Linear stability of a vortex ring revisited

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Abstract We revisit the stability of an elliptically strained vortex and a thin axisymmetric vortex ring, embedded in an inviscid incompressible fluid, to three-dimensional disturbances of infinitesimal amplitude. The results of Tsai & Widnall (1976) for an elliptically strained vortex are simplified by providing an explicit expression for the disturbance flow field. A direct relation is established with the elliptical instability. For Kelvin's vortex ring, the primary perturbation to the Rankine vortex is a dipole field. We show that the dipole field causes a parametric resonance instability between axisymmetric and bending waves at intersection points of the dispersion curves. It is found that the dipole effect predominates over the straining effect for a very thin core. The mechanism is attributable to stretching of the disturbance vortex lines in the toroidal direction.

When water is pushed through a hole,
The ring vortex plays a key role;
When the core is quite thin,
Res'nant waves are packed in;
To grasp them, why, that is the goal!

1. Introduction

Vortex rings are invariably susceptible to wavy distortions, leading sometimes to violent wiggles and eventually to disruption. We revisit the linear stability problem of a thin vortex ring. It is widely accepted that the *Moore-Saffman-Tsai-Widnall instability* is responsible for genesis of unstable waves. Remember that this is an instability for a straight vortex tube subjected to a straining field in a plane perpendicular to the tube axis (Moore & Saffman 1975, Tsai & Widnall 1976).

When viewed locally, a thin vortex ring looks like a straight tube. For simplicity, we restrict our attention to the *Rankine vortex*, a circular core of uniform vorticity. The Rankine vortex supports a family of neutrally stable three-dimensional waves of infinitesimal amplitude, being