Discrete groups, symmetric flows and hydrodynamic blowup

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Abstract Discrete group theory is applied to certain well-known flows with discrete symmetries. A flow is associated with particular discrete point or space group if flow components possess the same symmetries as the irreducible representations of the group. The flows examined are or have been considered candidates for finite-time blowup. The properties of the groups are related to the characteristics of the flows.

Singularity? Yes, I conceive it,
The problem is how to achieve it;
With array octahedral
Like a modern cathedral –
When you see it, you better believe it!

1. Introduction

This work is motivated by the observation that most of the incompressible flows that have been suggested as candidates for spontaneous generation of finite-time singularities possess some discrete symmetries. The theory of discrete point and space groups, which has been useful in categorising and predicting properties of molecules and crystals, is introduced here with the goal of doing the same for symmetric flows.

A flow will be defined as "being associated with" a discrete group if the flow variables, the components of velocity and vorticity and pressure, can be related to the group's irreducible representations, a term defined in the next section.

The properties of discrete groups and associated flows, may be correlated with finite-time collapse in the flows. It may help answer the question of whether flow solutions have global regularity or blowup in finite time, which is an open question for solutions of the Navier Stokes and Euler for incompressible flows.