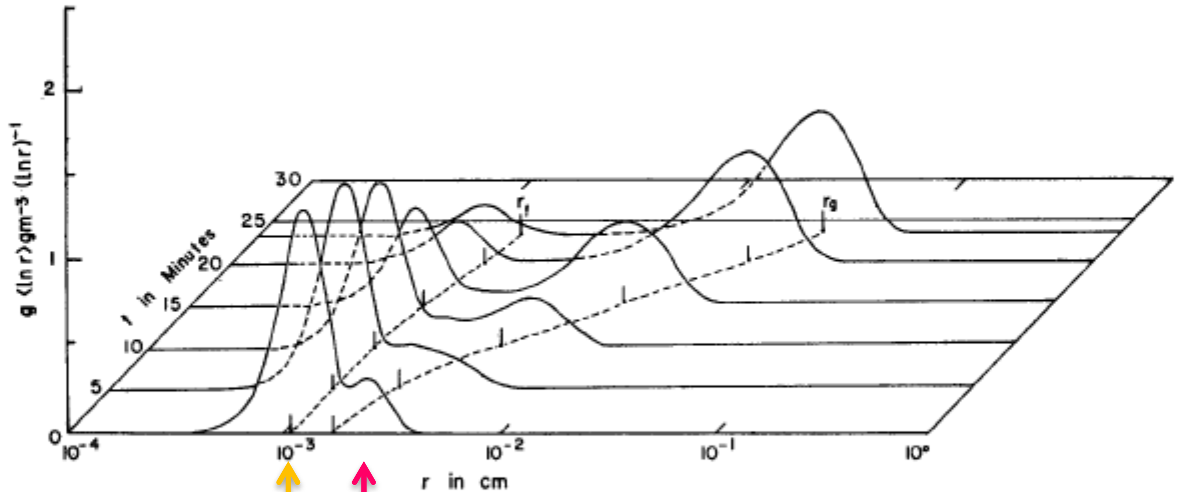


FIG. 7. Time evolution of the initial spectrum given in Fig. 3, with only S2-S2 interactions being allowed.

Collisions between all droplets are possible



S_1 S_2

FIG. 3. As in Fig. 2 except for $r_f^0 = 20$ μm.

Collisions between S_1 droplets are allowed

AUTOCONVERSION

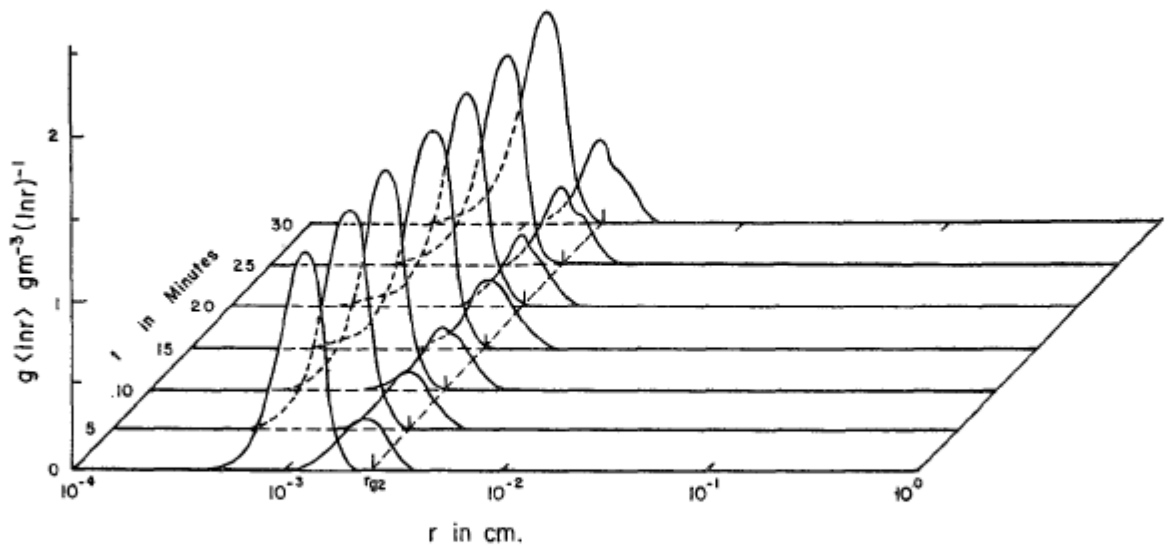


FIG. 5. Time evolution of the initial spectrum given in Fig. 3, with only S1-S1 interactions being allowed.

Collisions between S_1 and S_2 drops are possible
ACCRETION

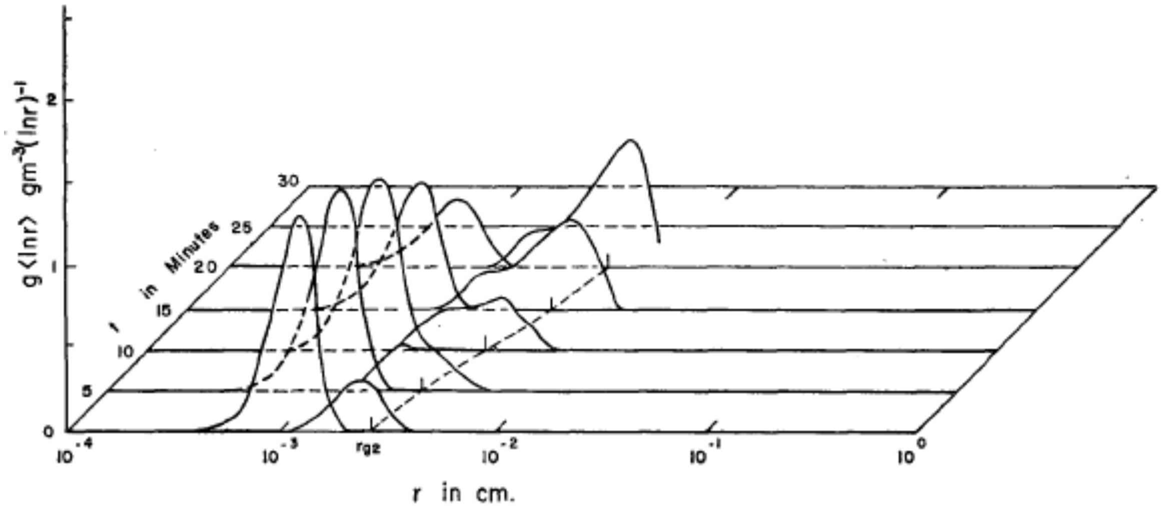


FIG. 6. Time evolution of the initial spectrum given in Fig. 3, with only S2-S1 interactions being allowed.

Collisions between S_2 drops are possible

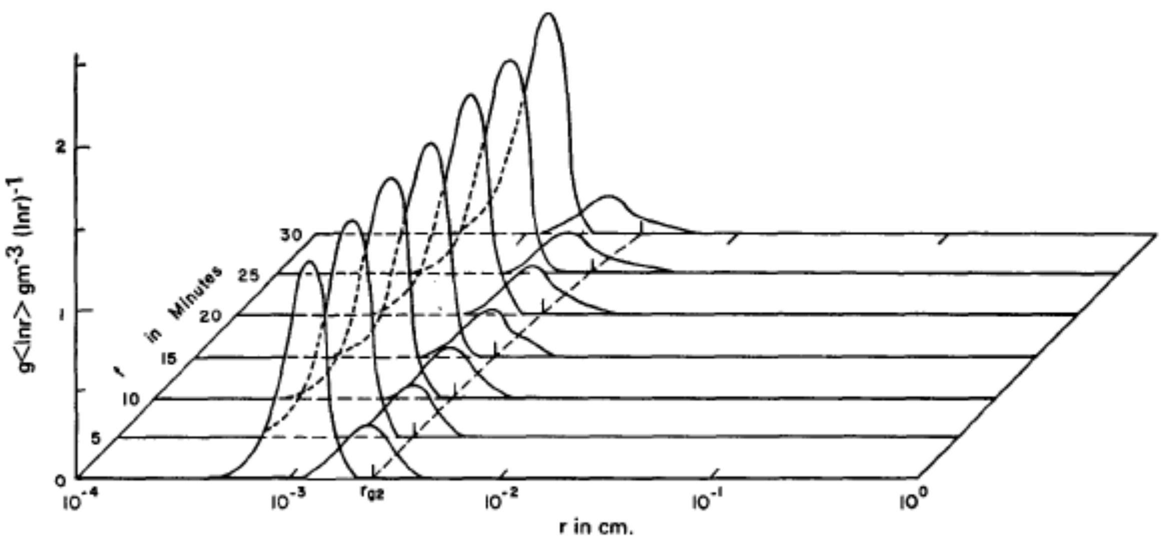
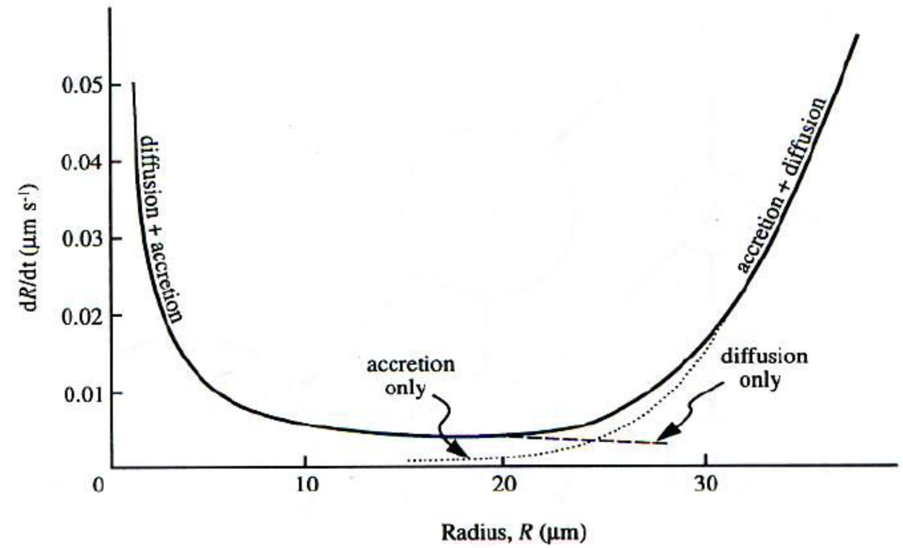


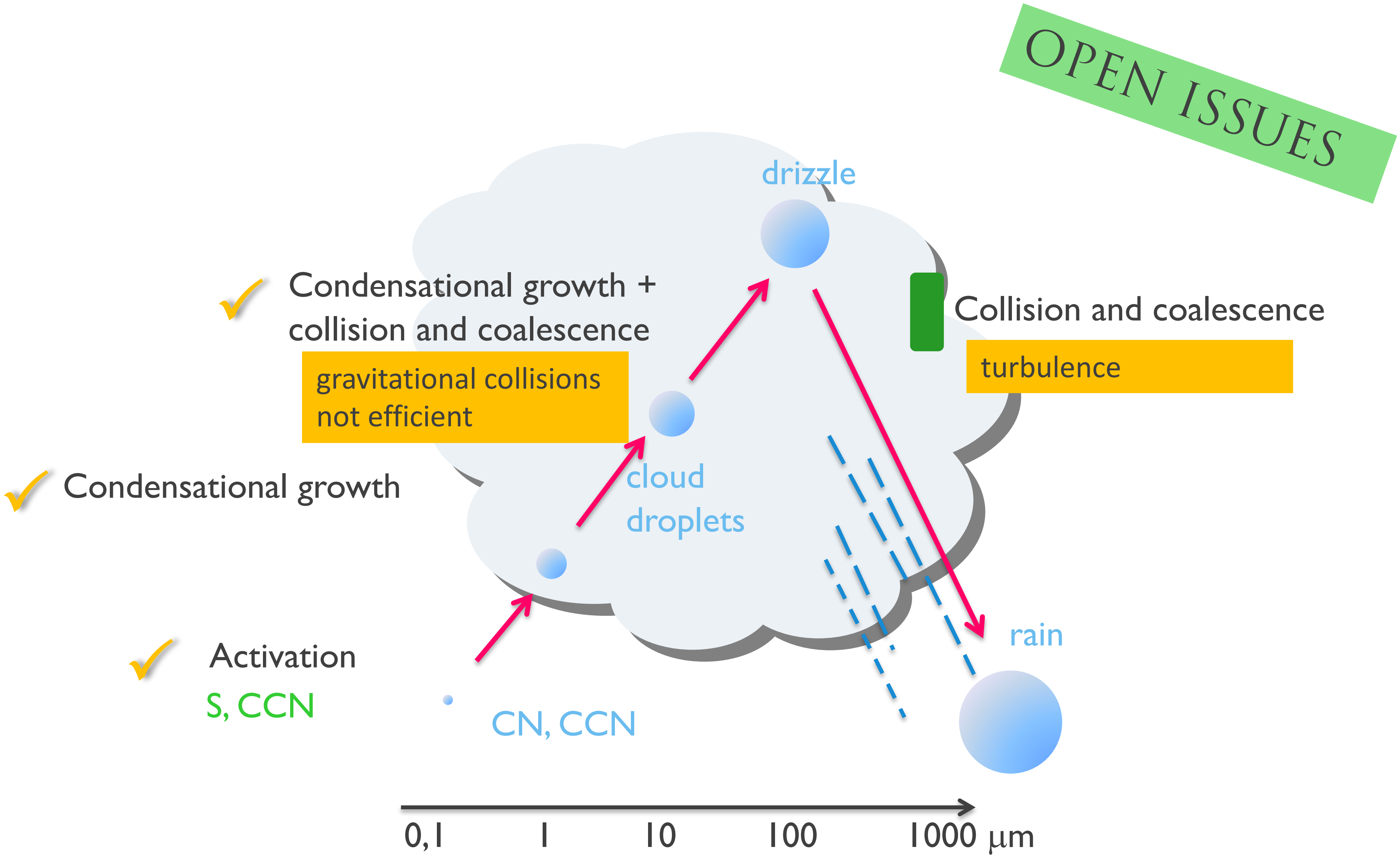
FIG. 7. Time evolution of the initial spectrum given in Fig. 3, with only S2-S2 interactions being allowed.

The size-gap problem

- It is difficult to explain the rapid growth of cloud droplets in the size range 15-40 μm in radius for which neither the diffusional mechanism nor the collision-coalescence mechanism is effective (i.e. the condensation-coalescence bottelneck or the size gap)



- Several mechanisms have been proposed, including:
  - Entrainment of dry air into the cloud
  - The effect of giant aerosol particles
  - Turbulent fluctuations of the water-vapor supersaturation
  - The turbulent collision-coalescence



Open issues
drizzle
Condensational growth +
collision and coalescence
gravitational collisions
not efficient
Collision and coalescence
turbulence
Condensational growth
cloud
droplets
rain
Activation
S, CCN
CN, CCN
0,1
1
10
100
1000 μm

Basic mechanisms of turbulent enhancement of gravitational collision/coalescence

- Turbulence modifies local droplet concentration (preferential concentration effect)
- Turbulence modifies relative velocity between droplets
- Turbulence modifies hydrodynamic interactions when two drops approach each other

Basic mechanisms of turbulent enhancement of gravitational collision/coalescence

Geometric collisions,
(no hydrodynamic interactions)

- Turbulence modifies local droplet concentration (preferential concentration effect)
- Turbulence modifies relative velocity between droplets
- Turbulence modifies hydrodynamic interactions when two drops approach each other

Basic mechanisms of turbulent enhancement of gravitational collision/coalescence

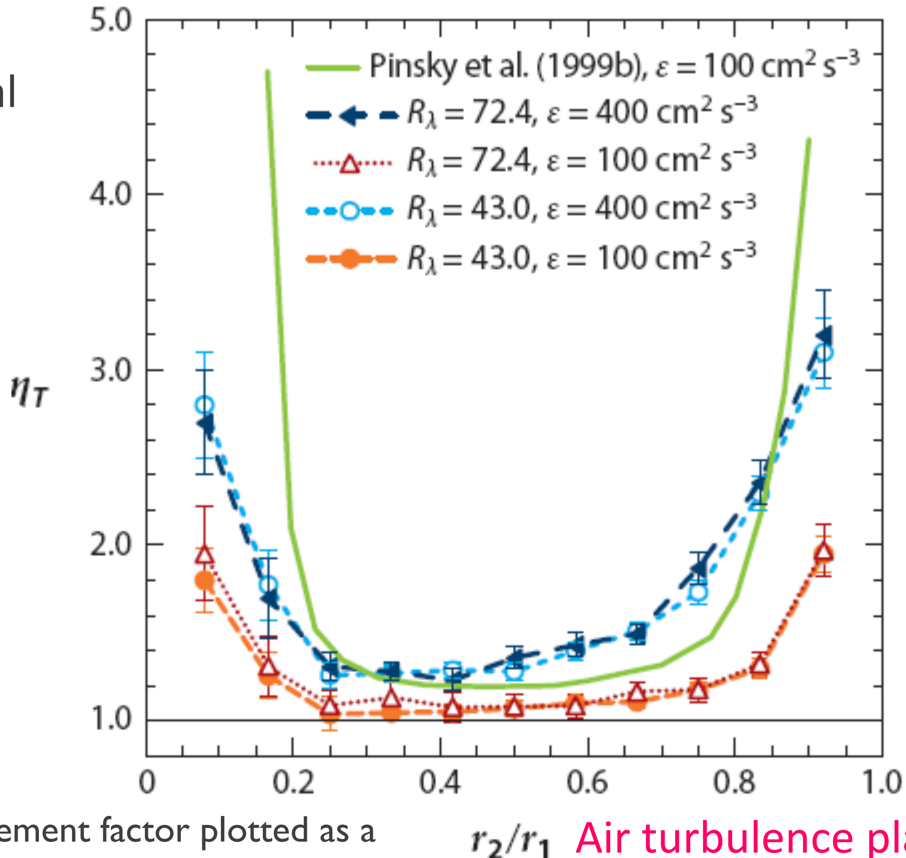
- Turbulence modifies local droplet concentration (preferential concentration effect)
- Turbulence modifies relative velocity between droplets
- Turbulence modifies hydrodynamic interactions when two drops approach each other

Collision efficiency

The effect of turbulence on collision efficiency

(the net enhancement factor – the ratio of the turbulent collection kernel and the hydrodynamic - gravitational collection kernel)

when $r_2/r_1 \ll 1$, the gravitational kernel may be small owing to small collision



when $r_2/r_1 \rightarrow 1$, the gravitational kernel is small owing to small differential sedimentation

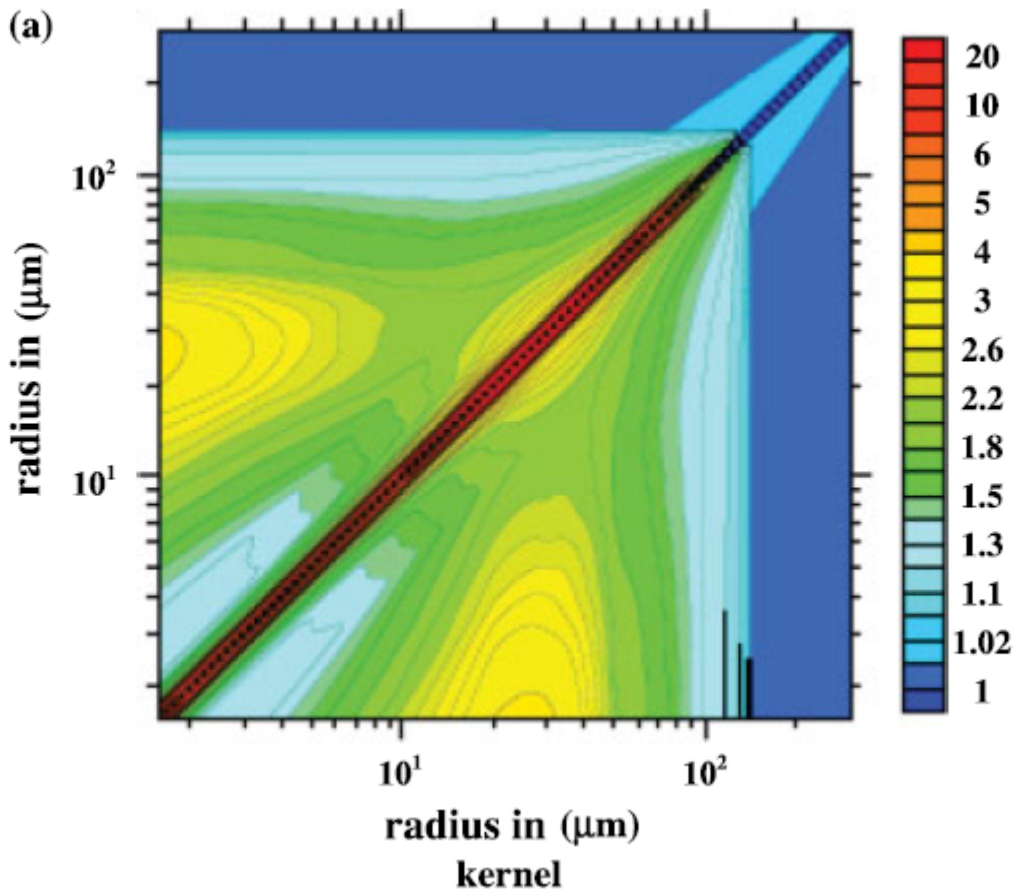
Grabowski and Wang, ARFM 2013

The net enhancement factor plotted as a function of the radius ratio r_2/r_l, with the larger droplet 30 μm in radius. ε is the flow viscous dissipation rate, and R_λ is the Taylor microscale Reynolds number of the simulated background turbulent airflow.

Air turbulence plays an important role in enhancing the gravitational collision kernel when the collision efficiency is small.

The enhancement typically ranges from 1 to 5!

Ratio of the turbulent and gravitational collection kernels

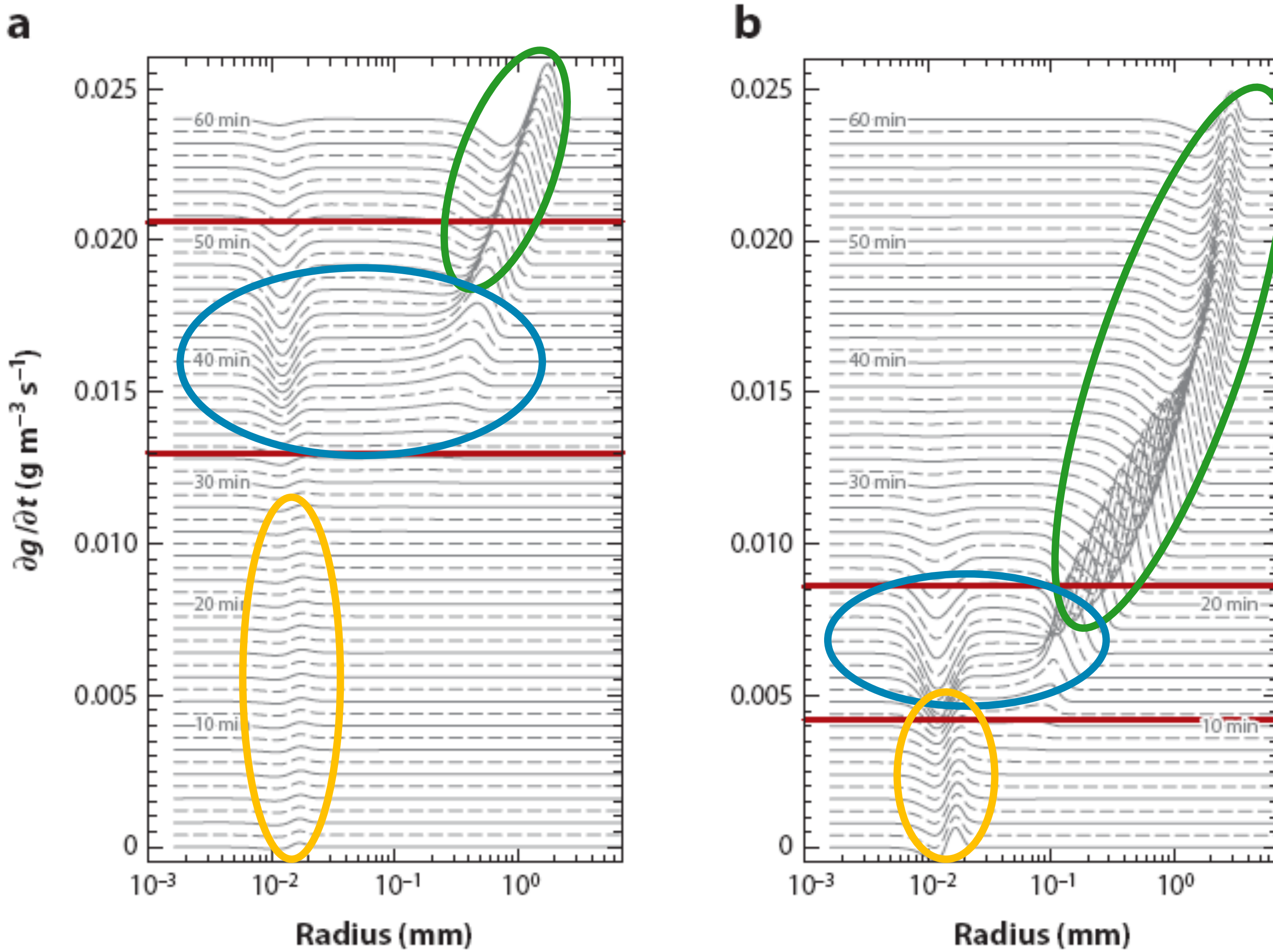


Wang and Grabowski, Atmos. Sci. Lett., 2009)

1- autoconversion

2 – accretion

3 – Hydrometeor self-collection (Berry and Reinhardt, 1974)



Without turbulence With turbulence

Xue, Y., L.P. Wang, and W. W. Grabowski, 2008)

SUMMARY

- Small-scale turbulence alone does not produce a significant broadening of the cloud-droplet spectrum during diffusional growth.
- The coupled small-scale and larger-scale turbulence, combined with larger-scale flow inhomogeneity, entrainment, and fresh activation of CCN above the cloud base , creates different growth histories for droplets. This leads to a significant spectral broadening.
- The effect of turbulence on the collision-coalescence growth is significant.
- Turbulence of moderate magnitudes leads to a significant acceleration of warm rain initiation.