

Motion of vortex lines in quantum mechanics

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Abstract In quantum theory, vortex lines arise in the hydrodynamic interpretation of the wave equation. In this interpretation, which is originally due to Madelung, the flow of the probability density for a single particle is described in terms of the hydrodynamic variables. For the sake of simplicity, the standard time-dependent Schrödinger equation, and the related vortex lines embedded in the probability fluid of the quantum particle, are considered here. A vortex line in this case is simply the curve defined by equating the wave function to zero. The linearity of the Schrödinger equation enables us to obtain a large family of exact time-dependent analytic solutions for the wave functions with vortex lines. Moreover, the method is general enough to allow for various initial configurations of the vortex lines.

Although the equation of motion of the quantum mechanical probability fluid is different in its literal form from the equations describing the real physical fluid, we believe that the evolution of the vorticity in the quantum and in the real fluid share the same qualitative features that can be described in terms of the topology of the vortex lines configurations. The general phenomena such as the switch-over, creation and annihilation of vortices can be observed in the quantum mechanical fluid.

*I'll talk about quantum switch-over
As felt going from Calais to Dover;
The vortex lines shift
Reconnect and then lift ...
I'm sorry my lecture's now over!*

1. Introduction

The principle of probability conservation in non-relativistic quantum mechanics gives rise to a hydrodynamic interpretation of the quantum