Interaction of Localized Packets of Vorticity with Turbulence

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Abstract

Continuing our recent efforts (Leonard, 2000), we study the evolution of small-scale elements or packets of vorticity as they evolve in an incompressible turbulent flow. Such objects are candidates for important structures in the inertial range and in the dissipation range of scales. In this paper we are primarily concerned with the process by which energy is exchanged between these packets and the larger scales. The spectral properties of this exchange are also investigated. Such information could be of interest, for example, in sub-grid modelling for large-eddy simulations.

We assume that the packet remains small enough so that the velocity gradient tensor is spatially constant over the domain of the packet and that the vorticity field of the packet is weak so that self-induced motions are negligible. The velocity gradient tensor will however, fluctuate in time, typical of that experienced by a material element in a velocity field producing chaotic advection. We use rapid distortion theory to determine the packet evolution.

Let the total velocity field be given by

$$\hat{\boldsymbol{u}} = \boldsymbol{\mathcal{U}} + \boldsymbol{u} \tag{0.1}$$

where $\boldsymbol{\mathcal{U}}$ and $\boldsymbol{\Omega}$ is the velocity field of the large-scale turbulence and $\boldsymbol{u}$ the field of the packet. The total energy of the flow is

$$\begin{aligned} \mathcal{E}_T(t) &= \frac{1}{2}\int |\boldsymbol{\mathcal{U}} + \boldsymbol{u}|^2 d\boldsymbol{x} \\ &= \frac{1}{2}\int |\boldsymbol{\mathcal{U}}|^2 d\boldsymbol{x} + \int \boldsymbol{\mathcal{U}}\cdot\boldsymbol{u}\, d\boldsymbol{x} + \frac{1}{2}\int |\boldsymbol{u}|^2 d\boldsymbol{x} \end{aligned} \tag{0.2}$$

The terms on the RHS above represent the large-scale energy, the cross energy, and the self energy, $\mathcal{E}_s$, of the packet.

In our previous study we found that $\mathcal{E}_s(t) \sim \sigma^3 a^2 \, e^{\lambda_1 t}$ where σ and a are length and velocity scales, respectively, of the original packet and λ_1 is the largest finite-time Lyapunov exponent for the material element occupied by the packet. Thus, the self energy increases at the same rate as a material line element. In wavenumber space this energy increase appears in small scales $k > \frac{1}{\sigma}$ and well as in larger scales $k < \frac{1}{\sigma}$.

In the present paper we show that the growth in $\mathcal{E}_s(t)$ is at the expense of the cross energy. Furthermore, there is generally a much larger exchange in energy between the large-scale term and the cross term due to the presence of the packet. We also discuss the spectral content of this larger energy exchange.

Reference

Leonard, A. 2000. 'Evolution of localized packets of vorticity and scalar in turbulence' In: *Turbulence Structure and Vortex Dynamics*, eds. J. Hunt & J. Vassilicos, Cambridge Press, 127-139