Vortex Sheet Roll-Up: Chaos and Ring Merger

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Vortex sheets are commonly used in fluid dynamics to model thin shear layers in slightly viscous flow. This work reviews two recent applications of the vortex-blob method to compute vortex sheet roll-up.

Figure 1 presents the roll-up of a vortex sheet in planar and axisymmetric flow, leading to a vortex pair and a vortex ring, respectively [1]. At early times the roll-up proceeds smoothly, but at late times the sheet develops irregular small-scale features; a wake is shed behind the vortex ring and gaps form in the spiral core in both cases. These features are due to resonance bands and a heteroclinic tangle in the dynamics [2]. The vortex sheet flow resembles a chaotic Hamiltonian system, although the chaos is induced here by self-sustained oscillations in the vortex core rather than external forcing.

Figure 2 presents a simulation of vortex ring merger computed by a 3-D Lagrangian particle method [3]. The rings are formed by the roll-up of two initially flat circular-disk vortex sheets. The particle velocities are evaluated using a treecode algorithm that reduces the cost from $O(N^2)$ to $O(N \log N)$, where N is the number of particles [4]. The present implementation uses Taylor approximation in Cartesian coordinates, variable order approximation, and nonuniform rectangular cells. Although the material surfaces representing the rolled-up sheets do not reconnect, the associated vorticity isosurfaces do reconnect as the rings approach each other.

References

[1] Krasny, R. & Nitsche, M. (2001). The onset of chaos in vortex sheet flow. submitted for publication.

[2] Rom-Kedar, V., Leonard, A. & Wiggins, S. (1990). An analytical study of transport, mixing and chaos in an unsteady vortical flow. *J. Fluid Mech.* **314**, 347–394.

[3] Lindsay, K. & Krasny, R. (2001). A particle method and adaptive treecode for vortex sheet motion in 3-D flow. submitted for publication.

[4] Barnes, J. & Hut, P. (1986). A hierarchical $O(N \log N)$ force-calculation algorithm. *Nature* **324**, 446–449.

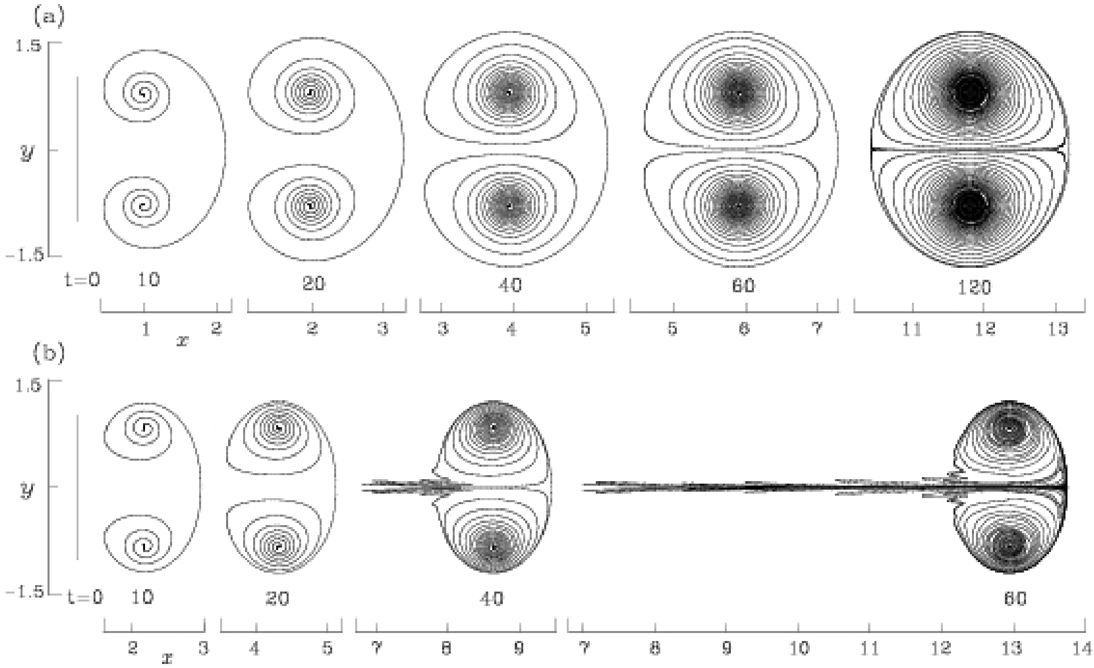

Figure 1: Computed vortex sheet roll-up at the indicated times t. (a) planar vortex pair; (b) axisymmetric vortex ring.

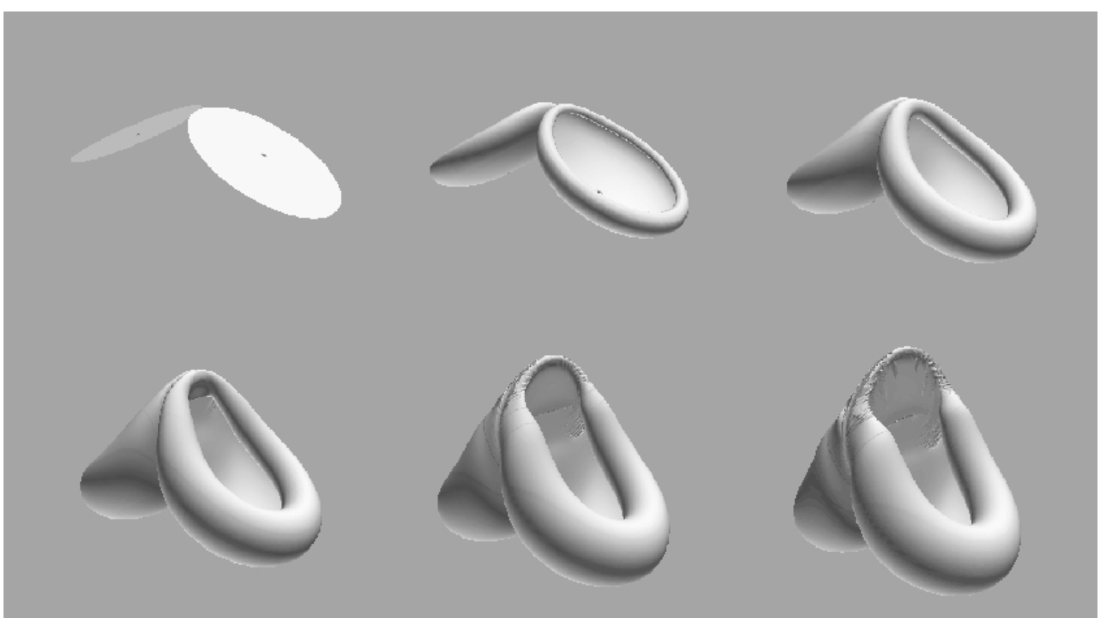

Figure 2: Simulation of vortex ring merger. The rings are formed by the roll-up of two initially flat circular-disk vortex sheets. t = 0, 1, 2, 3, 4, 4.5. This figure visualizes the material surfaces representing the sheets.