Steady Stokes flow in a trihedral corner

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The flow in a trihedral corner induced by a non-zero velocity distribution over one of the corner's sides is considered in the Stokes approximation. Similar problems were studied by Hills & Moffatt [1] and Shankar [2]. An algorithm of solution developed in the present study is based on the method of superposition. The velocity and pressure fields are presented as sums of three vector and scalar fields, respectively,

$$\mathbf{U} = r^n \sum_{i=1}^{3} \mathbf{u}^{(i)}(\theta^{(i)}, \phi^{(i)}), \qquad P = r^{n-1} \sum_{i=1}^{3} p^{(i)}(\theta^{(i)}, \phi^{(i)}), \tag{1}$$

where $(r, \theta^{(i)}, \phi^{(i)})$, $i = 1, 2, 3$ are three spherical coordinate systems with a common origin at the corner's vertex. These coordinate systems are chosen in such a way that i-th corner's wall occupies the domain $0 \le r < \infty$, $\theta^{(i)} = \pi/2$, $0 \le \phi^{(i)} \le \alpha_i$, $i = 1, 2, 3$ in a corresponding coordinate system. It is not necessary for the corner to be a canonical domain. Hence the developed approach may be applied to a more general class of the trihedral corners. However in what follows we restrict our consideration to the corner of a cubic cavity (figure 1,a). The intermediate mathematical treatments and the final representation of the solution are considerably simplified in this case. The functions $p^{(i)}$ are surface spherical harmonics, whereas $\mathbf{u}^{(i)}$ are expressed via three surface spherical harmonics by the Lamb's general solution. Choosing the spherical harmonics in the form of Fourier series with respect to $\phi^{(i)}$, one can present the velocity as follows

$$\begin{aligned} u_r^{(i)} &= \sum_{m=1}^{\infty} q_m^{(i)}(\theta^{(i)}) \sin(2m\phi^{(i)}), \quad u_{\theta^{(i)}}^{(i)} = \sum_{m=1}^{\infty} s_m^{(i)}(\theta^{(i)}) \sin(2m\phi^{(i)}), \\ u_{\phi^{(i)}}^{(i)} &= C^{(i)} \frac{P_n^{-1}(\cos\theta^{(i)})}{P_n^{-1}(0)} + \sum_{m=1}^{\infty} t_m^{(i)}(\theta^{(i)}) \cos(2m\phi^{(i)}), \end{aligned} \tag{2}$$

where $q_m^{(i)}$, $s_m^{(i)}$, $t_m^{(i)}$ are expressed via Legendre associated functions of the first kind. Satisfaction of the boundary conditions leads to a triple infinite system of linear algebraic equations for the unknown coefficients of the solution.

We show that the local behaviour of the velocity field near the corner's edges, where a discontinuity of the boundary velocity is assumed, coincides with the Goodier-Taylor solution for a two-dimensional wedge. Numerical study of streamline patterns in the flows, induced either by uniform translation of a corner's side in the direction perpendicular to its bisectrix (figure 1,b) or by uniform rotation of a side about the vertex of the corner, confirms existence of the corner eddies near the quiet edge. The first integral of motion is found for these flows that reduces the number of independent coordinates. If the wall rotates about a center displaced from the vertex, the induced flow is essentially three-dimensional. In the antisymmetric velocity field, there appears a stagnation line composed of stagnation points of different types (figure 1,c). Otherwise the three-dimensionality manifests itself in non-closed spiral shape of streamlines (figure 1,d).

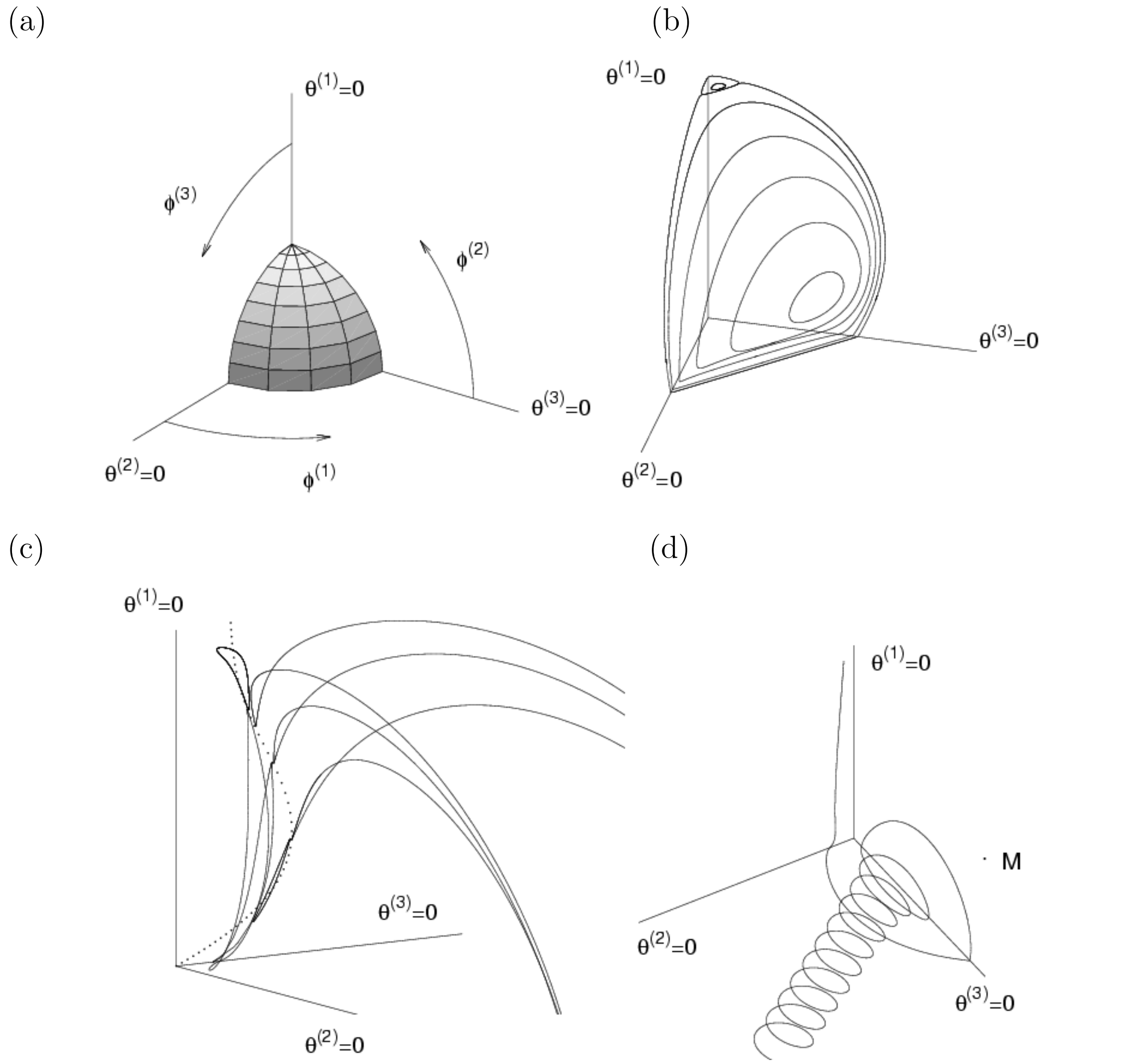

Figure 1: (*a*) Geometry of the problem, coordinate curves of the system (r, $\theta^{(1)}$, $\phi^{(1)}$) are shown on a sphere; (*b*) see in the text; (*c*) the centre of rotation of the bottom wall is at its bisectrix, (*d*) the centre M of rotation of the bottom wall is at the line perpendicular to the bisectrix.

References

[1] Hills, C. P., & Moffatt, H. K. (2000). Rotary honing: a variant of the Taylor paint-scraper problem. *J. Fluid Mech.* **418**, 119–135.

[2] Shankar, P. N. (2000). On Stokes flow in a semi-infinite wedge. *J. Fluid Mech.* **422**, 69–90.