Vortex Sheet Roll-Up: Chaos and Ring Merger

Robert Krasny

University of Michigan, Department of Mathematics
krasny@math.lsa.umich.edu

Keith Lindsay

National Center for Atmospheric Research, Climate and Global Dynamics
klindsay@cgd.ucar.edu

Monika Nitsche

University of New Mexico, Department of Mathematics
nitsche@math.unm.edu

Vortex sheets are commonly used in fluid dynamics to model thin shear layers in slightly viscous flow. This work reviews two recent applications of the vortex-blob method to compute vortex sheet roll-up.

Figure 1 presents the roll-up of a vortex sheet in planar and axisymmetric flow, leading to a vortex pair and a vortex ring, respectively [1]. At early times the roll-up proceeds smoothly, but at late times the sheet develops irregular small-scale features; a wake is shed behind the vortex ring and gaps form in the spiral core in both cases. These features are due to resonance bands and a heteroclinic tangle in the dynamics [2]. The vortex sheet flow resembles a chaotic Hamiltonian system, although the chaos is induced here by self-sustained oscillations in the vortex core rather than external forcing.

Figure 2 presents a simulation of vortex ring merger computed by a 3-D Lagrangian particle method [3]. The rings are formed by the roll-up of two initially flat circular-disk vortex sheets. The particle velocities are evaluated using a treecode algorithm that reduces the cost from $O(N^2)$ to $O(N \log N)$, where $N$ is the number of particles [4]. The present implementation uses Taylor approximation in Cartesian coordinates, variable order approximation, and nonuniform rectangular cells. Although the material surfaces representing the rolled-up sheets do not reconnect, the associated vorticity isosurfaces do reconnect as the rings approach each other.

References


Figure 1: Computed vortex sheet roll-up at the indicated times $t$. (a) planar vortex pair; (b) axisymmetric vortex ring.

Figure 2: Simulation of vortex ring merger. The rings are formed by the roll-up of two initially flat circular-disk vortex sheets. $t = 0, 1, 2, 3, 4, 4.5$. This figure visualizes the material surfaces representing the sheets.