A third-order topological invariant for three magnetic fields

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If you notice a third-order link
You can colour it green, brown and pink;
Invariants follow,
As hills follow hollow;
So a field triply-linked cannot shrink.

Abstract The topology of divergence-free fields is important in many parts of physics, e.g. in magnetohydrodynamics, plasma physics, hydrodynamics, superfluids etc. With the focus on applications in magnetohydrodynamics, our principal aim is the characterisation of magnetic fields by the means of invariants.

In this report an introduction to the problem of finding higher order invariants is given. Then a third-order link integral of three magnetic fields is presented, which can be shown to be a topological invariant and therefore an invariant in ideal magnetohydrodynamics. This integral generalises the known third-order link invariant derived from the Massey triple product, which could only be applied to isolated flux tubes. As an example three magnetic fields not confined to flux tubes are given that possess a third-order linkage.

1. Introduction

In recent years topological considerations have become increasingly important in the study of physical problems. In plasma physics for example the topology of magnetic fields plays an important role when dealing with problems such as the stability and time evolution of a plasma or when estimating its energy content.

The structure of magnetic fields is studied with the aid of topological measures of complexity, i.e. measures which are invariant under diffeo-

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