Simulation of vortex sheet roll-up: chaos, azimuthal waves, ring merger

Robert KRASNY[1], Keith LINDSAY[2] & Monika NITSCHE[3]

[1] *University of Michigan, Department of Mathematics, Ann Arbor, Michigan 48109-1109 USA*
krasny@umich.edu

[2] *National Center for Atmospheric Research, Climate and Global Dynamics Boulder, Colorado 80307-3000 USA*

[3] *University of New Mexico, Department of Mathematics and Statistics Albuquerque, New Mexico 87131-1141 USA*

Abstract This article reviews some recent simulations of vortex sheet roll-up using the vortex blob method. In planar and axisymmetric flow, the roll-up is initially smooth but irregular small-scale features develop later in time due to the onset of chaos. A numerically generated Poincaré section shows that the vortex sheet flow resembles a chaotic Hamiltonian system with resonance bands and a heteroclinic tangle. The chaos is induced by a self-sustained oscillation in the vortex core rather than external forcing. In three-dimensional flow, an adaptive treecode algorithm is applied to reduce the CPU time from $O(N^2)$ to $O(N \log N)$, where N is the number of particles representing the sheet. Results are presented showing the growth of azimuthal waves on a vortex ring and the merger of two vortex rings.

Vortex blob methods discrete,
Applied to roll-up of a sheet,
Will persuade any cynic
That heteroclinic
Tangles give insights quite neat.

1. Introduction

Vortex sheets are commonly used in fluid dynamics to model thin shear layers in slightly viscous flow. This article reviews some recent simulations of vortex sheet roll-up in planar, axisymmetric, and three-dimensional flow Krasny & Nitsche 2001; Lindsay & Krasny 2001. Vortex sheet simulations encounter difficulties due to Kelvin-Helmholtz instability and singularity formation Moore 1979 and the present work deals with these issues by applying the vortex blob method Chorin & Bernard 1973; Anderson 1985; Krasny 1987. This approach regularises