Finite time singularities in a class of hydrodynamic models

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Abstract Models of inviscid incompressible fluid are considered, with the kinetic energy (i.e., the Lagrangian functional) taking the form $\mathcal{L} \sim \int k^{\alpha}|\mathbf{v}_{\mathbf{k}}|^2 d^3\mathbf{k}$ in 3D Fourier representation, where α is a constant, $0 < \alpha < 1$. Unlike the case $\alpha = 0$ (the usual Eulerian hydrodynamics), a finite value of α results in a finite energy for a singular, frozen-in vortex filament. This property allows us to study the dynamics of such filaments without the necessity of a regularisation procedure for short length scales. The linear analysis of small symmetrical deviations from a stationary solution is performed for a pair of anti-parallel vortex filaments and an analog of the Crow instability is found at small wave-numbers. A local approximate Hamiltonian is obtained for the nonlinear long-scale dynamics of this system. Self-similar solutions of the corresponding equations are found analytically. They describe the formation of a finite time singularity, with all length scales decreasing like $(t^* - t)^{1/(2-\alpha)}$, where t^* is the singularity time.

We consider the fate of string links,
That can tangle with all sorts of kinks;
I won't try to disguise
What we regularise;
It's more subtle than anyone thinks!

1. Introduction

This talk is based on the recent article by Ruban, Podolsky, & Rasmussen (2001). Here we take the point of view that infinite curvature of frozen-in vortex lines is in some sense a more fundamental characteristics of hydrodynamic singularity than infinite value of the vorticity maximum. To illustrate this statement, we consider a class of models of an incompressible inviscid fluid, different from Eulerian hydrodynamics, such that finite energy solutions with infinitely thin frozen-in vortex filaments of finite strengths are possible. Thus, we deal with a situation