Asymptotic structure of fast dynamo eigenfunctions

B. J. BAYLY

Mathematics Dept., University of Arizona
Tucson AZ 85721, USA
bjb@math.arizona.edu

Abstract The eigenfunctions of the kinematic dynamo problem exhibit complicated spatial structure when the magnetic diffusivity is small. When the base flow is spatially periodic, we may study this structure by examining the Fourier components of the eigenfunction at large wavevectors. In this regime we may seek a WKB form in terms of slowly-varying functions of wavevector. The resulting hierarchy of equations may be systematically analysed for both zero and small nonzero diffusivities.

Eigenfunctions of dynamos fast
Have fine structure of gradient vast;
I ask you to gaze
At equations of phase,
And singular things that don't last.
—HKM 2001

A Professor in old Zakopane
Wrote limericks clever and fane,
Then read them aloud
To an exuberant crowd
Whose response was to throw lots of mane.
—BJB 2001

1. Introduction

This symposium is devoted to understanding the origin and role of the near-singular events and structures observed in real flows and numerical simulations. The direct approach, pursued by most participants at this meeting, is to start from nonlinear evolution equations (exact or approximated or modelled) and ask under what circumstances the solutions become infinite at some points in space and time.

An alternative, less direct, approach is to start from the linearised equations for perturbations to known solutions, and formulate hydrodynamic or hydromagnetic stability problems. The resulting eigenproblems are again nonlinear, because the unknown eigenvalue multiplies the unknown eigenfunction, but in a different way from the original evolution equations. These eigenproblems may also have singular solutions, but the singularities will be different from the singularities of the evolution equations. Despite the indirectness of this formulation, it is likely that