Sufficient condition for finite-time singularity and tendency towards self-similarity in a high-symmetry flow

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Abstract A highly symmetric Euler flow, first proposed by Kida (1985), and recently simulated by Boratav and Pelz (1994) is considered. It is found that the fourth order spatial derivative of the pressure (p_{xxxx}) at the origin is most probably positive. It is demonstrated that if p_{xxxx} grows fast enough, there must be a finite-time singularity (FTS). For a random energy spectrum $E(k) \propto k^{-\nu}$, a FTS can occur if the spectral index $\nu < 3$. Furthermore, a positive p_{xxxx} has the dynamical consequence of reducing the third derivative of the velocity u_{xxx} at the origin. Since the expectation value of u_{xxx} is zero for a random distribution of energy, an ever decreasing u_{xxx} means that the Kida flow has an intrinsic tendency to deviate from a random state. By assuming that u_{xxx} reaches the minimum value for a given spectral profile, the velocity and pressure are found to have locally self-similar forms similar in shape to what are found in numerical simulations. Such a quasi self-similar solution relaxes the requirement for FTS to $\nu < 6$. A special self-similar solution that satisfies Kelvin's circulation theorem and exhibits a FTS is found for $\nu = 2$.

On the singular blow-up of Euler,
I am an inveterate toiler;
I look near the null,
You may think this is dull,
But there's no need to be such a spoiler!

1. Introduction

Consider the three-dimensional (3D) Euler equation

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p, \tag{1}$$

with the divergence-free condition $\nabla \cdot \mathbf{v} = 0$. The self-consistent pressure p must satisfy $\nabla^2 p = -\nabla \cdot (\mathbf{v} \cdot \nabla \mathbf{v})$. An important question is whether the solution of (1) can become singular in finite time for a smooth initial condition with finite energy. Some useful and rigorous constraints on