Corotating five point vortices in a plane

Tatsuyuki NAKAKI

Faculty of Mathematics, Kyushu University
6–10–1 Hakozaki, Fukuoka 812–8581 JAPAN
nakaki@math.kyushu-u.ac.jp

Abstract Let us consider the motion of assembly of point vortices in the two-dimensional incompressible ideal fluid. The motion is described by an ordinary differential equation, and we focus our attention on the relative equilibria which imply the corotation of vortices. In this paper, we analyse the stability of the equilibria when five point vortices satisfy some condition on the strength and initial configuration. When parameters, which appear in the above condition, belong to some range, we show that the equilibria are unstable and that the solution, which locates near the equilibria at the initial time, exhibits the relaxation oscillation. The stable equilibria appear in a *narrow* parameter range. For some equilibria, the stability is shown by a computer-assisted proof.

On the problem of vortices five,
For long I've continued to strive;
Sometimes they're stable,
And then I am able
To show how they jiggle and jive.

1. Introduction

We consider the motion of assembly of point vortices in the two-dimensional Euler fluid. When several vortices are in the fluid, every vortex drifts away with the flow due to the other vortices. Such a phenomenon is described by the following ordinary differential equation when the fluid occupies the whole plane:

$$\frac{d}{dt}\overline{z_j} = \frac{1}{2\pi i}\sum_{k\neq j}\frac{\Gamma_k}{z_j - z_k}, \tag{1}$$

where $z_j = z_j(t)$ and Γ_j are the complex position at time t and the strength of jth vortex, respectively. The complex conjugate of z is denoted by $\overline{z}$ and $i = \sqrt{-1}$.

The equation (1) is analysed for long times and many results are already known. We briefly summarise some of them. When two point vortices are in the fluid, the motion of vortices is easily analysed as is