Advection–diffusion of a passive scalar in the flow of a decaying vortex

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Here's a blob that's subjected to swirl;
It's a problem for somebody virile!
But right at the core
Where it turns more and more,
That's where I get in a whirl.

1. Introduction

The shear due to the presence of a vortex is known to have strong effects on the advection–diffusion of a passive scalar. If there is an initial gradient of the scalar concentration with the spatial scale large compared with the size of the vortex, then this gradient will decay on a time-scale much faster than normal diffusion, making the scalar concentration almost uniform in the vicinity of the vortex (Rhines & Young 1983). The process is governed by the advection–diffusion equation,

$$\frac{\partial \Sigma}{\partial t} + \mathbf{u} \cdot \nabla \Sigma = \kappa \nabla^2 \Sigma, \tag{1}$$

which is a linear equation for the scalar field $\Sigma(\mathbf{x}, t)$.

The application of the linear advection–diffusion equation extends beyond the description of tracers carried by the fluid. In flows with a symmetry the same equation governs other important physical quantities. In the evolution of a weak magnetic field in two-dimensional MHD, for which both the velocity and the field vectors lie in the same plane, the evolution of the field is equivalent to the advection–diffusion of a flux function (Bajer 1998). The annihilation of the gradient of the flux function near a vortex implies the cancellation of the field — the phenomenon known in MHD as 'flux expulsion' (see Bajer, Bassom & Gilbert 2001 and the references therein). Similarly, the streamwise ve-