Optimal two-dimensional perturbations in a stretched shear layer

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A vortex sheet during roll-up and the braid regions between adjacent vortices in a shear flow are typical examples of shear layers stretched along the streamwise direction. These situations can be simply described by the following velocity field :

$$\mathbf{U} = \left(\gamma x + U_0(t) \ \mathrm{erf}\left(\frac{y}{a(t)} \right), -\gamma y, 0 \right) \ , \tag{1}$$

where erf is the error function and

$$U_0(t) = \exp(-\gamma t) \ ,$$
$$a(t) = \sqrt{exp(-2\gamma t) + 2(1 - \exp(-2\gamma t))/(\gamma Re)} \ .$$

Here, both initial width and initial velocity difference of the shear flow are normalized. Therefore the basic flow evolution depends only on two parameters: the strain rate γ (assumed uniform and constant) and the Reynolds number Re.

The goal of the paper is to study the 2D stability properties of such a flow. For this purpose, we search the 2D perturbations which maximize the gain of energy after a time t_f. These perturbations are the so-called optimal perturbations of a generalized stability analysis [1]. They naturally depend on the value t_f. For an unstretched non-viscous shear layer ($\gamma = 0$, $Re = \infty$) and large t_f, this analysis is equivalent to a normal mode analysis. For finite time t_f, optimal perturbations are usually associated with transient effects: the gain is larger than one would have obtained with the most unstable mode alone.

For stretched viscous shear layer, the time-dependence of the basic flow forbid any normal mode analysis. Maximizing a gain for the perturbation amplitude is therefore a natural approach to study the linear stability of such a flow. Moreover, this approach has the advantage to capture both the time-variation effects of the basic flow and transient effects associated with the non-self-adjoint character of the evolution operator.

Numerical results are obtained by a pseudo-spectral integration of the perturbation equations using a similar iterative technique as in [2] to get the optimal perturbation. Typical results for the gain are displayed on figure 1. This figure demonstrates two characteristics: large energy gains are obtained and the optimal streamwise wavenumber increases as γ grows. Other numerical results will be presented as well as asymptotic results obtained in the limit of small γ and large Reynolds numbers.

The results will be also applied to the two examples cited above and compared with other available data.

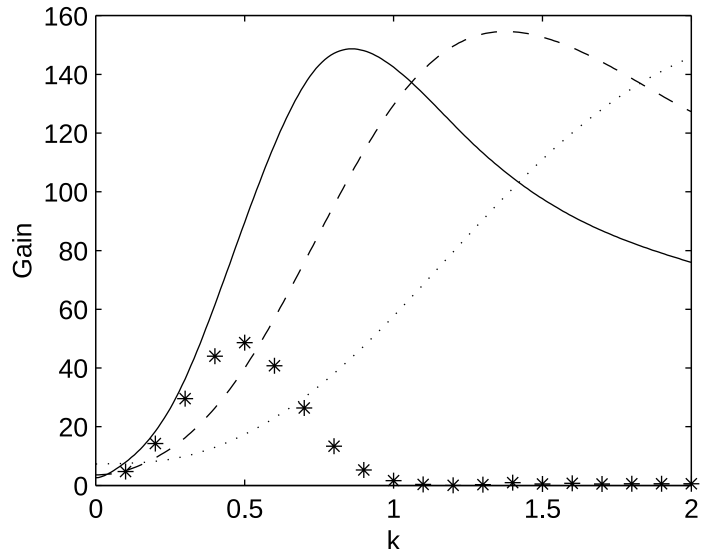

Figure 1: Energy gain versus perturbation streamwise wavenumber for $t_f = 9$, $Re = 10000$ and $\gamma = 0.01$ (solid line), $\gamma = 0.05$ (dashed line) and $\gamma = 0.1$ (dotted line). The stars are $G = \exp(\sigma(k)t_f)$ where $\sigma(k)$ is the growth rate for a non-diffusing unstretched shear layer.

References

[1] Farrell, B. F. & Ioannou, P. J. 1996 Generalized stability theory. Part I: Autonomous Operators. Part II: Nonautonomous operators. *J. Atmos. Sci.*, **53**(14), 2025–2053.

[2] Andersson, P., Berggren, M. & Henningson, D. S. 1999 Optimal disturbances and bypass transition in boundary layers. *Phys. Fluids*, **11**(1), 134–150.