Complexity measures of tangled vortex filaments

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Abstract We introduce and test measures of geometric and topological complexity to quantify morphological aspects of a tangle of vortex filaments. The tangle is produced by standard numerical simulation of superfluid turbulence in Helium II. Complexity measures such as linking number, writhing number, average crossing number and helicity are computed, and their relation to the energy of the fluid is investigated.

We found a complexity measure –
It really is quite a treasure –
For a vortex entangled,
By methods new-fangled;
I'll explain if you have enough leisure.

1. Introduction

Complex systems of filaments occur frequently in nature. Examples range from vortex structures to magnetic flux tubes to polymers, proteins and DNA. We would like to relate the morphological complexity of such systems with physical properties, such as energy. The aim of this paper is twofold. First we introduce candidate measures of geometric and topological complexity; secondly, we choose superfluid turbulence as a convenient benchmark, compute these measures and compare them to energy.

2. Vortex dynamics and superfluid turbulence

Superfluid turbulence (Barenghi 2001) consists of a disordered, apparently random tangle of vortex filaments. This state of turbulence is particularly simple if compared to traditional hydrodynamics turbulence. Firstly, the superfluid is inviscid. Secondly, all vortex filaments