Extraction and analysis of coherent vortex tubes in turbulent mixing layers using the orthogonal wavelet decomposition

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Many turbulent flows exhibit organised structures, i.e. vortex tubes which are responsible for the intermittency of the flow, evolving in an unorganised random background. A separation of the flow into these two components is a prerequisite for a more sound physical modelling of turbulence.

In [2] we introduced such a method to extract vortices out of two-dimensional turbulent flows, in [3] we generalized this technique for three-dimensional flows. This vortex extraction method is based on an orthogonal wavelet decomposition of the vorticity field, a subsequent thresholding of the wavelet coefficients and a reconstruction from those whose modulus is above the given threshold. Its value is motivated by mathematical theorems yielding optimal min-max estimators for denoising of intermittent signals [1]. It depends on the number of data points and the Reynolds number only. We showed that few strong wavelet coefficients represent the organised part of the flow, i.e. the coherent vortices. The remaining many weak wavelet coefficients represent the background flow which is structureless and may be modelled by a stochastic process.

Here we apply the wavelet method to a three dimensional turbulent mixing layer calculated by high resolution direct numerical simulation [4]. In the following we focus on the extraction of two vortex tubes, i.e. ribs in between two rollers. In Fig. 1 we plot the isosurfaces of the vorticity modulus for the total flow, the coherent part (3% of the wavelet coefficients) and the incoherent part (97% of the wavelet coefficients). We observe that the coherent flow almost perfectly preserves the vortex tube present in the total flow, while the incoherent flow is structureless.

The corresponding longitudinal energy spectra in Fig. 2 show that the coherent part exhibits the same power-law behaviour as the total flow over the whole inertial range. In contrast, the incoherent flow has a flat energy spectrum, i.e. an equipartition of energy, which means in other words that it is decorrelated. The PDFs of vorticity show that the coherent part preserves the same strongly non Gaussian behaviour as the total flow, in particular its extreme values. For the incoherent flow we observe a strongly reduced variance of the vorticity PDF which shows an exponential behaviour.

These results give the motivation to develop a new turbulence model, called CVS (Coherent Vortex Simulation), where the evolution of the coherent part of the flow (vortex tubes) is deterministically computed in an adaptive wavelet basis, while the the influence of the incoherent flow onto the coherent one is statistically modelled [2, 5].

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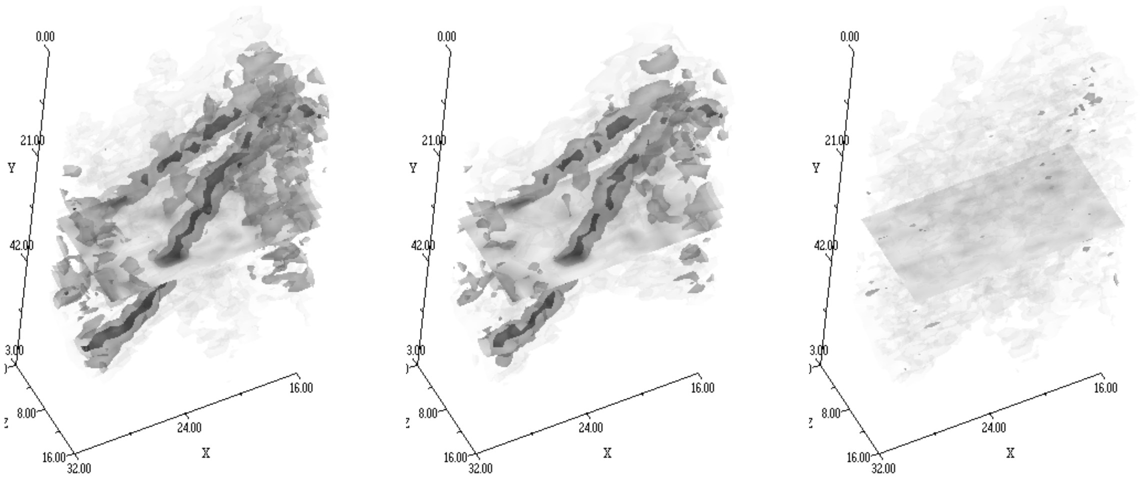
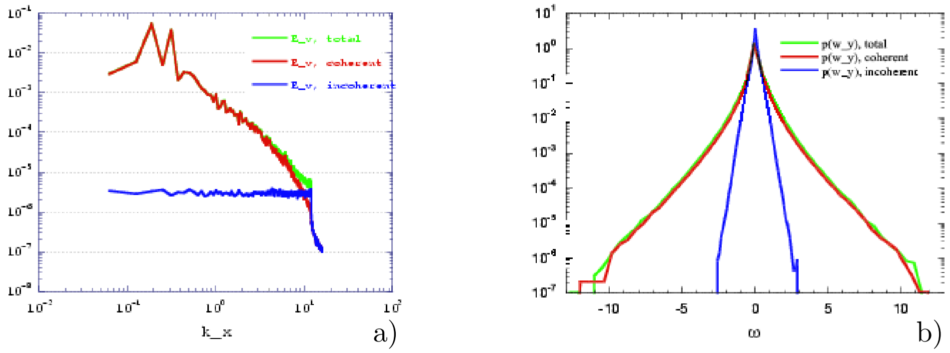


a) b) c)

Figure 1: Vortex tubes in a turbulent mixing layer. Isosurfaces of vorticity modulus for a) total flow, b) coherent part, c) incoherent part using the same grey scale.



a) b)

Figure 2: Corresponding energy spectra in longitudinal direction a) and vorticity PDFs b).

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