Corotating five point vortices in a plane

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We consider the motion of assembly of point vortices in the two-dimensional Euler fluid. This problem can be described by an ordinary differential equation, which is analyzed for long times. We briefly summarize some known results. When two point vortices are in the fluid, the motion of vortices is easily analyzed as is shown in textbooks on fluid mechanics. A qualitative analysis of three point vortices are done by Aref [1]. When the number of vortices is greater than three, the analysis of motion is done for special cases. For example, Morikawa & Swenson [4], and Cabral & Schmidt [3] treat the problem of vortices at the vertices and center of a regular polygon. They show the stability of corotation of the vortices.

In this talk, we treat the case where vortices are at the vertices and center of a *diamond*: Let $b > 0$ and Γ_v be parameters. We consider the time evolution of five point vortices $\{z_j\}_{j=1,\dots,5}$, where $z_j = z_j(t)$ is the position of jth vortices at time t. At the initial time, we impose $z_1(0) = -z_3(0) = 1$, $z_2(0) = -z_4(0) = b$ and $z_5(0) = 0$, which mean that vortices $z_1, \dots, z_4$ are at the vertices and z_5 is at the center of a diamond. The shape of the diamond is described by the parameter b. We assume that $\Gamma_1 = \Gamma_3 = 1$, $\Gamma_2 = \Gamma_4 = \Gamma_v$ and $\Gamma_5 = \Gamma_c$, where Γ_j is the strength of the vortices z_j $(j = 1, \dots, 5)$, and Γ_c is determined so that the five point vortices corotate.

The purpose of the talk is to discuss the stability of the corotation. The dynamics of a passive tracer in a velocity field of the vortices are also considered from numerical point of view.

At first we treat the simple situation where $\Gamma_v = 1$ and $b = 1$. In this case the diamond becomes a square and vortices of equal strength are at the vertices. We can easily find that the five vortices corotates for any Γ_c. Nakaki [5] shows that the corotation is unstable and exhibits a relaxation oscillation for $\Gamma_c < -0.5$. Especially, when $\Gamma_c = -1.5$, the five vortices becomes stationary and we can clearly observe the relaxation oscillation; the vortices do not move for a while, however, they begin to move suddenly and approach to the another stationary configuration. The dynamics of the passive tracer in the field of this vortices shall be shown from numerical point of view. We note that, for the four vortices problems, the dynamics of tracer is already analyzed in [2].

Next let us focus our attention to the case where $0 < b < 1$. Our numerical simulation suggests that the corotation is stable only on a narrow parameter range. We shall show the parameter range U where the corotation is unstable in linearized sense. When the parameters do not belong to U, the corotation corresponds to an elliptic equilibrium. To show the stability of the elliptic equilibrium, a computer assisted proof is applied. We use the interval computations on a workstation, and succeed in showing that the corotation is stable for some parameters. For a stable corotation of vortices, we numerically discuss the dynamics of the passive tracer.

References

[1] Aref, H. (1979). Motion of three vortices. *Phys. Fluids* **22**, 393–400.

[2] Boatto, S. & Pierrehumbert, R. T. (1999). Dynamics of a passive tracer in a velocity field of four identical point vortices. *J. Fluid Mech.* **394**, 137–174.

[3] Cabral, H. E. & Schmidt, D. S. (1999). Stability of relative equilibria in the problem of $N + 1$ vortices. *SIAM J. Math. Anal.* **31** (2), 231–250.

[4] Morikawa, G. K. & Swenson, E. V. (1971). Interacting motion of rectilinear geostrophic vortices. *Phys. Fluids* **14**, 1058–1073.

[5] Nakaki, T. (1999). Behavior of point vortices in a plane and existence of heteroclinic orbits. *Dynam. Contin. Discrete Impuls. Systems* **5**, 159–169.