Energy, helicity and crossing number relations for complex flows

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Algebraic, geometric and topological measures based on crossing number relations can be used to quantify morphological complexity of ideal tangles of magnetic or vortex filaments [4]. These measures can be related to the energy M and the helicity $\mathcal{H}$ of fluid systems [2], [3]. In the case of perfectly conducting fluids Moffatt (1992) showed that magnetic energy is bounded by helicity according to the inequatlity

$$2M \geq q_0 |\mathcal{H}_M| \ , \tag{1}$$

where q_0 is a positive constant. By using the results of [2] for volume-preserving flows we show that

$$q_0 = \left(\frac{16}{\pi V}\right)^{1/3} \ , \tag{2}$$

where V is the total magnetic volume. Moreover, for a homogeneous tangle of vortex filaments we also show that the average crossing number $\bar{C}$ of the tangle is bounded by

$$|\mathcal{H}| \leq q_1 \bar{C} \leq \sqrt{2K\Omega} \ , \tag{3}$$

where q_1 is a quadratic functional of the circulation and K and Ω denote kinetic energy and enstrophy of the system. These results may find useful applications in the study of relationships between energy and complexity of vortex flows. Work in this direction is in progress [5].

References

[1] Freedman, M.H. (1988) A note on topology and magnetic energy in incompressible perfectly conducting fluids. *J. Fluid Mech.* **194**, 549–551.

[2] Freedman, M.H. & He, Z.-X. (1991) Divergence-free fields: energy and asymptotic crossing number. *Ann. Mathematics* **134**, 189–229.

[3] Moffatt, H.K. (1992) Relaxation under topological constraints. In *Topological Aspects of the Dynamics of Fluids and Plasmas* (ed. H.K. Moffatt *et al.*), pp. 3–28. Kluwer, Dordrecht, The Netherlands.

[4] Ricca, R.L. (2001) Tropicity and complexity measures for vortex tangles. In *Quantized Vortex Dynamics and Superfluid Turbulence* (ed. C.F. Barenghi *et al.*), in press. Lecture Notes in Physics, Springer.

[5] Barenghi, C.F., Ricca, R.L. & Samuels, D.C. (2001) Submitted.