A singularity-free model of the local velocity gradient and acceleration gradient structure of turbulent flow

Brian CANTWELL

Department of Aeronautics and Astronautics,
Stanford University
cantwell@stanford.edu

Abstract Research on the fine scale structure of turbulence has led to a greatly improved understanding of the basic geometry of the local flow patterns associated with kinetic energy dissipation. One model of the local flow that has been considered previously is based on a simplification of the transport equation for the velocity gradient tensor called the Restricted Euler Equation. This equation is exactly solvable and, although the solution reproduces many of the geometrical features observed in direct numerical simulations of turbulence, the solution also exhibits a finite time singularity. It is well known that the velocity and acceleration gradients in free turbulent flows actually decrease continuously with time when measured by a Lagrangian observer. For example, an observer convecting with the flow on the dividing streamline of an ensemble averaged turbulent plane mixing layer would measure large scale gradients that decrease in proportion to $1/time$ and microscale gradients that decrease like $1/\sqrt{time}$ (Cantwell 1981). The power law in time associated with this decay can generally be estimated using dimensional analysis together with classical balances relating turbulent kinetic energy production and dissipation. This paper will describe a procedure for removing the singularity in the Restricted Euler model while maintaining the convenience of an exact solution. The resulting system is useful for generating large ensembles for statistical modelling. The new model is matched to decay rates derived from dimensional analysis and accurately predicts many of the geometrical features of both the velocity and acceleration gradient tensors. Probability density functions for both gradient fields generated by the model are compared with results from direct numerical simulation.

1. Introduction

Once the Reynolds number of a viscous flow is large enough to produce instability and once the amplitude of the instability is large enough to produce turbulence then further amplification ceases and the overall behaviour of the flow tends to be independent of the viscosity. Define

u_0 – Integral velocity scale characterising the overall motion
δ – Integral length scale characterising the overall motion