Point collapse in octahedral, vortical flows

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Octahedral flows are the set of flows which are invariant under the finite octahedral group of transformations[1]. The $8C_3$ rotational symmetries about lines through opposite vertices infer the velocity-field permutation $u(x, y, z) = v(z, x, y) = w(y, z, x)$. At zero-planes, across which there are the reflectional symmetries $3\sigma_n$, the normal velocity is odd and the parallel components are even.

A subset of octahedral flows, in which a compact vorticity distribution exists on the zero planes, are considered as candidates for finite-time blowup of the unforced, Euler equations for three-dimensional, incompressible flows. Numerical simulations of these octahedral, vortical flows suggest that an inner solution develops about the origin with the Leray scaling[2], a self-similar, point collapse.

The reasons why octahedral, vortical flows could blowup spontaneously in finite time will be discussed in this talk. Firstly, the origin of octahedral flows is a highly degenerate critical point. A Taylor series expansion of the velocity in x shows only odd terms are nonzero with the linear terms being zero. The reflectional symmetries, $3\sigma_n$, pin the incoming and outgoing manifolds to the axes (lines through opposite vertices), whereas the three-fold rotational symmetries, $8C_3$, restrict all axis flows to be either incoming *or* outgoing. Analysis of the third order terms shows that if the axes are incoming, then the diagonals (lines through the centroids of opposite faces) are outgoing (or vice-versa).

The vortical flow is then constructed to have vortex tubes extending normally from the reflectional symmetry planes (zero-planes). The vorticity in the plane must be positive in the first quadrant and the centroid be above the diagonal. The vortex lines have a large radius of curvature with respect to the distance from the centroid to the origin. Such a flow is then dipolar and makes the axes all incoming manifolds.

The rotational symmetries, $8C_3$, allow an induced strain rate from the image dipoles to have as its maximum direction the axial direction of the fundamental vortex dipole. Thus, restrictions of vortex-line curvature surrounding self-stretching of a vortex tube[3] have been removed. They are replaced, however, by the constraint that the images must continually move closer to the fundamantal in time. The curvature of the vortex lines will become singular, if the tube collapses into the acute-angled corner set up by the symmetries.

Thus, all the ingredients for point collapse are in this picture: vortical flows with octahedral symmetries can be constructed with six dipoles straddling each incoming manifold each immersed in a strain rate field with the most positive eigendirection being parallel to the vorticity. Numerical simulations suggest that this highly constructed flow does keep its configuration and does collapse in a self-similar manner[4], see the figure. New results will be presented which, with hope, further support this scenario.

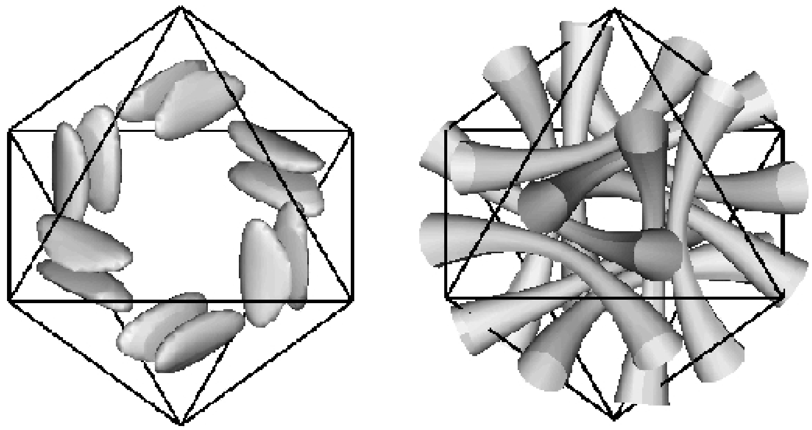

Figure 1: On the left is an isosurface plot of vorticity magnitude from a pseudospectral simulation with initial condition $u = \sin x(\cos 3y \cos z - \cos y \cos 3z)$. The time and magnitude are: $t = 1.5$, $|\omega| = .8|\omega|_\infty$. On the right is the location of vortex filaments at a late time in a simulation. The vortex filament calculation is meant to be a 1-d model of the full field simulation. The surface indicates the size of the core at each point on the polygonal curve. Shown on both is an octahedron which indicates the enforced symmetries.

References

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