Higher order topological invariants

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In MHD magnetic topology has become an important tool to study the evolution and stability of plasma flows. A prominent example is the magnetic field on the surface of the sun, which usually shows a complicated topology, i.e. the field lines are knotted, linked, and twisted. As long as a plasma evolves under an ideal Ohm's law magnetic flux and connectivity are frozen into the fluid flow. Therefore any linkage or knottedness of field lines, shortly referred to as magnetic topology, is conserved. In an ideal plasma magnetic topology strongly constraints the evolution and limits a possible release of magnetic energy.

Since magnetic topology is so important we would like to characterize it as precise as possible. In this sense magnetic helicity provides a lowest (second) order integral invariant describing magnetic topology, but it does not detect more complicated higher order links within a field configuration. Magnetic helicity for instance does not distinguish the Borromean rings from unlinked rings. Unfortunately, higher order invariants for continuous magnetic fields have not yet been found, even though there are strong indications, e.g. from studying magnetic configurations restricted to isolated flux tubes, that an infinite series of integral invariants to characterize magnetic topology should exist. In our research we so far focused our attention on third order invariants. New results on third order invariants shall be presented and examples for their application shall be given.