Evidence for singularity formation in a class of stretched solutions of the equations for ideal MHD

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Abstract A class of stretched solutions of the equations for incompressible, ideal $3D$-MHD are studied using Elsasser variables $\mathbf{V}^{\pm} = \mathbf{U} \pm \mathbf{B}$. This class takes the form $\mathbf{V}^{\pm} = \left(\mathbf{v}^{\pm},\, v_3^{\pm}\right)$ where $\mathbf{v}^{\pm} = \mathbf{v}^{\pm}(x,y,t)$ and $v_3^{\pm}(x,y,z,t) = z\gamma^{\pm}(x,y,t) + \beta^{\pm}(x,y,t)$. The chosen domain is of a tubular form which is infinite in the z-direction with periodic cross-section. This follows a previous study by the authors on this same class of solutions for the $3D$ Euler equations. In both cases the systems are of infinite energy. Strong numerical evidence for a finite time singularity in the Euler case was subsequently confirmed by a rigorous analytical proof by Constantin. In the MHD case, pseudo-spectral computations of the $2D$ partial differential equations for $\gamma^{\pm}$, $\mathbf{v}^{\pm}$ and $\beta^{\pm}$ valid on the cross-sectional domain provide evidence for a finite time blow-up in both the fluid and magnetic variables although an analytical proof for the existence of this singularity remains elusive.

We consider the problem of blow-up,
The vortices rapidly grow up;
They flourish like petals
On flowers among nettles;
You can purchase them down at the Co-op.

1. Introduction

The question whether a singularity develops in the vorticity field of the three-dimensional incompressible Euler equations is an important one. The vorticity $\boldsymbol{\omega}$ undoubtedly accumulates into local structures such as vortex tubes and sheets, but whether this process occurs sufficiently rapidly for $\boldsymbol{\omega}$ to become singular in a finite time has been an issue which has yet to be settled conclusively. Numerical and analytical evidence suggests that the close interaction of vortex tubes at very strong angles